Izvestiya: Mathematics

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To cite this article: I A Cheltsov 2004 *Izv. Math.* **68** 429

Izvestiya: *Mathematics* **68**:2 429–434

Izvestiya RAN: *Ser. Mat.* **68**:2 215–221 DOI 10.1070/IM2004v068n02ABEH000481

Conic bundles with big discriminant loci

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Abstract. We prove that conic bundles with sufficiently big discriminant locus cannot be birationally transformed into fibrations whose generic fibre has numerically trivial canonical divisor.

Let $\pi\colon X \to S$ be a flat morphism such that the generic fibre of π is $\mathbb{P}^1$ and X, S are smooth. In what follows all varieties are assumed to be projective, normal and defined over $\mathbb{C}$. Suppose that $\operatorname{Pic}(X) \cong \pi^*(\operatorname{Pic}(S)) \oplus \mathbb{Z}$. Then π is called a *standard conic bundle*. Consider the subset Δ_π of S such that the scheme-theoretic fibres of π over the points of Δ_π are not isomorphic to $\mathbb{P}^1$. The subset Δ_π is called the *discriminant locus* of π. One can show that $\Delta_\pi \subset S$ is a reduced divisor with normal crossing singularities. Therefore Δ_π is often called the *discriminant divisor* of π. Moreover, the scheme-theoretic fibres of π over smooth points of Δ_π are isomorphic to the union of two smooth rational curves intersecting each other transversally at one point, and the scheme-theoretic fibres of π over singular points of Δ_π are isomorphic to a smooth rational curve taken with multiplicity two. As shown in [1], every fibration into rational curves is equivalent[1] to a standard conic bundle. In this paper we will prove the following theorem.

Theorem 1. *Let $\pi\colon X \to S$ be a standard conic bundle with discriminant Δ_π such that the divisor $4K_S + \Delta_\pi$ is big. Then X is not birationally isomorphic to a fibration whose general fibre is a smooth variety having numerically trivial canonical divisor.*

Corollary 2. *Let $\pi\colon X \to \mathbb{P}^n$ be a standard conic bundle with discriminant Δ_π such that the degree of $\Delta_\pi \subset \mathbb{P}^n$ is greater than $4(n+1)$. Then X is not birationally isomorphic to an elliptic fibration.*

In certain sense, the hypotheses of Theorem 1 cannot be weakened.

Example 3. Suppose that $X \cong \mathbb{P}^1 \times S$ and $S \cong E_1 \times \cdots \times E_k \times T$, where E_i is a smooth elliptic curve and T is a smooth variety of general type. Then the projection $\pi\colon X \to S$ is a standard conic bundle with empty Δ_π, the divisor $4K_S + \Delta_\pi$ is not big, and the projection $\tau\colon X \to \mathbb{P}^1 \times E_1 \times \cdots \times E_j \times T$ is a fibration whose general fibre is a product of smooth elliptic curves.

[1] Two fibrations $\tau\colon U \to Z$ and $\overline{\tau}\colon \overline{U} \to \overline{Z}$ are equivalent if there are birational maps $\alpha\colon U \dashrightarrow \overline{U}$ and $\beta\colon Z \dashrightarrow \overline{Z}$ such that $\overline{\tau} \circ \alpha = \beta \circ \tau$ and α induces an isomorphism of generic fibres of τ and $\overline{\tau}$.

AMS 2000 *Mathematics Subject Classification.* 14M220, 14E05.

Birational transformations into elliptic fibrations were used in [2], [3] to prove the potential density [2] of rational points on Fano 3-folds. The following result was proved in these papers.

Theorem 4. *The rational points are potentially dense on all smooth Fano 3-folds with the possible exception of the family of double coverings of $\mathbb{P}^3$ ramified in a smooth sextic surface.*

The possible exception appears in Theorem 4 because the double covering of $\mathbb{P}^3$ ramified in a smooth sextic is the only smooth Fano 3-fold which is not birationally isomorphic to an elliptic fibration, as shown in [4]. The following result was proved in [1].

