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# Singularities of 3-Dimensional Varieties Admitting an Ample Effective Divisor of Kodaira Dimension Zero

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ABSTRACT. For a normal threefold $X$ with an effective Cartier divisor $H$, which is a minimal model of Kodaira dimension zero, we prove that either $X$ is a generalized cone over $H$, or $X$ has quadruple singularities and $H$ is either a K3 surface, or an Enriques surface.

## Introduction

In [1–4] Fano investigated threefolds in projective spaces with hyperplane sections that are K3 minimal surfaces and Enriques surfaces. His study was continued in [5–11].

Note that if $Y$ is a smooth minimal algebraic surface, then the following conditions are equivalent:

(1) $\varkappa(Y)=0$;
(2) $K_Y \equiv 0$;
(3) $12K_Y \sim 0$;
(4) $Y$ is one of the following types:
  a) an Abelian variety, $h^1(\mathcal{O}_Y)=2$, $h^2(\mathcal{O}_Y)=1$, $K_Y \sim 0$;
  b) a K3 surface, $h^1(\mathcal{O}_Y)=0$, $h^2(\mathcal{O}_Y)=1$, $K_Y \sim 0$;
  c) an Enriques surface, $h^1(\mathcal{O}_Y)=0$, $h^2(\mathcal{O}_Y)=0$, $2K_Y \sim 0$;
  d) a bielliptic surface, $h^1(\mathcal{O}_Y)=1$, $h^2(\mathcal{O}_Y)=0$, $12K_Y \sim 0$.

See, e.g., [12].

Therefore, it is natural to study the more general problem of investigating properties of threefolds admitting an ample divisor, which is a minimal smooth surface of Kodaira dimension zero. It happens that with the exception of the case when the ample divisor is a K3 surface, such a variety always has singularities.

If singularities are allowed, then a cone over the corresponding surface is the simplest example of a variety in question. Moreover, we call a variety $X$ containing an ample effective irreducible reduced Cartier divisor $H$ a ***generalized cone over*** $H$ if $X$ is the result of contraction of the exceptional section in $\mathbb{P}(\mathcal{O}_H \oplus \mathcal{O}_H(H|_H))$. By definition, a generalized cone over any variety contains this variety as an ample effective irreducible reduced Cartier divisor.

Our main result is Theorem 5.1 (see §5).

Taking into account the results of [13] and [14], the classification of varieties with an ample effective Cartier divisor that is either a K3 surface or an Enriques surface, is equivalent, in practice, to the classification of Fano varieties with canonical isolated singularities of integer Fano index.

All the varieties are over $\mathbb{C}$. The basic definitions, notation, and notions are described in [15].

## §1. Isolated singularities

**1.1. Lemma.** *Let $H$ be an ample effective Cartier divisor on a normal variety $X$. If $H$ is smooth, then* $\mathrm{Sing}(X) \cap H = \varnothing$ *and the singularities of $X$ are isolated.*

**Proof.** Suppose there is a curve in $\mathrm{Sing}(X)$. Since $H$ is ample, we have $\mathrm{Sing}(X) \cap H \neq \varnothing$. But $H$ is a Cartier divisor. Hence, $\mathrm{Sing}(H) \neq \varnothing$.

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**1.2. Corollary.** *Suppose $H$ is an ample effective Cartier divisor on a normal threefold $X$, and $H$ is a smooth minimal surface with $\varkappa(H) = 0$. If $\mathrm{Sing}(X) = \varnothing$, then $X$ is a Fano threefold and $H$ is a K3 surface.*

**Proof.** If $\mathrm{Sing}(X) = \varnothing$, then, by the accessory formula, we have $12(K_X + H)|_H \sim 0$, and, by the Lefschetz theorem on hyperplane sections, we obtain $(K_X + H) \equiv 0$. The Kleiman test implies that $-K_X$ is ample, i.e., $X$ is a Fano variety, therefore $\mathrm{Pic}^0(X) = 0$ and $\mathrm{Pic}(X)$ has no torsion (see, e.g., [7]). Hence, $K_X + H \sim 0$ and $H$ is a K3 surface (see, e.g., [7]).

