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A Fano 3-fold with a unique elliptic structure

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Abstract. An example of a Fano 3-fold that has a unique representation as an elliptic fibration is presented. No other examples of rationally connected varieties with such a property are known so far.

Bibliography: 5 titles.

All varieties in this paper are assumed to be projective and defined over $\mathbb{C}$. The main definitions, notation, and concepts can be found in [1].

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§ 1. Introduction

In this paper we shall study properties of the following 3-fold.

Main object. Let $\theta\colon X \to \mathbb{P}^3$ be a double cover ramified over a sextic S such that X has one singular point O, which is a simple double point.

The birational structure of X was studied in [2], where the following result was proved.

Birational rigidity of *X*.

$$\operatorname{Bir} X = \operatorname{Aut} X,$$

and X is not birationally isomorphic to

(1) *Mori* 3*-folds*[1] *that are not isomorphic to X*,
(2) *conic bundles*,
(3) *fibrations of surfaces of Kodaira dimension $-\infty$.*

Besides birational rigidity X has other interesting properties.

Elliptic structure on *X*. Let $f\colon W \to X$ be a blow up of the singular point O. Then the linear system $|-K_W|$ is free and the morphism

$$\varphi_{|-K_W|}\colon W \to \mathbb{P}^2$$

is an elliptic fibration.

[1]Mori 3-folds are Fano 3-folds with terminal $\mathbb{Q}$-factorial singularities and Picard group $\mathbb{Z}$.

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Note that the image in $\mathbb{P}^3$ of a fibre of the elliptic fibration so constructed is a line passing through the point $\theta(O)$.

Convention. We identify fibrations that are birationally equivalent (as fibrations).

The aim of this paper is to prove the following result.

Main theorem. *X cannot be birationally transformed into other elliptic fibrations.*

Note that each rationally connected surface is birationally isomorphic to infinitely many distinct elliptic fibrations.

Important remark. X is the only known example of a rationally connected variety that can be birationally represented in a unique way (in the class of birationally isomorphic varieties) as an elliptic fibration.

Our methods also describe other properties of X.

K3 structures on X. Let $\mathcal{P}$ be a pencil in $|-K_X|$ and assume that the commutative diagram

$$\begin{array}{ccc} & W & \\ {}^{f}\swarrow & & \searrow{}^{g} \\ X & \overset{\varphi_{\mathcal{P}}}{\dashrightarrow} & \mathbb{P}^1 \end{array}$$

resolves the indeterminacy of the map $\varphi_{\mathcal{P}}$. Then the general fibre of g is a smooth K3 surface.

The following result complements the Main theorem.

Additional theorem. *X is not birationally isomorphic to*

(1) *Fano* 3*-folds with canonical singularities that are not biregular to X*,
(2) *fibrations into surfaces of Kodaira dimension zero*, *except for the* K3 *fibrations constructed above.*

§ 2. Auxiliary objects

This chapter introduces objects that will be used in the proof of the Main theorem.

Movable log pair. A *movable log pair*

$$(X, M_X) = \left(X, \sum_{i=1}^{n} b_i \mathcal{M}_i\right)$$

is a variety X together with a formal finite linear combination of linear systems $\mathcal{M}_i$ without fixed components such that all $b_i \in \mathbb{Q}_{\geqslant 0}$.

Note that (X, M_X) can be regarded as a usual log pair.

Observation. The strict transform of M_X is defined in a natural way for each birational map.

We shall assume that the log canonical divisors of all the log pairs considered are $\mathbb{Q}$-Cartier divisors. Hence discrepancies, terminality, canonicity, log terminality, and log canonicity can be defined for movable log pairs in a similar way to the usual ones.

Centre of canonical singularities. A proper irreducible subvariety $Y \subset X$ is a *centre of canonical singularities* of (X, M_X) if there exist a birational morphism $f\colon W \to X$ and an f-exceptional divisor $E \subset W$ such that

$$a(X, M_X, E) \leqslant 0 \quad \text{and} \quad f(E) = Y.$$

Set of centres of canonical singularities. We denote by $CS(X, M_X)$ the set of centres of canonical singularities of (X, M_X).

