From computational geometry and statistics to public transport: Results and methods in continuous location

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Continuous location problems

Given a set of demand points in the plane $\mathbb{R}^2$ or in higher dimensional spaces $\mathbb{R}^n$, continuous location problems aim to find points, lines, hyperplanes, circles or spheres in $\mathbb{R}^n$ such that the distances to the demand points are minimized. Distances are often measured by norms. In some applications, also metrics and gauges make sense as distance measures. As objective function, the sum of distances is most common, but also the maximum distance or any ordered median function is possible. Applications of continuous location problems are numerous and range from Operations Research to applications in computational geometry and statistics. Sometimes, applications are rather unexpected, such as in timetabling in public transport or as the building block of an algorithm in robust optimization.

Goal of the lecture

In this lecture, continuous location problems will be defined and analyzed. A classification scheme will be provided which allows to state many continuous location problems in an easy way. We will see that there exist approaches that can be used for many different continuous location problems, i.e., not only for the location of points, but also for the location of dimensional structures such as lines, hyperplanes, circles and spheres. The approaches to be used depend on the underlying distance measure and on the objective function and include the FDS approach in which a finite can-

didate of points is identified and the usage of linear programming which is applied when distances are measured by block norms or polyhedral gauges.

We will start with the classical problem to locate points and then turn to the location of lines and hyperplanes, and to the location of circles ans spheres. We will illustrate some of the problem types using examples from different applications areas.