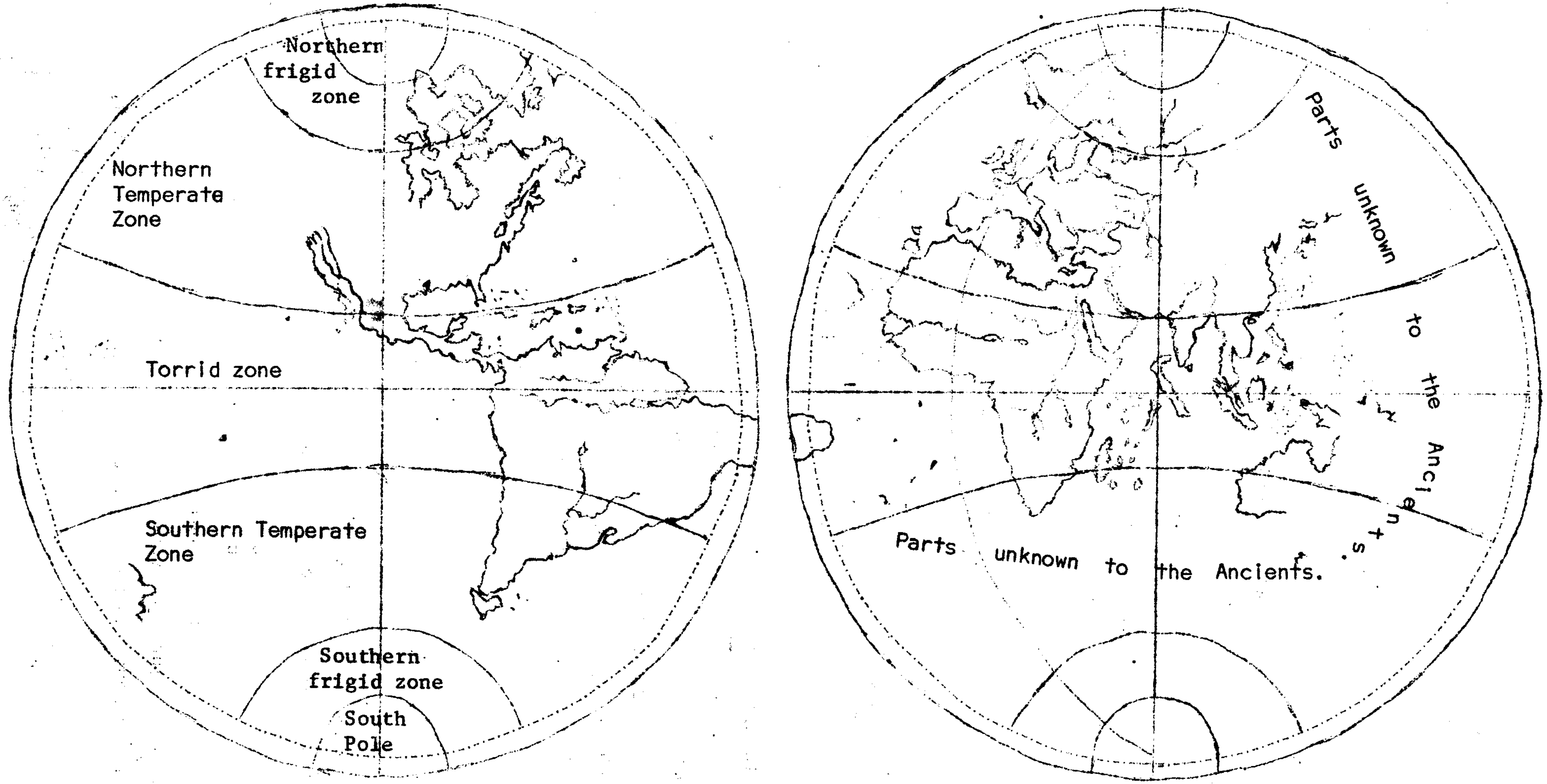
JAMES COOK MATHEMATICAL NOTES

Number 14. January 1978.



A map of the known world at the end of the seventeenth century, from Edward Well's *New Sett of Maps*, Oxford, 1701.

