Hub Networks: What to take into account

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Summary

Hub location problems (HLPs) determine a large family in Location Analysis, which has been intensively studied in the last decades, motivated both by its theoretical interest and relevant applications. The recent surveys and book chapters dedicated to these models [1–3] are a clear sign of the increasing interest they raise among the community. In HLPs there is a service demand, which is expressed as a set of commodities, i.e., a a set of origin/destination (OD) pairs, each of them associated with a given quantity that must be sent from its origin to its destination. HLPs combine strategic decisions on the design of a network with operational decisions on the routing of flows through the activated network aiming at satisfying the existing demand. The design decisions include the location of the facilities (hubs) as well as the activation of the links that will be used for routing the flows. The operational decisions determine how to distribute the flows through the activated links in such a way that every commodity is routed through at least one hub. The objective usually includes the minimization of the setup costs of the activated hubs and links, plus the routing costs of the flows sent through the network. One characteristic of HLPs is that the flows routed through the activated inter-hub links benefit from economies of scale, meaning that their routing cost have a discount factor.

Applications of HLPs abound in transportation and telecommunications where it is important to efficiently route commodities between multiple origins and destinations. Then, it may be preferable to replace (many) direct connections between OD pairs by (fewer) indirect but privileged links that use the selected hubs as transshipment, consolidation, or sorting nodes.

Fundamental HLPs studied in earlier work, e.g., p-median or p-center HLPs, have evolved to more sophisticated and realistic models that incorporate additional ingredients. For instance, in most HLP applications to distribution or transportation systems, a revenue (prize) is obtained for

serving the demand of a given commodity. Still, classical HLPs ignore such revenues and impose that the demand of every commodity be served. Broadly speaking, this requirement expresses the implicit assumption that the overall prize for serving all the commodities (which is constant) compensates the overall costs of the solution, i.e., the overall net profit is positive. Since such hypothesis does not necessarily hold, HLPs should integrate additional strategic decisions on the commodities that are served and consider a profit-oriented objective that measures the tradeoff between the revenues of the served commodities and the overall network design and transportation costs.

There are further issues that should be taken into account involving HLPs with profits. The first one is that, in fact, commodities may have alternative demand levels, each of them associated with a different revenue per served demand unit. Then, the decision concerning service to a given commodity should not reduce to whether or not the commodity is served, but should include the demand level at which service is provided to the commodity. Moreover, when alternative service levels are considered for the commodities, the demands w_{ij}^l, $l \in L$, associated with the service levels of a given commodity will be affected by the unit prizes of the commodity, q_{ij}^l, $l \in L$. The usual dependence is that w_{ij}^l, is a decreasing function on q_{ij}^l, i.e., demand will decrease as the unit prize increases. Further dependencies may exist between the unit revenues q_{ij}^l and *quality* of service, i.e., travel times W_{ij}^l, for the different service levels $l \in L$. In its turn, travel times are strongly affected by congestion at the hubs, as the time needed for *traversing* an activated hub will depend on the amount of flow circulating through the hub.

We will consider models for HLPs with profits that (progressively) incorporate the above ingredients. We will study some properties, possible variable elimination criteria, and valid inequalities that may be used to reinforce some formulations as well as their associated separation problems.

References

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