The next result follows from [3].

Uniqueness theorem. *A canonical model is unique if it exists.*

For an arbitrary movable log pair (X, M_X) we consider a birational morphism $f\colon W \to X$ such that the log pair

$$(W, M_W) = (W, f^{-1}(M_X))$$

has canonical singularities.

Iitaka map and Kodaira dimension. If the linear system $|n(K_W + M_W)|$ is non-empty for $n \gg 0$, then the map

$$I(X, M_X) = \varphi_{|n(K_W+M_W)|} \circ f^{-1} \quad \text{for } n \gg 0$$

is called the *Iitaka map* of (X, M_X) and

$$\varkappa(X, M_X) = \dim(I(X, M_X)(X))$$

is called the Kodaira dimension of (X, M_X). Otherwise $I(X, M_X)$ is considered to be undefined everywhere and $\varkappa(X, M_X) = -\infty$.

One can prove the following result.

Correctness theorem. *The map $I(X, M_X)$ and the quantity $\varkappa(X, M_X)$ do not depend on one's choice of the morphism f.*

Note that the Iitaka map and the Kodaira dimension of a movable log pair depend a priori on the positive integer $n \gg 0$ involved in their definition. One can show that the Kodaira dimension does not depend on this number. Moreover, in dimension 3 it follows from the Log Abundance (see [1]) that the Iitaka map also depends only on the properties of the movable log pair. We shall mainly use movable log pairs and shall call them simply log pairs.

§ 3. Log Calabi–Yau structures

We now outline relations between the previous chapter and the Main theorem. We shall use the notation of § 1. One can show that

$$\operatorname{Pic} X = \mathbb{Z}K_X \quad \text{and} \quad K_X \sim \theta^*(\mathcal{O}_{\mathbb{P}^3}(-1)).$$

We fix a log pair (X, M_X) and choose $\lambda \in \mathbb{Q}_{>0} \cup \{+\infty\}$ such that

$$K_X + \lambda M_X \sim_{\mathbb{Q}} 0,$$

where $\lambda = +\infty$ for $M_X = \varnothing$.

Definition. In the case $\lambda = 1$ we call (X, M_X) a *log Calabi–Yau* 3-*fold*.

Core theorem. *Assume that* (X, M_X) *is a log Calabi–Yau* 3-*fold. Then* (X, M_X) *is canonical,* $\varkappa(X, M_X) = 0$, *and*

$$\mathrm{CS}(X, M_X) = \begin{cases} \{O\}, & \\ \{\mathrm{Bs}\,\mathcal{P}\} & \text{for the pencil } \mathcal{P} \text{ in } |-K_X| \text{ such that } O \notin \mathrm{Bs}\,\mathcal{P}, \\ \{\mathrm{Bs}\,\mathcal{P}, O\} & \text{for the pencil } \mathcal{P} \text{ in } |-K_X| \text{ such that } O \in \mathrm{Bs}\,\mathcal{P}, \\ \varnothing. & \end{cases}$$

It turns out that both the Main and the Additional theorems can easily be deduced from the Core theorem. We can obtain a rather precise description of the boundary M_X on the basis of the Core theorem in the case when the singularities of the log pair (X, M_X) are not terminal.

Refinement of the Core theorem. *Assume that* (X, M_X) *is a log Calabi–Yau* 3-*fold and the log pair* (X, M_X) *is not terminal. Then*

$$M_X = \psi^{-1}(M_Y),$$

where the rational map $\psi\colon X \dashrightarrow Y$ *is the composite of* θ *and the projection from* $\theta(\mathrm{CS}(X, M_X))$.

Proof. Let Z be the union of curves in $\mathrm{CS}(X, M_X)$ if the last set contains a curve. Otherwise let $Z = O$.

Note that in the case when $\mathrm{CS}(X, M_X)$ contains the point O we have

$$\mathrm{mult}_O(M_X) = 1.$$

This follows from Corti's theorem (see § 6).