## §2. $\mathbb{Q}$-Gorenstein property

**2.1. Definition.** Let $D = \sum n_E E$ be the Weil divisor on a normal variety $X$. The *sheaf* $\mathcal{O}_X(D)$ is defined as follows:

$$\Gamma(U, \mathcal{O}_X(D)) = \{f \in \mathrm{Rat}(X);\ v_E(f) + n_E \geq 0\ \forall E \in U,\ \mathrm{codim}\, E = 1\}.$$

Here $v_E(f)$ denotes the order of $F$ in $E$.

**2.2. Lemma.** *Let $D$ be the Weil divisor on a normal variety $X$. Then $\mathcal{O}_X(D)$ is a reflexive sheaf of rank one.*

**Proof.** See [16, Proposition 1.6].

**2.3. Lemma.** *Let $\mathcal{G}$ be a reflexive sheaf on the variety $X$ Then there exists an exact sequence*

$$0 \to \mathcal{G} \to \mathcal{E} \to \mathcal{F} \to 0,$$

*where $\mathcal{E}$ is locally free, and $\mathcal{F}$ is a subsheaf of a locally free sheaf.*

**Proof.** Consider an arbitrary locally free resolvent for $\mathcal{G}^*$: $\mathcal{J} \to \mathcal{I} \to \mathcal{G}^* \to 0$. The dual resolvent will be $0 \to \mathcal{G} \to \mathcal{I}^* \to \mathcal{F}^*$. This sequence is exact in the first two terms. The quotient of $\mathcal{I}^*$ over $\mathcal{G}$ is embedded in $\mathcal{F}^*$.

**2.4. Lemma.** *Suppose $X$ is a normal variety, $H$ is an ample divisor on $X$, $\mathcal{G}$ is a reflexive sheaf on $X$. Then $H^1(\mathcal{G} \otimes \mathcal{O}_X(-nH)) = 0$ for $n \gg 0$.*

**Proof.** Consider the part of the cohomology exact sequence associated with the exact sequence from Lemma 2.3:

$$H^0(\mathcal{F} \otimes \mathcal{O}_X(-nH)) \to H^1(\mathcal{G} \otimes \mathcal{O}_X(-nH)) \to H^1(\mathcal{E} \otimes \mathcal{O}_X(-nH)).$$

The required equality follows from $H^0(\mathcal{F} \otimes \mathcal{O}_X(-nH)) = 0$ for $n \gg 0$, since $\mathcal{F}$ is a subsheaf of a locally free sheaf, and $H^1(\mathcal{E} \otimes \mathcal{O}_X(-nH)) = 0$ for $n \gg 0$, since $X$ is normal.

**2.5. Lemma.** *Suppose $X$ is a normal variety with isolated singularities, $D$ is a Weil divisor on $X$, $H$ is an ample Cartier divisor on $X$. If $D|_Y \sim 0$ for $Y \in |nH|$ smooth and $n \gg 0$, then $D \sim 0$.*

**Proof.** Consider the sequence:

$$0 \to \mathcal{O}_X(D) \otimes \mathcal{O}_X(-nH) \to \mathcal{O}_X(D) \to \mathcal{O}_Y \to 0.$$

This sequence is exact, since all these sheaves are free in a neighborhood of $Y$, and the sequence is trivial outside $Y$.

We have $H^0(\mathcal{O}_X(D) \otimes \mathcal{O}_X(-nH)) = 0$ for $n \gg 0$, since $\mathcal{O}_X(D)$ is a subsheaf of a locally free sheaf. By Lemma 2.4, $H^1(\mathcal{O}_X(D) \otimes \mathcal{O}_X(-nH)) = 0$. Further, $H^0(\mathcal{O}_Y) = \mathbb{C}$, since $Y$ is connected. Therefore $H^0(\mathcal{O}_X(D)) = \mathbb{C}$, i.e., the linear system of Weil divisors $|D|$ contains an effective divisor, hence $D \sim 0$.