Theorem 5. *Suppose that $\pi\colon X \to S$ is a standard conic bundle with discriminant locus Δ_π, the divisor $4K_S + \Delta_\pi$ is quasi-effective, $\tau\colon Y \to Z$ is a conic bundle and $\psi\colon X \dashrightarrow Y$ is a birational map. Then there is a birational map $\theta\colon S \dashrightarrow Z$ such that $\tau \circ \psi = \theta \circ \pi$.*

Let $\pi\colon X \to S$ be a standard conic bundle with discriminant locus Δ_π such that the divisor $4K_S + \Delta_\pi$ is big. Then $|n(4K_S + \Delta_\pi)|$ gives a generically finite rational map $\varphi_{|n(4K_S+\Delta_\pi)|}$ for $n \gg 0$. Suppose that there is a birational map $\rho\colon X \dashrightarrow Y$ such that Y is smooth and admits a fibration $\tau\colon Y \to Z$ whose generic fibre has numerically trivial canonical divisor. We will use the technique of [1] to derive a contradiction.

We consider the complete linear system $\mathcal{D}_Y = |\pi^*(D)|$, where D is a very ample divisor on Z. Let $\mathcal{D}_X$ be the proper transform of $\mathcal{D}_Y$ on X. The condition $\operatorname{Pic}(X) \cong \pi^*(\operatorname{Pic}(S)) \oplus \mathbb{Z}$ implies that $K_X + \frac{1}{n}\mathcal{D}_X \equiv \pi^*(R_S)$ for some $n \in \mathbb{N}$ and some $\mathbb{Q}$-divisor R_S on S. The following result is well known.

Proposition 6. *We have*

$$R_S \equiv K_S + \frac{1}{4}\Delta_\pi + \frac{1}{4n^2}\,\pi_*(\mathcal{D}_X^2), \qquad -\pi_*(K_X^2) \equiv 4K_S + \Delta_\pi.$$

Proof. Consider an irreducible curve $C \subset S$. We must prove that

$$4R_S \cdot C = (4K_S + \Delta_\pi) \cdot C + \frac{1}{n^2}\,\pi_*(\mathcal{D}_X^2) \cdot C$$

and $-\pi_*(K_X^2) \cdot C = (4K_S + \Delta_\pi) \cdot C$. However, every curve on S is numerically equivalent to a linear combination of two curves that are complete intersections of sufficiently general very ample divisors on S. Hence we may assume that C is a complete intersection $\bigcap_{i=1}^{k-1} H_i$, where k is the dimension of S, and the H_i are sufficiently general very ample divisors on S. In particular, C is smooth and irreducible, C intersects the discriminant Δ_π transversally at smooth points and C properly intersects $\pi_*(\mathcal{D}_X^2)$. The surface $\widehat{C} = \pi^{-1}(C)$ is a smooth complete intersection of smooth divisors $\widehat{H}_i = \pi^{-1}(H_i) \sim \pi^*(H_i)$. Hence we have $\big(K_X + \frac{1}{n}\mathcal{D}_X\big)\big|_{\widehat{C}} \equiv \widehat{\pi}^*(R_S|_C)$ and

$$\frac{1}{n^2}\,\mathcal{D}_X^2 \cdot \widehat{C} = K_X^2 \cdot \widehat{C} - 2K_X \cdot \widehat{\pi}^*(R_S|_C) = K_X^2 \cdot \widehat{C} + 4R_S \cdot C,$$

[2]The rational points of a variety X defined over a number field $\mathbb{F}$ are potentially dense if there is a finite extension $\mathbb{K}/\mathbb{F}$ such that the set of $\mathbb{K}$-rational points is Zariski dense.

where $\widehat{\pi} = \pi|_{\widehat{C}} \colon \widehat{C} \to C$. Moreover,

$$\mathcal{D}_X^2 \cdot \widehat{C} = \pi_*(\mathcal{D}_X^2) \cdot C, \qquad K_X^2 \cdot \widehat{C} = \pi_*(K_X^2) \cdot C.$$