Consider the linear system $\mathcal{H}$ of surfaces in $|-K_X|$ containing Z. We choose a birational morphism $f\colon W \to X$ such that the linear system $f^{-1}(\mathcal{H})$ is free, the 3-fold W is smooth, and f is an isomorphism outside Z. We set

$$g = \varphi_{\mathcal{H}} \circ f \quad \text{and} \quad (W, M_W) = (W, f^{-1}(M_X)).$$

We fix a sufficiently general divisor D in $f^{-1}(\mathcal{H})$.

Four cases are now possible: $\theta(Z)$ does not lie in S; $\theta(Z)$ is a line in S not passing through the point $\theta(O)$; $\theta(Z)$ is a line in S passing through the point $\theta(O)$; $Z = O$.

Assume that $\theta(Z) \not\subset S$. We may also assume that W contains precisely one f-exceptional divisor lying over the generic point of each irreducible component of Z. Then

$$M_W|_D \sim_{\mathbb{Q}} \sum_{i=1}^{k} c_i F_i|_D,$$

where all the $f(F_i)$ are points on X and all the c_i are rational. Hence M_W lies in the fibres of g and

$$I(X, M_X) = g \circ f^{-1}.$$

Assume that $\theta(Z)$ is a line in S not passing through the point $\theta(O)$. We may also assume that f is the composite of the blow up of Z and the blow up of a section of the exceptional surface of the first blow up. Then

$$M_W|_D \sim_{\mathbb{Q}} a(X, M_X, E_2)E_2|_D,$$

where E_2 is the exceptional surface of the second blow up. On the other hand, it is easy to see that $E_2|_D$ is a smooth rational curve on the smooth K3 surface D. Thus,

$$a(X, M_X, E_2) = 0.$$

Hence M_W lies in the fibres of g and

$$I(X, M_X) = g \circ f^{-1}.$$

Assume that $\theta(Z)$ is a line in S passing through the point $\theta(O)$. We may also assume that f is the composite of the blow up of O, the blow up of the proper transform of Z, and the blow up of a section of the exceptional surface of the second blow up. Then

$$M_W|_D \sim_{\mathbb{Q}} (a(X, M_X, E)E + a(X, M_X, E_2)E_2)|_D,$$

where E and E_2 are the exceptional surfaces of the first and the third blow ups, respectively. On the other hand, $E|_D$ and $E_2|_D$ are two smooth rational curves on the smooth K3 surface D that intersect transversally at one point. Hence

$$a(X, M_X, E) = a(X, M_X, E_2) = 0.$$

It easily follows from this that M_W lies in the fibres of g and

$$I(X, M_X) = g \circ f^{-1}.$$

Assume now that $Z = O$. Then g is an elliptic fibration. We may also assume that f is the blow up of O. For a sufficiently general fibre C of g,

$$M_W \cdot C = 0.$$

Hence M_W lies in the fibres of g and

$$I(X, M_X) = g \circ f^{-1}.$$

§ 4. Iitaka maps

We shall use the notation of the introduction.

Corollary to the Core theorem. *The following relations hold*:

$$\lambda = 1 \iff \varkappa(X, M_X) = 0,$$
$$\lambda < 1 \iff \varkappa(X, M_X) > 0,$$
$$\lambda > 1 \iff \varkappa(X, M_X) = -\infty.$$

Log pairs with $\varkappa(X, M_X) = 1$ or 2 can be explicitly described.