**2.6. Lemma.** *Suppose $D$ is a Weil divisor, and $H$ is an ample smooth Cartier divisor on a normal variety $X$. If $D|_H \sim 0$, then $D|_Y \sim 0$ for generic $Y \in |nH|$ and $n \gg 0$.*

**Proof.** Introduce the following notation: $C = Y \cap H = nH|_H = H|_Y$ is an irreducible smooth curve; $f\colon X_{\mathrm{main}} \to X$ is a resolution of singularities of $X$ such that $f$ is an isomorphism outside $\mathrm{Sing}(X)$; $\hat{H} = f^{-1}(H) = f^*(H)$; $\hat{Y} = f^{-1}(Y) = f^*(Y)$. Then $D|_Y H|_Y = D|_H nH|_H = DC = 0$, and since the divisor $H$ is ample on $Y$, by the Hodge index theorem, either $D|_Y D|_Y < 0$, or $D|_Y \equiv 0$. Note that, generally speaking, $\mathrm{Sing}(X) \cap Y = \varnothing$ and therefore,

$$D|_Y D|_Y = f^{-1}(D)|_{\hat{Y}} f^{-1}(D)|_{\hat{Y}} = nf^{-1}(D)|_{\hat{H}} f^{-1}(D)|_{\hat{H}} = nD|_H D|_H = 0.$$

Summing, we obtain $D|_Y \equiv 0$, $D|_C \sim 0$.

Consider the exact sequence of sheaves and the first terms of the associated cohomology sequence:

$$0 \to \mathcal{O}_Y(D|_Y - H|_Y) \to \mathcal{O}_Y(D|_Y) \to \mathcal{O}_C \to 0,$$
$$0 \to H^0(\mathcal{O}_Y(D|_Y)) \to H^0(\mathcal{O}_C) \to H^1(\mathcal{O}_Y(D|_Y - H|_Y)).$$

By the Kleiman test, the divisor $(H - D)|_Y$ is ample on $Y$ and therefore $h^1(\mathcal{O}_Y((D - H)|_Y)) = 0$ by the cohomology vanishing theorem (see, e.g., [15, Theorem 1-2-5]). The curve $C$ is connected, and the equality $h^0(\mathcal{O}_C) = 1$ implies $h^0(\mathcal{O}_Y(D|_Y)) = 1$, whence $D|_Y \sim 0$.

**2.7. Theorem.** *Suppose $H$ is an ample effective Cartier divisor on a normal threefold $X$, and $H$ is a smooth minimal surface with $\varkappa(H) = 0$. Then $X$ is $\mathbb{Q}$-Gorenstein and $-K_X \sim_{\mathbb{Q}} H$.*

**Proof.** Since $X$ is normal, $K_X$ is well defined as a Weil divisor. Since $\mathrm{Sing}(X) \cap H = \varnothing$, the following accessory formula for the divisor $H$ is valid:

$$12(K_X + H)|_H \sim 0.$$

Lemmas 2.6 and 2.5 yield $12(K_X + H) \sim 0$.

**2.8. Remark.** Under the assumptions of Theorem 2.7, we have the following possibilities:

(1) $H$ is an Abelian variety, $X$ is Gorenstein;
(2) $H$ is a K3 surface, $X$ is Gorenstein;
(3) $H$ is an Enriques surface, $X$ is **2**-Gorenstein;
(4) $H$ is a bielliptic surface, $X$ is **12**-Gorenstein.