Hence $4R_S \cdot C = -\pi_*(K_X^2) \cdot C + \frac{1}{n^2}\,\pi_*(\mathcal{D}_X^2) \cdot C$. But we have $-K_X^2 \cdot \widehat{C} = (4K_S + \Delta_\pi) \cdot C$ by the adjunction formula and because $K_{\widehat{C}}^2 = 8 - 8g(C)$ on the smooth ruled surface $\widehat{C}$.

Let $f \colon V \to X$ be a birational morphism such that the map $\rho \circ f$ is a morphism and the variety V is smooth. Let $\mathcal{D}_V$ be the proper transform of $\mathcal{D}_X$ on V. Then $\mathcal{D}_V$ is free, and we have the equivalence

$$K_V + \frac{1}{n}\,\mathcal{D}_V \equiv (\pi \circ f)^* \Big(K_S + \frac{1}{4}\,\Delta_\pi + \frac{1}{4n^2}\,\pi_*(\mathcal{D}_X^2) \Big) + \sum_{i=1}^{r} a_i E_i,$$

where E_i is an f-exceptional divisor and $a_i \in \mathbb{Q}$.

Lemma 7. *There is an f-exceptional divisor E_j such that $a_j < 0$.*

Proof. Suppose that all the a_i are non-negative. Let C be the proper transform on V of a sufficiently general curve lying in the fibres of $\tau \colon Y \to Z$. Then we have

$$0 = \Big(K_V + \frac{1}{n}\,\mathcal{D}_V \Big) \cdot C = (\pi \circ f)^* \Big(K_S + \frac{1}{4}\,\Delta_\pi + \frac{1}{4n^2}\,\pi_*(\mathcal{D}_X^2) \Big) \cdot C + \sum_{i=1}^{r} a_i E_i \cdot C$$

and $\sum_{i=1}^{r} a_i E_i \cdot C \geqslant 0$. On the other hand, we have

$$\Big(K_S + \frac{1}{4}\,\Delta_\pi + \frac{1}{4n^2}\,\pi_*(\mathcal{D}_X^2) \Big) \cdot \pi \circ f(C) = \Big(K_S + \frac{1}{4}\,\Delta_\pi \Big) \cdot \pi \circ f(C) > 0$$

since the divisor $K_S + \frac{1}{4}\Delta_\pi$ is big.

Therefore the log-pair $\big(X, \frac{1}{n}\mathcal{D}_X\big)$ is not canonical. Hence it is natural to try to find a standard conic bundle equivalent to the conic bundle π such that the singularities of the corresponding birationally equivalent log-pair are canonical. In the classical terminology, we must *untwist the maximal singularities* of the pair $\big(X, \frac{1}{n}\mathcal{D}_X\big)$. If such an *untwisted* conic bundle exists, then it is easy to see that the proofs of Lemmas 7 and 16 complete the proof of Theorem 1. Actually, in dimension three this conic bundle almost exists,[3] but its existence is not easy to prove (see [5]). However, the *untwisted* conic bundle may apriori not exist in higher dimensions. The main idea of [1] is to *untwist* at least those singularities of the log-pair $\big(X, \frac{1}{n}\mathcal{D}_X\big)$ that correspond to the divisors E_j with $a_j < 0$. An important remark, which was also made in [1], is that non-canonical singularities of the resulting *untwisted* log-pair *are not seen* by the fibration τ. This means that the pullback of a sufficiently general curve in the fibres of τ on an appropriate birational model of X has trivial intersection with the corresponding exceptional divisors, and the proof of Lemma 7 still works. The following result is Proposition 3.5 of [1].

[3]It may be singular and hence non-standard in our definition.