Description theorem I. *Let $\varkappa(X, M_X) = 1$. Then the log pair (X, M_X) is not canonical, $I(X, M_X)$ is the composite of θ and the projection from some line in $\mathbb{P}^3$, and*

$$M_X = I(X, M_X)^{-1}(\mathbb{P}^1).$$

Description theorem II. *Let $\varkappa(X, M_X) = 2$. Then the log pair (X, M_X) is not canonical, $I(X, M_X)$ is the composite of θ and the projection from the point $\theta(O)$ in $\mathbb{P}^3$, and*

$$M_X = I(X, M_X)^{-1}(\mathbb{P}^2).$$

Proof of Description theorems I and II. The Core theorem yields the canonicity of the log pair $(X, \lambda M_X)$. Thus,

$$\varkappa(X, M_X) \geqslant \varkappa(X, \lambda M_X) = 0.$$

Assume that $(X, \lambda M_X)$ is terminal. We take $\delta \in \mathbb{Q} \cap (\lambda, 1)$ such that $(X, \delta M_X)$ is still terminal. Then

$$3 = \varkappa(X, \delta M_X) \leqslant \varkappa(X, M_X) \leqslant 2.$$

Hence

$$\mathrm{CS}(X, \lambda M_X) \neq \varnothing.$$

The assertion now follows from the refinement of the Core theorem.

What can be said about log pairs of Kodaira dimension $-\infty$?

Description theorem III. *If $\varkappa(X, M_X) = -\infty$, then* $\mathrm{CS}(X, M_X) = \varnothing$.

Proof. $(X, \lambda M_X)$ is canonical by the Core theorem and the assertion follows from the inequality $\lambda > 1$.

Note that the birational rigidity of X follows from Description theorem III.

§ 5. Birational geometry of *X*

In this section we prove both the Main and the Additional theorems together with the birational rigidity of X using results of the previous section, the Core theorem, and the refinement of the Core theorem.

Theorem A. *X is not birational to a fibration with general fibre of Kodaira dimension $-\infty$.*

Proof. Assume that ρ is a birational transformation of X into a fibration $\tau\colon Y \to Z$ such that the general fibre of τ has Kodaira dimension $-\infty$. We take a 'sufficiently big' very ample divisor H on Z and choose $\mu \in \mathbb{Q}_{>0}$ such that

$$(X, M_X) = (X, \mu\rho^{-1}(|\tau^*(H)|))$$

is a log Calabi–Yau 3-fold. By construction

$$\varkappa(X, M_X) = -\infty,$$

which contradicts the Core theorem.

Theorem B. $\operatorname{Bir} X = \operatorname{Aut} X$ *and* X *is not birational to any Fano* 3*-fold with canonical singularities that is not biregular to* X*.*

Proof. We shall prove a slightly stronger result. Assume that we have a birational map $\rho\colon X \dashrightarrow Y$ such that Y has canonical singularities and big and nef anticanonical divisor. We claim that ρ is an isomorphism.

It is well known that $|-nK_Y|$ is free for $n \gg 0$. We consider the log pairs

$$(Y, M_Y) = \left(Y, \frac{1}{n}\,|-nK_Y|\right) \quad \text{and} \quad (X, M_X) = (X, \rho^{-1}(M_Y)).$$

The corollary to the Core theorem shows that (X, M_X) is a log Calabi–Yau 3-fold and the refinement of the Core theorem yields the terminality of (X, M_X). Hence we can take $\zeta \in \mathbb{Q}_{>1}$ such that both log pairs $(X, \zeta M_X)$ and $(Y, \zeta M_Y)$ are canonical models. The Uniqueness theorem shows that ρ is an isomorphism.

Theorem C. *All fibrations birational to* X *with general fibre of Kodaira dimension zero are described in* § 1*.*

Proof. Let ρ be a birational transformation of the 3-fold X into a fibration $\tau\colon Y \to Z$ such that the Kodaira dimension of the general fibre of τ is zero. Consider a 'sufficiently big' very ample divisor H on Z. The equality

$$\varkappa\bigl(X, \rho^{-1}(|\tau^*(H)|)\bigr) = \dim Z$$

and Description theorems I and II bring us to the required result.

§ 6. Proof of Core theorem

In this section we prove the Core theorem. We shall use the notation of the introduction. We fix a log Calabi–Yau 3-fold (X, M_X).

The global strategy: (1) show that $\mathrm{CS}(X, M_X)$ contains no points with the possible exception of O; (2) prove that (X, M_X) is canonical in O; (3) describe curves in $\mathrm{CS}(X, M_X)$.