## §3. Terminal modification

**3.1. Lemma.** *Let $X$ be a normal threefold. There exists a birational morphism $f\colon X_{\mathrm{term}} \to X$ such that $X_{\mathrm{term}}$ has terminal $\mathbb{Q}$-factorial singularities and $K_{X_{\mathrm{term}}}$ is $f$-numerically effective.*

**Proof.** See, for example, [17].

**3.2. Lemma.** *Under the assumptions of Lemma 3.1, if $X$ is $\mathbb{Q}$-Gorenstein, then we always have $K_{X_{\mathrm{term}}} \sim_{\mathbb{Q}} f^*(K_X) - B$, where $B$ is an effective $\mathbb{Q}$-divisor.*

**Proof.** See, for example, [18, Proposition 2.18].

**3.3. Remark.** Under the assumptions of Lemma 3.2, the following possibilities can arise:

(1) $f$ is an isomorphism, $X$ has terminal $\mathbb{Q}$-factorial singularities;
(2) $f$ is an isomorphism in codimension one, and $K_{X_{\mathrm{term}}} \sim_{\mathbb{Q}} f^*(K_X)$, $X$ has terminal singularities;
(3) $K_{X_{\mathrm{term}}} \sim_{\mathbb{Q}} f^*(K_X)$, $X$ has canonical singularities;
(4) $B$ is an effective nonzero $\mathbb{Q}$-divisor.

**3.4. Corollary.** *If the singularities of $X$ are isolated, then the morphism $f$ contracts any of its exceptional divisors to a point.*

## §4. Extremal rays on $X_{\text{term}}$

Let $H$ be an ample Cartier divisor on a normal threefold $X$, which is a smooth minimal surface with $\varkappa(H) = 0$. Consider the terminal modification $f\colon X_{\text{term}} \to X$. By Lemma 1.1, $\operatorname{Sing}(X) \cap H = \varnothing$ and we can denote $\hat{H} = f^{-1}(H) = f^*(H)$. By Theorem 2.7, $X$ is $\mathbb{Q}$-Gorenstein and $-K_X \sim_{\mathbb{Q}} H$. Then Lemma 3.2 implies

$$K_{X_{\text{term}}} \sim_{\mathbb{Q}} -\hat{H} - B.$$

Suppose the singularities of $X$ are not canonical. Then, by Remark 3.3, $B$ is an effective nonzero $\mathbb{Q}$-divisor, and the $\mathbb{Q}$-divisor $K_{X_{\text{term}}} + \hat{H} \sim_{\mathbb{Q}} -B$ is not numerically effective. Therefore there exists a 1-face $R \in \mathrm{NE}(X)$ such that $-BR < 0$. But $-BR = K_{X_{\text{term}}}R + \hat{H}R$, and the divisor $\hat{H}$ is numerically effective. Hence, $K_{X_{\text{term}}}R = \hat{H}R - BR < 0$, i.e., $R$ is an extremal ray in the sense of Mori.

By [15, Theorem 3-2-1], there exists a morphism $g\colon X_{\text{term}} \to Y$ onto a normal variety $Y$ such that $-K_{X_{\text{term}}}$ is $g$-ample and for any curve $C \in X_{\text{term}}$ $g(C)$ is a point iff $C \in R$.

**4.1. Lemma.** *For any curve $C \in R$, we have $K_{X_{\text{term}}}C < -1$.*

**Proof.** $\hat{H}C > 0$ for any curve $C \in R$, since otherwise the equality $\hat{H}C = 0$ would imply that $f(C)$ is a point and $K_{X_{\text{term}}}R \geq 0$, because $K_{X_{\text{term}}}$ is $f$-numerically effective, but $K_{X_{\text{term}}}R < 0$. Taking into account the fact that $\hat{H}$ is a Cartier divisor, we obtain $\hat{H}C \geq 1$ and $K_{X_{\text{term}}}C = -\hat{H}C - BC < -1$.

**4.2. Remark.** The following possibilities can arise:

(1) the morphism $g$ contracts the curve to a point;
(2) the morphism $g$ contracts the divisor to a curve;
(3) the morphism $g$ contracts the divisor to a point;
(4) the dimension of $Y$ equals two;
(5) $Y$ is 1-dimensional.