MATRIX OF ZEROS AND ONES (JCMN 13)

The $n \times n$ matrix A $(n \geq 3)$ has all its elements zero except that $a_{rs} = 1$ when $|r - s| = 1$. If M_j and m_j are the greatest and least elements of A^j show that $m_j = 0$ and $M_j \to \infty$ as $j \to \infty$. Since A is the incidence matrix of the graph:

and the (r, s) element of A^j is the number of j-step paths from node r to node s, the result is clear, the number being zero if $r + s + j$ is odd. Simple inequalities like $M_j \geq 2^{j/2}$ can be found to indicate the way in which this value tends to infinity. Solutions from *H.O. Davies*, *J.B. Parker* and *R.B. Potts*.

QUADRATICS AGAIN (JCMN 13)

For what complex b and c does the quadratic $z^2 + bz + c = 0$ have roots of equal modulus? If this is so and if p and q are the roots, in what circumstances does $p^k = q^k$ for some integer k? Solutions from *H.O. Davies* and *R.B. Potts* are geometrical, using the fact that complex numbers $u + v$ and $u - v$ have the same modulus when u and v as vectors are perpendicular, that is one is a real multiple of i times the other. The roots p and q may be expressed as $-(b/2)(1 \pm d^{1/2})$ where $d = 1 - 4c/b^2$ and so the condition is that d must be real and ≤ 0, or that $4c/b^2 \geq 1$ (of course $b = 0$ is a trivial special case).

If $p^k = q^k$ and $\exp(2\pi in/k) = p/q = (1+i\sqrt{-d})/(1-i\sqrt{-d})$ then $b^2 = 4c \cos^2 n\pi/k$.

AFTER DINNER MATHEMATICS

from *C.F. Moppert*

A circle with radius 1 rolls inside a circle with radius 2. Using school geometry, show that every point of the circumference of the small circle describes a diameter of the large one.

The inventor of π was not Thagoras, but an Englishman called Cumference, a relative of Falstaff. He was subsequently knighted. (But the floating π is attributed to Archie Medes of the stall behind the Adelaide Railway Station — Editor.)

MEA CULPA

As there were eight mistakes in JCMN 13 the editor has resolved to do better in 1978.

RANK ONE MATRICES (JCMN 13)

The square matrix A has rank one. Show that it is similar to a diagonal matrix if and only if its square is non-zero.

A must be of the form uv' where u and v are non-zero column vectors. The only solution of the eigenvalue equation $uv'x = Ax = \lambda x$ is $\lambda = v'u$ with eigenvector u. Since $A \neq 0$, if it is similar to a diagonal matrix then $v'u \neq 0$, and therefore $A^2 = uv'uv' = v'uA \neq 0$. Conversely if $A^2 \neq 0$ then u is not orthogonal to v. Take a new basis with u as first element and the others all orthogonal to v, then A becomes diagonal, with only the top left element non-zero. Alternatively the result comes easily from the theory of the Jordan Canonical Form.

Solutions from *R.L. Agacy, E.S. Barnes* and *R.B. Potts*.

A PARTY GAME

This is for any number of players. In fact you can even play it on your own if you are stranded on a desert island with a sandy beach. The players take it in turn to add either 0 or X to a sequence, and the object is not to make two identical adjacent segments of length two or more. For example suppose that the play has reached

0 X 0 0 0 X 0.

The next player has to lose a point because putting 0 will make 0 X 0 0 repeated, and putting X will make 0 X repeated. Of these two alternatives a clever player may choose the first because it gives the position

0 X 0 0 0 X 0 0

and thereby puts the next player in the same position of having to lose a point, because an extra 0 will make X 0 0 0 repeated and an extra X will make 0 0 X repeated.

For mathematicians here is a problem. Prove or disprove that sooner or later one of the players must lose a point. In algebraic terms, if d maps the natural numbers into a two element set, prove that for some natural numbers n and $r \geq 2$, for all $s \leq r$, $d(n-s+1) = d(n-r-s+1)$.

