A tour of perspective through the ordered median location problems

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Description and content

The Ordered Median location problem, see [18], has been recognized as a powerful tool from a modeling point of view within the field of Location Analysis. Actually, this problem provides a common framework for most of the classical location problems (median, center, k-centrum, centdian, trimmed-mean, among others) as well as for others which have not been studied before.

The goal of the ordered median location problem is to minimize the ordered weighted average of the distances or transportation costs, between the clients/demand points and the server, once we have applied rank dependent compensation factors on them. These rank dependent weights allow, for instance, to compensate unfair situations. Indeed, if a solution places a set of facilities so that the accessibility cost of a demand point at j is in the s-th position in the ordered sequence of cost between each client and its corresponding server and the cost of a demand point at j' is in the t-th position with $s < t$, the model tries to favor j with respect to j' by assigning weights $\lambda_s \leq \lambda_t$. (Note that these weights do not penalize site j' but instead they compensate site j because these lambdas reduce the dispersion of the costs.) In order to incorporate this ordinal information in the overall transportation cost, the objective function applies a correction factor to the transportation cost for each demand point (to reach the facility) which is dependent on the position of that cost relative to similar costs from other demand points. For example, a different penalty might be applied if the transportation cost of a demand point at j was the 5th-most expensive cost rather than the 2nd-most expensive, see [4, 15, 18, 21, 27]. It is even possible to neglect some costs by assigning a zero penalty. This adds a “sorting”-problem to the underlying location problem, making formulation and solution more challenging.

This type of objective function has been extensively studied and successfully applied in a variety of problems within the literature of Location Analysis. [21] and [20] characterize the structure of optimal solutions sets. [27], [2, 3], [11], [19], [7–9] and [28], among others, develop algorithms for different continuous ordered median location problems. In addition, there are nowadays some successful approaches available when the framework space is either discrete (see [4, 6, 10, 15, 16, 24–26]) or a network (see [1, 13, 12, 17, 23, 22]).

This presentation analyzes the ordered median location problem in two different frameworks: continuous and discrete; where some classical but also new results have been collected. For each solution space we study general properties that lead to solution algorithms. In the continuous case, we present a compact mixed integer second order cone formulation able to solve problems of moderate size and a novel set partitioning formulation that allows the application of a branch and price algorithm for solving these problems. Finally, for the discrete case we revisit different formulations based on natural variables and explore their improvements using a set covering based formulation. We also present the two most recent formulations one based on a brach-and-price algorithm [5] and another one exploiting monotony of the lambda parameters [14] that provides the best performance among the available solution methods up to date.

Conclusions

The ordered median function and its corresponding Ordered Median Location Problem are powerful tools from a modeling point of view within the area of Location Analysis. We have included some of most recent developments considering two different framework spaces: continuous and discrete. Our aim has been to include in this talk self contained material to guide potential researcher on this inspiring family of problems. The list of references can also help in pointing to the right sources and giving further details on some topics.

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