Proposition 8. *One can find a birational morphism $\sigma\colon \overline{S} \to S$, a birational map $\lambda\colon \overline{X} \to X$ and a standard conic bundle $\overline{\pi}\colon \overline{X} \to \overline{S}$ with discriminant $\Delta_{\overline{\pi}}$ such that $\sigma\circ\overline{\pi} = \pi\circ\lambda$, the morphism σ is a composite of blow-ups of smooth subvarieties and the centre on $\overline{X}$ of the discrete valuation related to the divisor E_i is a subvariety $C_i \subset \overline{X}$ such that $\overline{\pi}(C_i) \subset \overline{S}$ is a smooth divisor and either $\overline{\pi}(C_i)$ intersects $\Delta_{\overline{\pi}}$ transversally or $\overline{\pi}(C_i) \subset \Delta_{\overline{\pi}}$.*

Let $g\colon W \to \overline{X}$ be a birational morphism such that $\gamma = f^{-1}\circ\lambda\circ g$ is regular and W is smooth. Let $\mathcal{D}_W$ and $\mathcal{D}_{\overline{X}}$ be the proper transforms of $\mathcal{D}_X$ on W and $\overline{X}$ respectively. Then

$$K_W + \frac{1}{n}\,\mathcal{D}_W \equiv g^*\left(K_{\overline{X}} + \frac{1}{n}\,\mathcal{D}_{\overline{X}}\right) + \sum_{i=1}^{r} b_i\overline{E}_i + \sum_{i=1}^{l} c_iG_i,$$

where b_i and c_i are rational numbers, the divisors G_i are g-exceptional and $\gamma(\overline{E}_i) = E_i$.

Lemma 9. *Suppose that $b_j < 0$. Then $g(\overline{E}_j) = C_j \subset \overline{X}$ is a subvariety of codimension two, the morphism $\overline{\pi}|_{C_j}\colon C_j \to \overline{\pi}(C_j)$ is birational, the subvariety $\overline{\pi}(C_j) \subset \overline{S}$ is a smooth divisor that intersects $\Delta_{\overline{\pi}}$ transversally and the induced double covering of $\overline{\pi}(C_j)\cap\Delta_{\overline{\pi}}$ splits.*

Proof. The inequality $b_j < 0$ implies that $\operatorname{mult}_{C_j}(\mathcal{D}_{\overline{X}}) > n$. Let L_P be a reduced irreducible component of the fibre of $\overline{\pi}$ over a point $P \in C_j$ such that $L_P\cap\overline{E}_j \neq \varnothing$. We have $\mathcal{D}_{\overline{X}}\cdot L_P = n$ if $P \in \Delta_{\overline{\pi}}$, and $\mathcal{D}_{\overline{X}}\cdot L_P = 2n$ otherwise. Thus $\overline{\pi}|_{C_j}$ is birational and $\overline{\pi}(C_j) \not\subset \Delta_{\overline{\pi}}$.

The following result is Lemma 2.2 of [1].

Lemma 10. *Let $\chi\colon \check{S} \to \overline{S}$ be a blow-up of a smooth subvariety $T \subset \Delta_{\overline{\pi}}$ such that the induced double covering of T splits. Then there is a standard conic bundle $\check{\pi}\colon \check{X} \to \check{S}$ such that $\check{\pi}$ is equivalent to $\overline{\pi}$, the discriminant $\Delta_{\check{S}}$ of $\check{\pi}$ is the proper transform of $\Delta_{\overline{S}}$ on $\check{S}$ and $\check{\pi} = \overline{\pi}$ over $\overline{S}\setminus T$.*

Corollary 11. *We may assume that $\overline{\pi}(C_j)\cap\Delta_{\overline{\pi}} = \varnothing$ whenever $b_j < 0$.*

If $b_j < 0$, then the morphism $\overline{\pi}|_{C_j}$ is birational and C_j is a quasi-section of the restriction of $\overline{\pi}$ to $\overline{\pi}(C_j)$, but aprioiri $\overline{\pi}|_{C_j}$ need not be biregular and C_j need not be a section of the restriction of $\overline{\pi}$.