To implement the global strategy we require several auxiliary results. The following result is established in [4].

Shokurov's Connectedness theorem. *Let*

(1) $f\colon W \to X$ *be a morphism of normal varieties such that* $f_*(\mathcal{O}_W) = \mathcal{O}_X$;
(2) $D = \sum_{i=1}^n d_i D_i$ *be a divisor such that* D_i *is* f*-exceptional whenever* $d_i < 0$;
(3) $-(K_W + D)$ *be* $\mathbb{Q}$*-Cartier,* f*-nef, and* f*-big*;
(4) $g\colon V \to W$ *be a log resolution of* (W, D)*.*

Then the divisor

$$\sum_{a(W,D,E) \leqslant -1} E$$

is connected in the neighbourhood of each fibre of $f \circ g$*.*

Corti's lemma ([5], Theorem 3.1). *Let P be a smooth point on a surface H and assume that for some non-negative rational numbers a_1 and a_2,*

$$P \in \mathrm{LCS}(H, (1-a_1)\Delta_1 + (1-a_2)\Delta_2 + M_H),$$

where the boundary M_H is movable and the irreducible reduced curves Δ_1 and Δ_2 intersect normally at the point P. Then

$$\mathrm{mult}_P(M_H^2) \geqslant \begin{cases} 4a_1a_2 & \text{if } a_1 \leqslant 1 \text{ or } a_2 \leqslant 1, \\ 4(a_1+a_2-1) & \text{if } a_1 > 1 \text{ and } a_2 > 1. \end{cases}$$

Corollary to Corti's lemma. *Let $P \in \mathrm{LCS}(H, M_H)$, where P is a smooth point of the surface H and the log pair (H, M_H) is movable. Then*

$$\mathrm{mult}_P(M_H^2) \geqslant 4.$$

Corti's theorem ([5], Theorem 3.11). *Let $O \in \mathrm{CS}(X, M_X)$, where O is a simple double point of a 3-fold X and the log pair (X, M_X) is movable. Then*

$$\mathrm{mult}_O(M_X) \geqslant 1.$$

What do we do now?

The local strategy: (1) use Shokurov's connectedness theorem and Corti's lemma to show that $\mathrm{CS}(X, M_X)$ does not contain smooth points of X; (2) derive from Corti's theorem the canonicity of the log pair (X, M_X) at the point O.

Lemma I. $\mathrm{CS}(X, M_X)$ *contains no smooth points of* X.

Proof. Assume that $\mathrm{CS}(X, M_X)$ contains a smooth point P. Consider the log pair

$$(X, B_X) = (X, H_X + M_X),$$

where H_X is a sufficiently general hyperplane section of X passing through P. By construction,

$$P \in \mathrm{LCS}(X, B_X).$$

Hence Shokurov's connectedness theorem yields

$$P \in \mathrm{LCS}(H_X, M_X|_{H_X}).$$

Next, the corollary to Corti's lemma shows that

$$\mathrm{mult}_P(M_X^2) = \mathrm{mult}_P((M_X|_{H_X})^2) \geqslant 4.$$

On the other hand,

$$2 = -K_X \cdot M_X^2 \geqslant \mathrm{mult}_P(M_X^2).$$

The inequality

$$2 = -K_X \cdot M_X^2 \geqslant 2\,\mathrm{mult}_P^2(M_X)$$

in combination with Corti's theorem easily yields the following result.

Lemma II. *(X, M_X) is canonical at O.*

Thus, to prove the Core theorem we may assume that $\mathrm{CS}(X, M_X)$ contains some irreducible reduced curve C.

The local strategy: (1) show that $\theta(C)$ is a line in $\mathbb{P}^3$; 2) prove that all components of $\theta^{-1}(\theta(C))$ belong to $\mathrm{CS}(X, M_X)$.

The inequality

$$2 = -K_X \cdot M_X^2 \geqslant \mathrm{mult}_C(M_X^2)(-K_X) \cdot C \geqslant -K_X \cdot C \geqslant \deg\theta(C)$$

brings us to the following result.

