James Cook Math Notes Issue #1

POLYNOMIALS, their zeros and their derivatives.

Let $f(z)$ be any polynomial (with real or complex coefficients). The zeros of the derivative f' must be in the convex hull of the zeros of f.

Let $f(z) = \sum c_n z^n / n!$ and in the complex plane let S be the set $\{ x + iy \; ; \; y^2 \le x^2 \}$. If the coefficients c_n are all real and if the zeros of f are all in S then (taking logarithms and differentiating twice) it follows that $c_1^2 \ge c_0 c_2$. If in addition the zeros of f are all real then so are the zeros of f' etc. Consequently $c_n^2 \ge c_{n-1} c_{n+1}$.

PROBLEM: If f has real coefficients and its zeros are all in the set S described above, then does the derivative f' have the same property?

Let ABC be a triangle, and M the centroid. Two points L and N may be defined as follows. Treating the plane as the complex plane, take a cubic $f(z)$ with zeros at the vertices of the triangle, then L and N are the zeros of $f'(z)$. Clearly M is the mid-point of LN. Is there any simple geometrical characterisation of the line LN?

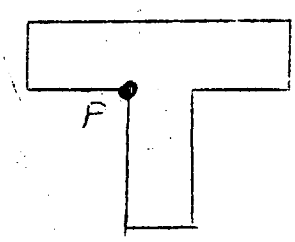
Points P and P' may be constructed so that BPP' and CPP' are both equilateral, then LN bisects (internally or externally) the angle PMP'. Similarly it bisects angles QMQ' and RMR'.

Another property of LN is that it is a principal axis of inertia of the body consisting of particles of equal mass at the vertices A, B and C.

OBJECTIVE TESTING

The Australian Council for Educational Research is a body that conducts tests on school pupils in order to determine the award of Federal Government Scholarships. One of their questions was as follows.

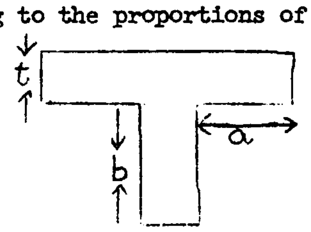
The T-shaped figure has an area of 5 square units. A straight line is drawn through P so as to divide the figure into two parts of equal area. In how many ways can this be done?



(A) 1 (C) 3

(B) 2 (D) It is not possible.

All the four suggested answers are wrong. The number of ways of bisecting the figure by a line through P may be 1, 2 or 3 according to the proportions of the figure.



If a = b = t the answer is one

If a = 2t and b = 3t there are two ways.

If a = 2t and b = 4t there are three ways. The educational experts of the A.C.E.R. do not reveal which answer they

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A LITTLE PUZZLE

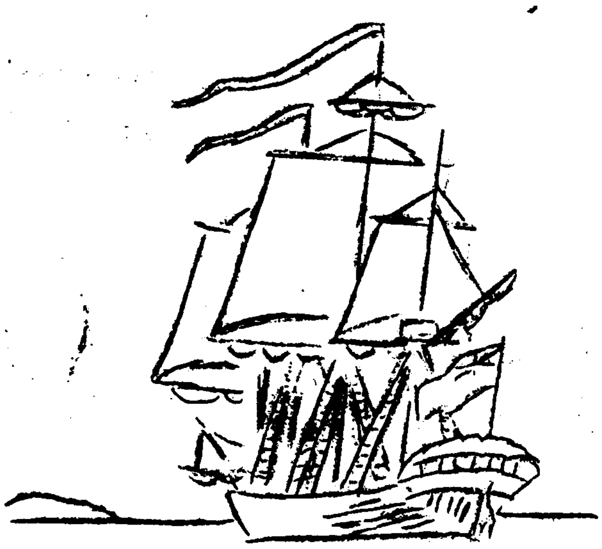
Is it obvious that

$$\begin{vmatrix} x & y & z & t \\ y & -x & t & -z \\ z & t & -x & -y \\ t & -z & -y & x \end{vmatrix} \geq 0 \ ?$$

TRISECTING ANGLES WITH RULER AND COMPASS

The ancients knew that this could be done with the aid of the conchoid of Nicomedes (an algebraic curve of the fourth degree), or by using two conics. Recently Mr. Leo Livingood of Townsville wrote to me suggesting another approach. Consider all the possible circular arcs joining the two points (1, 0) and (-1, 0). Each has a trisecting point in the right half plane, and to trisect angles all that we need is the locus of these points, and it is easy to show that the locus is the right hand branch of the hyperbola $y^2 = (x + 1)(3x - 1)$.

The moral of this story is that if geometers really believed in the third dimension they would provide themselves with lathes and planing machines as well as the traditional straight edge and pair of compasses. Then they could construct conics and trisect angles. What algebraic equations could they solve?



Bicentenary of
Captain Cook's voyage
in HMS Resolution
1772 - 1775

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