**4.3. Lemma.** *Let $g\colon X \to Y$ be a birational contraction of the extremal ray on a threefold with $\mathbb{Q}$-factorial terminal singularities. If for some point $x \in Y$ the set $g^{-1}(x)$ is a curve, then $K_X g^{-1}(x) \geq -1$.*

**Proof.** See, for example, [17, (2.3.2)].

**4.4. Lemma.** *Cases* 4.2(1) *and* 4.2(2) *are impossible.*

**Proof.** Let $x \in Y$ be such that $\dim(g^{-1}(x)) > 0$. By Lemma 4.3, $K_{X_{\text{term}}}g^{-1}(x) \geq -1$, and, by Lemma 4.1, $K_{X_{\text{term}}}g^{-1}(x) < -1$.

**4.5. Lemma.** *Case* 4.2(3) *is impossible.*

**Proof.** Suppose $g$ contracts the divisor $D$ to a point. Obviously, $D$ does not belong to a fiber of the morphism $f$, and $C = n\hat{H}|_D$ for $n$ large enough is an effective 1-cycle on $X_{\text{term}}$, which is contained in $R$, but $BC = 0$.

**4.6. Lemma.** *Case* 4.2(5) *is impossible.*

**Proof.** Consider any effective irreducible divisor $E$ contracted to a point by the morphism $f$. Let $l$ be any curve in a fiber of the morphism $g|_E$. We have $K_{X_{\text{term}}}l \geq 0$, since $K_{X_{\text{term}}}$ is $f$-numerically effective, but $l \in R$ and $K_{X_{\text{term}}}l < 0$.

**4.7. Lemma.** *In case* 4.2(4), *the morphism $g$ is a $\mathbb{P}^1$-bundle over $H$.*

**Proof.** Let $C$ be a generic fiber of the morphism $g$. Then

$$2 = -K_{X_{\text{term}}}C = \hat{H}C + BC, \qquad \hat{H}C \geq 1, \quad BC > 0.$$

Hence, $\hat{H}C = 1$, $BC = 1$ and, therefore, the morphism $g$ has no multiple or reducible fibers, and the morphism $g|_{\hat{H}}\colon \hat{H} \to Y$ is birational. $\hat{H}$ contains no fibers, since $BC = 1$ and $\hat{H}$ does not intersect $B$. Taking into account the fact that $Y$ is a normal variety, we see that $g|_{\hat{H}}$ is an isomorphism.

The singularities of $X_{\text{term}}$ are rational (see, e.g., [15, Theorem 1-3-6]). Therefore (see, e.g., [19, (3.19)]) $X_{\text{term}}$ is a Cohen–Mackauley variety. Since all the fibers of $g$ are of the same dimension and $Y$ is smooth, the morphism $g$ is flat. The generic fiber of $g$ is $\mathbb{P}^1$, whence each fiber is $\mathbb{P}^1$ and $X_{\text{term}}$ is a $\mathbb{P}^1$-bundle over $Y$.

## §5. Main theorem

**5.1. Theorem.** *Suppose that $X$ is a normal threefold, $H$ is an ample effective Cartier divisor on $X$, and $H$ is a smooth minimal surface of Kodaira dimension zero. Then either $X$ is a generalized cone over $H$, or $H$ is either a K3 surface, or an Enriques surface.*

**Proof.** Suppose the singularities of $X$ are not canonical. Lemmas 4.4–4.7 imply that $X_{\text{term}}$ is a $\mathbb{P}^1$-bundle over $H$ with the section $\hat{H}$.

Applying the morphism $g_*$ to the exact sequence of sheaves

$$0 \to \mathcal{O}_{X_{\text{term}}} \to \mathcal{O}_{X_{\text{term}}}(\hat{H}) \to \mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}}) \to 0$$

and taking into account the fact that $R^0 g_*(\mathcal{O}_{X_{\text{term}}}) = \mathcal{O}_Y$, since $g$ is the contraction of the extremal ray and $R^1 g_*(\mathcal{O}_{X_{\text{term}}}) = 0$ by the vanishing theorem, we obtain the exact sequence

$$0 \to \mathcal{O}_Y \to R^0 g_*(\mathcal{O}_{X_{\text{term}}}(\hat{H})) \to R^0 g_*(\mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}})) \to 0. \qquad (*)$$

The mapping $g|_{\hat{H}}\colon \hat{H} \to Y$ is an isomorphism. Therefore $R^0 g_*(\mathcal{O}_{X_{\text{term}}}(\hat{H}))$ is a locally free sheaf of rank 2.