Lemma III. *$-K_X \cdot C \leqslant 2$, and $\theta(C)$ is either a conic or a line.*

Lemma IV. *$\theta(C)$ is a line.*

Proof. Assume the contrary. Then the above inequality shows that $-K_X \cdot C = 2$, $\theta(C)$ is a conic, $\theta|_C$ is an isomorphism, and

$$\mathrm{mult}_C(M_X) = 1.$$

We choose a sufficiently general divisor H in $|-K_X|$. H is a smooth K3 surface intersecting the curve C precisely at two distinct points, x_1 and x_2. Let $g\colon V \to H$ be the blow up of x_1 and x_2. Let $E_1 = g^{-1}(x_1)$ and $E_2 = g^{-1}(x_2)$. Then the linear system

$$|g^*(H|_H) - E_1 - E_2|$$

contains precisely one effective divisor D.

Note that D is a smooth curve of genus 2. On the other hand,

$$(g^{-1}(M_X|_H)) \sim_{\mathbb{Q}} g^*(H|_H) - E_1 - E_2$$

and

$$(g^{-1}(M_X|_H))^2 = 0.$$

Hence the linear system $|nD|$ has no fixed components for $n \gg 0$ and $D^2 = 0$. Thus, for some $n \gg 0$ the linear system $|nD|$ is free and

$$\varphi_{|nD|}(V) = \mathbb{P}^1.$$

Hence, for $k \in (1, n]$ the fibration $\varphi_{|nD|}$ has a multiple fibre kD. This means that the genus of the curve D must be 1.

We can now complete the proof of the Core theorem.

Proof of the Core theorem. Assume that

$$\mathrm{CS}(X, M_X) \neq \varnothing \quad \text{and} \quad \mathrm{CS}(X, M_X) \neq \{O\}.$$

It follows from Lemmas I–IV that $\mathrm{CS}(X, M_X)$ contains a smooth rational curve C such that $-K_X \cdot C = 1$. Moreover, $C \not\subset S$. Hence

$$\theta^{-1}(\theta(C)) = C \cup C',$$

where C' is a smooth rational curve such that $-K_X \cdot C' = 1$.

Consider now the pencil $\mathcal{H}$ of surfaces in $|-K_X|$ containing C and C'. We choose a birational morphism $f\colon W \to X$ such that the pencil $f^{-1}(\mathcal{H})$ is free, W is smooth, and f is an isomorphism outside C and C'. Setting

$$(W, M_W) = (W, f^{-1}(M_X))$$

we fix a sufficiently general divisor D in the pencil $f^{-1}(\mathcal{H})$.

Two cases are now possible: the curves C and C' pass through the point O; the curves C and C' do not pass through the point O. In the first case we must show that

$$C' \in \mathrm{CS}(X, M_X) \quad \text{and} \quad O \in \mathrm{CS}(X, M_X).$$

In the last case we must show that

$$C' \in \mathrm{CS}(X, M_X).$$

Assume that the curves C and C' pass through the point O. We may also assume that f is the composite of the blow up of O, the blow up of the proper transform of C, and the blow up of the proper transform of C'. Then

$$M_W|_D \sim_{\mathbb{Q}} (a(X, M_X, E)E + a(X, M_X, E_2)E_2)|_D,$$

where E and E_2 are the exceptional surfaces of the first and the third blow ups, respectively. On the other hand $E|_D$ and $E_2|_D$ are two smooth rational curves on the smooth K3 surface D that intersect transversally at one point. Hence

$$a(X, M_X, E) = a(X, M_X, E_2) = 0.$$

Assume now that the curves C and C' do not pass through the point O. We may also assume that f is the composite of the blow up of C and the blow up of the proper transform of C'. Then

$$M_W|_D \sim_{\mathbb{Q}} a(X, M_X, E_2)E_2|_D,$$

where E_2 is the exceptional surface of the second blow up. On the other hand $E_2|_D$ is a smooth rational curve on the smooth K3 surface D, which shows that

$$a(X, M_X, E_2) = 0.$$

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