THIRD TIME LUCKY (JCMN 13)

If I press the 1/x key on my calculator three times do I get the same result as pressing it once? More precisely if f(x) is the nearest integer to $g(x) = 10^{19}/x$ then is $f(f(f(n))) = f(n)$ for all integers n between 10^9 and 10^{10}?

Firstly when is $f(f(n)) \neq n$? It is when $g(f(n))$ is outside the interval $(n-\frac{1}{2}, n+\frac{1}{2})$, that is when f(n) is outside $(g(n+\frac{1}{2}), g(n-\frac{1}{2}))$. For this it is necessary that either $g(n-\frac{1}{2})-g(n)$ or $g(n) - g(n+\frac{1}{2})$ should be less than a half; but of these two differences the first is the larger and the second is $10^{19}/(2n^2+n)$. Therefore for

$f(f(n)) \neq n$ it is necessary that $n(n+\frac{1}{2}) \geq 10^{19}$. Put $\theta = 10^{19/2} = 3162277660 \cdot 16\ldots$. For $f(f(n)) \neq n$ it is necessary that either $n > \theta$ or $n < 10^{19}/n < n+\frac{1}{2}$. The only solution to this last inequality is $n = 3162277660 = n_o$ and $f(n_o) = n_o$. Therefore $f(f(n)) = n$ for all $n < \theta$. Further if $n > \theta$ then $f(n) < \theta$, and so $f(f(f(n))) = f(n)$ for all n between 10^9 and 10^{10}.

C.J. Smyth

STILL LUCKY?

Mathematics is the art of generalizing. Prove or disprove the following generalization of the result given above under the heading "Third time lucky". Let $g(x)$ be continuous strictly decreasing and convex in the interval in which it is defined, so that the inverse function $h(x)$ has the same properties. Let $f(x)$ and $k(x)$ be the nearest integers to $g(x)$ and $h(x)$ respectively. Then $f(k(f(n))) = f(n)$ for all integers n in the interval of definition of the function g.

THE FINITE ELEMENT METHOD

Consider the partial differential equation

$$\partial/\partial x(r^2 y_x) = r^2 y_{tt} \text{ or } y_{xx} + 2(r'/r)y_x = y_{tt}$$

where $r = r(x) > 0$. If $r(x)$ is constant the P.D.E. is the one-dimensional wave-equation and there are no difficulties. Therefore we replace the function $r(x)$ by a step-function and assume that an adequate approximation to the continuously varying $r(x)$ will be found by taking a step-function changing by a large number of small steps. Now to see what happens near a small step in $r(x)$, say $r(x) = b$ for $x < 0$ and $r(x) = b(1+\varepsilon)$ for $x > 0$. The general solution for $x < 0$ is $y = f(x-t) + g(x+t)$. Take the first term, the other may be treated similarly. The boundary condition at $x = 0$ is that $r^2 \partial y/\partial x$ and $\partial y/\partial t$ must both be continuous at $x = 0$ for all t, and elementary reasoning shows that (to first order in ε) there is a transmitted wave $(1-\varepsilon)\, f(x-t)$ to the right of the origin and the reflected wave $\varepsilon f(x+t)$ to the left. For the original P.D.E. there is a conservation equation $(\partial/\partial t)(\frac{1}{2}\, r^2(y_x{}^2 + y_t{}^2)) = (\partial/\partial x)(r^2 y_x\, y_t)$, the expression $\frac{1}{2}\, r^2(y_x{}^2 + y_t{}^2)$ is positive definite and is conserved, so that it has the two properties that make energy an important scientific concept, and so for simplicity we call it energy. If the step-function $r(x)$ has n steps then the total amount of energy diverted by partial reflection from the left-to-right waves is of order $n\varepsilon^2 f^2$ which tends to 0 as the step-function approximates to the true $r(x)$. The left-to-right wave $y = f(x-t)$ is therefore propagated past the discontinuity with only a change of scale, and in fact $r(x)\, f(x-t)$ is unaltered at the discontinuity (at least to first order in ε). Any second order changes in $r(x)\, f(x-t)$ will become negligible as the number of steps tends to infinity. Similar reasoning of course applies to the right-to-left wave $y = g(x+t)$. Therefore the finite-element method (if applied numerically) leads to a conclusion that is approximately equivalent to the assertion that $y = (f(x-t) + g(x+t))/r(x)$ is the general solution of the P.D.E. However substitution shows that it is not a solution except when $r(x)$ is linear. Would some of our experts on differential equations write to explain what went wrong? Putting the question more precisely, let x^2 in (1, 2) be approximated by step-functions $r_n(x)$ each of n steps, choose some boundary conditions, say $y = f(t)$ when $x = 1$ and $y_x + y_t = 0$ when $x = 2$. Let $y_n(x, t)$ be the solution of the P.D.E. above for $r = r_n(x)$, does $y_n(x, t)$ tend to the solution $y(x, t)$ for the P.D.E. with $r(x) = x^2$?