Remark 12. In the case when $\chi\colon \check{S} \to \overline{S}$ is a blow-up of a smooth subvariety $T \subset \overline{S}$ such that $T\cap\Delta_{\overline{\pi}} = \varnothing$, we see that the product $\check{X} = \overline{X}\times_{\overline{S}}\check{S}$ with the natural projection $\check{\pi}\colon \check{X} \to \check{S}$ is a standard conic bundle equivalent to $\overline{\pi}$ whose discriminant $\Delta_{\check{S}}$ is the proper transform of $\Delta_{\overline{S}}$ on $\check{S}$.

If $b_j < 0$, then the universal properties of blow-ups show that the quasi-section C_j of the restriction of $\overline{\pi}$ to $\overline{\pi}(C_j)$ can be regularized by blowing up smooth subvarieties as in the proof of Proposition 3.7 in [1]. Thus, if $b_j < 0$, we may assume that the morphism $\overline{\pi}|_{C_j}\colon C_j \to \overline{\pi}(C_j)$ is biregular, the subvariety C_j is smooth, $\overline{\pi}(C_j)\cap\Delta_{\overline{\pi}} = \varnothing$ and $\overline{\pi}(C_j)\cap\overline{\pi}(C_l) = \varnothing$ whenever $b_l < 0$.

Lemma 13. *Suppose that* $b_j < 0$ *and let a birational map* $\alpha\colon \overline{X} \dashrightarrow \widetilde{X}$ *be the composite of a blow-up of the smooth subvariety* C_j *and contraction of the proper transform of* $\overline{\pi}^{-1}(\overline{\pi}(C_j))$ *to a smooth subvariety* $\widetilde{C}_j \subset \widetilde{X}$ *such that there is a standard conic bundle* $\widetilde{\pi}\colon \widetilde{X} \to \overline{S}$ *with discriminant locus* $\Delta_{\overline{\pi}}$ *and* $\widetilde{\pi} \circ \alpha = \overline{\pi}$. *Then* $\operatorname{mult}_{\widetilde{C}_j}(\mathcal{D}_{\widetilde{X}}) < n$, *where* $\mathcal{D}_{\widetilde{X}} = \alpha(\mathcal{D}_{\overline{X}})$.

Proof. By construction, $\operatorname{mult}_{\widetilde{C}_j}(\mathcal{D}_{\widetilde{X}}) = 2n - \operatorname{mult}_{C_j}(\mathcal{D}_{\overline{X}}) < n$.

In particular, the log-pair $\big(\widetilde{X}, \frac{1}{n}\mathcal{D}_{\widetilde{X}}\big)$ in Lemma 13 is canonical at a generic point of $\widetilde{C}_j$.

Lemma 14. *In the notation of Lemma* 13, *let* $T \subset \widetilde{X}$ *be a subvariety of codimension two such that* $\widetilde{\pi}(T) = \overline{\pi}(C_j)$ *and* $\operatorname{mult}_T(\mathcal{D}_{\widetilde{X}}) > n$. *Then* $\operatorname{mult}_T(\mathcal{D}^2_{\widetilde{X}}) < \operatorname{mult}_{C_j}(\mathcal{D}^2_{\overline{X}})$.

Proof. This is an elementary property of the multiplicity of intersection.

Therefore we may iterate the birational transformation of Lemma 13 finitely many times (combining it with regularization of the corresponding quasi-section if necessary). This enables us to assume that the log-pair $\big(\overline{X}, \frac{1}{n}\mathcal{D}_{\overline{X}}\big)$ is canonical at generic points of subvarieties of codimension two in $\overline{X}$ that dominate the divisors $\overline{\pi}(C_j) \subset \overline{S}$ with $b_j < 0$. We note that this construction is two-dimensional in nature and goes back to [6].