The commutative diagram

$$\begin{array}{ccccccc}
 & 0 & & & & 0 & \\
 & \uparrow & & & & \uparrow & \\
0 \to & \mathcal{O}_{X_{\text{term}}} & \to & \mathcal{O}_{X_{\text{term}}}(\hat{H}) & \to & \mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}}) & \to 0 \\
 & \uparrow & & \uparrow \alpha & & \uparrow & \\
 & g^* g_* \mathcal{O}_{X_{\text{term}}} & \to & g^* g_* \mathcal{O}_{X_{\text{term}}}(\hat{H}) & \to & g^* g_* \mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}}) & \to 0 \\
 & \uparrow & & & & & \\
 & 0 & & & & &
\end{array}$$

implies that the mapping $\alpha\colon g^* g_* \mathcal{O}_{X_{\text{term}}}(\hat{H}) \to \mathcal{O}_{X_{\text{term}}}(\hat{H})$ is surjective, and it determines the morphism $\mathcal{A}\colon X_{\text{term}} \to \mathbb{P}(g_* \mathcal{O}_{X_{\text{term}}}(\hat{H}))$ over $Y$. Since $\hat{H}$ is $g$-ample and is a section of the morphism $g$, the morphism $\mathcal{A}$ is finite birational, and so is an isomorphism because of the normality of $\mathbb{P}(g_* \mathcal{O}_{X_{\text{term}}}(\hat{H}))$.

The sequence $(*)$ splits, since

$$\operatorname{Ext}^1(R^0 g_*(\mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}})), \mathcal{O}_Y) = \operatorname{Ext}^1(\mathcal{O}_{\hat{H}}(\hat{H}|_{\hat{H}})), \mathcal{O}_{\hat{H}}) = H^1(\mathcal{O}_{\hat{H}}(-\hat{H}|_{\hat{H}})),$$

but $H^1(\mathcal{O}_{\hat{H}}(-\hat{H}|_{\hat{H}})) = 0$, by the vanishing theorem. Hence,

$$R^0 g_* \mathcal{O}_{X_{\text{term}}}(\hat{H}) \cong \mathcal{O}_Y \oplus R^0 g_*(\mathcal{O}_{\hat{H}}(\hat{H})),$$

and since $Y \cong \hat{H} \cong H$, we obtain

$$X_{\text{term}} \cong \mathbb{P}(\mathcal{O}_H \oplus \mathcal{O}_H(H|_H)),$$

so that $X$ is the contraction of the exceptional section of $\mathbb{P}(\mathcal{O}_H \oplus \mathcal{O}_H(H|_H))$, i.e., a generalized cone.

Suppose the singularities of $X$ are canonical. Consider the exact sequence of sheaves and the associated cohomology sequence:

$$0 \to \mathcal{O}_X(-H) \to \mathcal{O}_X \to \mathcal{O}_H \to 0,$$
$$H^1(\mathcal{O}_X) \to H^1(\mathcal{O}_H) \to H^2(\mathcal{O}_X(-H)).$$

As a consequence of the vanishing theorem, using the rationality of the singularities of $X$ and Serre duality, we obtain:

$$H^1(\mathcal{O}_X) = H^2(\mathcal{O}_X(-H)) = 0.$$

Hence, $H^1(\mathcal{O}_H) = 0$ and $H$ is either an Enriques surface or a K3 sequence (c.f. [13, Proposition 2.2 and 3.3]).

**5.2. Corollary.** *Suppose $X$ is a normal threefold, and $H$ is an ample smooth effective Cartier divisor, which is either an Abelian variety or a bielliptic surface. Then $X$ is a generalized cone over $H$.*

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