TWO POINTS IN A TRIANGLE (JCMN 1, 2, 3 and 12)

These propositions of *C.A. Davis* may be proved with a sequence of lemmata as follows.

<u>Lemma 1.</u>

Of the triangles circumscribed to a fixed circle any one of smallest area is equilateral and has area $3\sqrt{3}/\pi$ times the area of the circle. Also this triangle touches the circle at the mid-points of the sides.

Proof. The existence follows from topological arguments, and any triangle not equilateral clearly does not give minimum area. The fact about mid-points is clear.

<u>Lemma 2.</u>

Of all the triangles circumscribed about a fixed ellipse the smallest has area $3\sqrt{3}/\pi$ times the area of the ellipse, and touches at the mid-points of the sides.

Proof. Use a little transformation geometry. A mapping of the form $(x,y) \to (\mu x,y)$ will transform the ellipse to a circle.

<u>Lemma 3.</u>

Of all the ellipses inscribed in a fixed triangle there is a largest, and it touches the sides at their mid-points. Call this the "maximal ellipse" of the triangle.

Proof. For any ellipse inscribed in any triangle the ratio of areas by Lemma 2 must satisfy triangle/ellipse $\geq 3\sqrt{3}/\pi$. Use an affine mapping of the given triangle to an equilateral triangle, inscribe a circle, and use the inverse mapping. This shows that there is an inscribed ellipse with the exact ratio of areas $3\sqrt{3}/\pi$. There cannot be two distinct inscribed ellipses with this same maximal area because they would both have to touch all three sides at their mid-points.

<u>Lemma 4.</u>

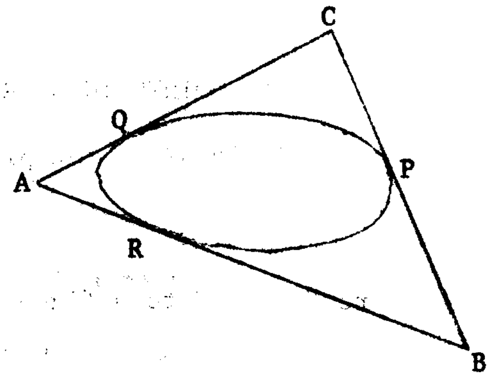
The points of contact of a triangle with its maximal ellipse have eccentric angles differing by 120^o.

Proof. Transform the ellipse to a circle as in Lemma 2. The triangle becomes equilateral.

Lemma 5.

Given a complex cubic $f(z)$, the maximal ellipse of the triangle formed by the zeros of f has its foci at the zeros of the derivative $f'(z)$.

Proof. By using a linear transformation of the complex variable we may take the axes to be the axes of the ellipse. The ellipse is then $x^2/a^2 + y^2/b^2 = 1$. Let the eccentric angles of the points of contact, P, Q and R be α, $\alpha + 120^\circ$ and $\alpha - 120^\circ$ respectively. The line QR is $(x/a)\cos\alpha + (y/b)\sin\alpha = -1/2$ and its pole A has coordinates $(-2a \cos \alpha, -2b \sin \alpha)$. Now change to complex numbers, let the points A, B and C be represented by u, v and w respectively.