Corollary 15. *The rational numbers* b_i *are non-negative.*

Let $R_{\overline{S}}$ be a $\mathbb{Q}$-divisor on $\overline{S}$ such that $K_{\overline{X}} + \frac{1}{n}\,\mathcal{D}_{\overline{X}} \equiv \overline{\pi}^*(R_{\overline{S}})$. Then we get

$$R_{\overline{S}} \equiv K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{\pi}} + \frac{1}{4n^2}\,\overline{\pi}_*(\mathcal{D}^2_{\overline{X}})$$

as in the proof of Proposition 6.

Lemma 16. *The divisor* $K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{S}}$ *is big.*

Proof. On the variety $\overline{S}$ we have

$$K_{\overline{S}} + \frac{1}{4}\overline{\Delta}_S \sim_{\mathbb{Q}} \sigma^*\left(K_S + \frac{1}{4}\Delta_S\right) + \Sigma,$$

where $\overline{\Delta}_S$ is the proper transform of Δ_S on $\overline{S}$ and Σ is a σ-exceptional $\mathbb{Q}$-divisor. Moreover, Σ is effective since Δ_S has only normal crossing singularities. Thus $K_{\overline{S}} + \frac{1}{4}\overline{\Delta}_S$ is big. On the other hand, the divisors $\overline{\Delta}_S$ and $\Delta_{\overline{S}}$ are reduced, effective and coincide outside the exceptional locus of σ. Hence the divisor $K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{S}}$ is big as well.

Let C be the proper transform on W of a sufficiently general curve in the fibres of $\tau\colon Y \to Z$. Then

$$0 = \left(K_W + \frac{1}{n}\,\mathcal{D}_W\right) \cdot C = (\overline{\pi} \circ g)^*\left(K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{\pi}} + \frac{1}{4n^2}\,\overline{\pi}_*(\mathcal{D}^2_{\overline{X}})\right) \cdot C + \sum_{i=1}^{r} b_i \overline{E}_i \cdot C + \sum_{i=1}^{l} c_i G_i \cdot C$$

and $\sum_{i=1}^{r} b_i E_i \cdot C \geqslant 0$. Moreover, we have $G_i \cdot C = 0$ because G_i is γ-exceptional, but

$$\left(K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{\pi}} + \frac{1}{4n^2}\,\overline{\pi}_*(\mathcal{D}^2_{\overline{X}})\right) \cdot \overline{\pi} \circ g(C) = \left(K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{\pi}}\right) \cdot \overline{\pi} \circ g(C) > 0$$

because the divisor $K_{\overline{S}} + \frac{1}{4}\Delta_{\overline{\pi}}$ is big. Therefore we have proved Theorem 1.

Remark 17. In the proof of Theorem 1, we may replace τ by a conic bundle and assume that the divisor $4K_S + \Delta_\pi$ is merely quasi-effective. This yields a proof of Theorem 5.

The following example is taken from [1].

Example 18. Let $T \subset \mathbb{P}^2$ be a reduced irreducible curve of degree greater than 12 with only simple double points such that the normalization $\check{T}$ of T has genus 1, $\omega\colon \widehat{T} \to \check{T}$ is an etale double covering and $\chi\colon S \to \mathbb{P}^2$ is the blow-up of double points of T. By Theorem 5.2 of [1], there is a standard conic bundle $\pi\colon X \to S$ with discriminant locus $\Delta_\pi \cong \check{T}$ such that the corresponding etale double covering is ω. The divisor $4K_S+\Delta_\pi$ is big, the group $H^3(X,\mathbb{Z})$ has no torsion and $h^3(X,\mathbb{C}) = h^1(\widehat{T}, \mathcal{P}_{\widehat{T}}) - h^1(T, \mathcal{P}_T) = 0$ (see [7]–[9]). Hence the group $H^3(X,\mathbb{Z})$ is trivial.

The author thanks V. A. Iskovskikh, A. V. Pukhlikov, V. G. Sarkisov and V. V. Shokurov for helpful conversations.

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Received 1/SEP/03
Translated by THE AUTHOR

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX