Facility location under uncertainty

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Many facility location problems involve strategic decisions that must hold for some considerable time after being implemented. During this time, unpredictable changes may occur in the underlying data. Sources of uncertainty in facility location include demand levels, travel time, cost for supplying the customers, location of the customers, presence or absence of the customers, and price for the commodities, to mention a few [2], [3]. In many cases it is desirable to make decisions that hedge against such uncertainty.

In this lecture an overview is provided on this topic. A major emphasis is put on how uncertainty can be embedded in optimization models. For illustrative purposes, several well-known conventional facility location problems are used. discrete setting is adopted, i.e., a finite set of potential locations for the facilities is assumed to be given. This is motivated by the practical relevance of this setting stemming from many successful applications of location analysis to areas such as logistics, transportation and routing.

A major aspect when dealing with uncertainty regards its representation. First, uncertain parameters may be discrete or continuous. Second, if no probabilistic behavior is known for the underlying parameters—or if it is irrelevant due to the nature of the problem—then a robustness measure can be adopted for assessing the performance of the system. In this case, one can resort to robust optimization models and methods. When probabilistic information is available—and it is of relevance—then uncertainty can be cast as a random vector with a given cumulative distribution function (CDF). If such a case one can make use of stochastic programming models and methods. In some applications, we may face ambiguity in terms of the underlying probability distribution. This motivates casting the problem in the context of adaptive distributionally robust optimiza-

tion [1]. The latter is also a way to unify robust optimization with polyhedral uncertainty and two-stage stochastic programming [4]. In some applications a decision maker is satisfied with solutions ensuring that some constraints hold with a given probability smaller than one while the other constraints remain fully satisfied, i.e., with probability one. In this case one faces a chance-constrained optimization problem.

In optimization under uncertainty, "scenario" is the term used for one possible realization of all the uncertain parameters. This notion is independent of whether or not probabilistic information is available. Nevertheless, if uncertain parameters can be represented by random variables a probability can often be associated with each scenario. Depending on the problem, we may have a finite or an infinite number of scenarios. This impacts the models and techniques that can be used. Note, however, that other features exist that impact the optimization models. One of them regards the identification of the ex ante and ex post decisions, i.e., the decisions to be implemented before uncertainty is revealed and those to be implemented afterwards. Another one regards the so-called attitude of the decision maker towards risk. Two attitudes are usually considered: risk neutral and risk averse.

This lecture starts by discussing uncertainty-related issues. Then, the focus is put on robust optimization models. The lecture proceeds with an overview of two-stage stochastic facility location problems. That discussion culminates with adaptive distributionally robust optimization. May the time allow, chance-constrained programming is also revisited in the context of facility location.

References

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