$$u = -2\, a \cos \alpha - 2\, i\, b \sin \alpha$$

$$v = -2\, a \cos (\alpha + 120^\circ) - 2\, i\, b \sin (\alpha + 120^\circ)$$

$$w = -2\, a \cos (\alpha - 120^\circ) - 2\, i\, b \sin (\alpha - 120^\circ)$$

Clearly $u + v + w = 0$. Now to find the sum of products two at a time,

$$vw = a^2(2 \cos 2\alpha - 1) + 4\, i\, a\, b \sin 2\alpha + b^2(2 \cos 2\alpha + 1)$$

and uw and uv may be found by adding 120° to α.

$$uv + vw + wu = 3b^2 - 3a^2$$

The cubic is $f(z) = z^3 + 3(b^2 - a^2)z - uvw$, and the zeros of the derivative are the foci $z = \pm (a^2 - b^2)^{1/2}$.

Q.E.D.

A THEOREM OF WIELANDT (JCMN 13)

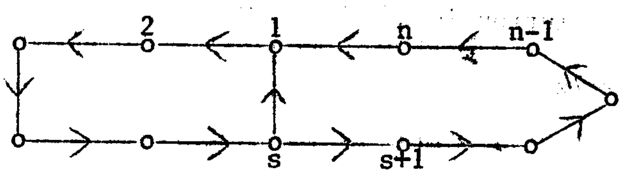
NON-NEGATIVE MATRICES (JCMN 13)

H. Kestelman points out that these problems are special cases ($s=n-1$ and $s=1$) of a theorem proved by Busacker and Saaty, as follows. Let G be a directed graph of n nodes, for any k the graph G^k has an edge (or arc) from r to s when G has a k-step path from r to s. A complete graph is one in which from each node there is an edge to every other node. Theorem: if G has a circuit of length s and if G^k is complete for some k then it is complete for $k = n + s(n-2)$.

Proof: Given $N \geq n + s(n - 2)$ we want a path from a starting point P to any other node T in N steps. Firstly we may go from P to some Q on the cycle in $q \leq n - s$ steps. Now consider the graph G^s, it has a sling at Q, that is an edge from Q to itself, and so in G^s we may find a path from Q to any other point R in exactly n-1 steps, that is in G there is a path from P to R in exactly q + s(n-1) steps. Choose R so that there is a path RT of length N - q - s(n - 1), then PQRT is of length N.

Q.E.D.

The relation between non-negative matrices and directed graphs should be clear, the r, s component of the n × n matrix A being non-zero when there is an edge from node r to node s in the directed graph G, then A^k corresponds similarly to G^k. The fact that if s is prime to n the n + s (n - 2) of the theorem is best possible may be shown by the graph G with edges (1, 2), (2, 3), ... (n-1, n), (n, 1) and (s, 1) (or the matrix with non-zero elements in these positions)



(Case n = 11, s = 6)

It is impossible to find a path of n + s (n - 2) - 1 steps from s + 1 to n because of a lemma in number theory that the number ns - s - n cannot be expressed as a sum of non-negative multiples of n and s.

ANYONE FOR A SPOT OF RIGOUR? (JCMN 13)

The note in our previous issue was about the partial differential equation $\partial/\partial x(r^2 y_x) = r^2 y_{tt}$ which may also be written $y_{xx} + (2r'/r)y_x = y_{tt}$, where $r = r(x) > 0$. Instead of going through the reasoning with a fine-tooth comb it would be more enlightening to look at the facts of the case. Did you believe the suggestion that non-constancy of the function r(x) leads to partial reflection of signals?

Many of our readers no doubt remember lecturing on elementary acoustics and telling the class that $\phi = (1/r)f(r-ct)$ is the solution of the wave equation $c^2\nabla^2\phi = \phi_{tt}$ (in spherical polar coordinates) corresponding to a disturbance caused by a point source at the origin. Because of the spherical symmetry we may imagine a suitable conical barrier put in to the space, so that the formula gives a solution for sound in a narrow cone, that is for a voice-pipe of radius proportional to length along the pipe. Armed with this knowledge we may go back to the pure mathematics of the original P.D.E. and check that if f is any function then $y = (1/x)f(x-t)$ satisfies $y_{xx} + (2/x)y_x = y_{tt}$. That is to say there is a non-constant function $r = r(x) = x$ for which a signal may be propagated in one direction only. In fact the signal may be confined to a region $a < t-x < b$, which is an interval of the x-axis moving at unit speed in the positive direction. The idea that non-constancy of r(x) causes partial reflections must be wrong.

JCMN14.

There may seem to be a contradiction between the existence of a solution confined to an interval moving at unit speed and the demonstration in the previous article that energy is propagated at speeds less than one (except when one of the two parameters u and v is zero and therefore $r(x)$ constant). However it is possible to reconcile these two facts, for the energy of the signal may be initially more concentrated in the front half of the moving interval, and slip back as the interval travels on its way.

Another way of looking at the case $r(x) = x$ is to observe that the differential operator may be factorized, in fact the P.D.E. may be written in the form

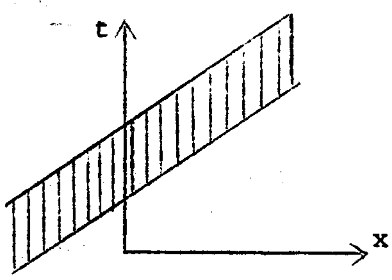
$$(\partial/\partial x - \partial/\partial t)(\partial/\partial x + \partial/\partial t)(xy) = 0$$

and it becomes clear that the general solution is of the form

$$y = (f(x-t) + g(x+t))/x$$

for any two functions f and g.

For each of the two cases $r(x) = \text{constant}$ and $r(x) = x$ we have found firstly that for any interval (a, b) there are solutions vanishing outside the region $a < t - x < b$;



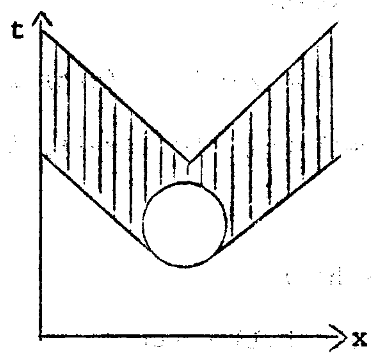
roughly speaking these solutions may be described as signals that travel from left to right without partial reflection. Similarly there are solutions vanishing outside a region $a < t + x < b$. Secondly solutions of these types form a complete set, in the sense that every solution is a linear combination of them. If the P.D.E. has these two properties we might call it "echo-free". One fact about these solutions may be noted; if a non-trivial solution vanishes outside the diagonal strip $a < t - x < b$ then it occupies the full length of the strip, this follows from the energy conservation equation

$$(\partial/\partial t)(r^2(y_x^{\,2} + y_t^{\,2})/2) = (\partial/\partial x)(r^2 y_x y_t)$$

which shows that $\int r^2(y_x^{\,2} + y_t^{\,2})dx$ is the same for all sections $t = \text{constant}$ of the diagonal strip.

The case where our P.D.E. is echo-free may be visualised as follows. Suppose that some phenomenon on a one-dimensional continuum satisfies the P.D.E., and suppose that the system initially at rest is perturbed by some influence acting in a bounded region of the x-t plane, then the resulting disturbance will be confined to the

V-shaped region indicated.



We have seen that $r(x)$ = constant and $r(x) = x$ make the P.D.E. echo-free, are there any other functions $r(x)$ with this property? If $r(x)$ has the property and $y(x, t)$ vanishes outside $a < t - x < b$ and satisfies the P.D.E. $(\partial/\partial t)(r^2 y_t) = (\partial/\partial x)(r^2 y_x)$ then by the theory of differential equations there is $Y(x, t)$ satisfying

$$Y_x = r^2 y_t \qquad \text{and} \qquad Y_t = r^2 y_x$$

and this Y will satisfy $\partial/\partial x(r^{-2} Y_x) = (\partial/\partial t)(r^{-2} Y_t)$ which is the P.D.E. obtained by putting $1/r$ for r in the original. Will $Y(x, t)$, with a suitable choice of the arbitrary constant that may be added, also vanish outside $a < t - x < b$? Y is constant in each of the two half-planes outside the diagonal strip, but are these two constants equal? This is not clear, perhaps one of our readers can answer this question, which consisely put is as follows. If y is defined and differentiable any number of times in the x, t plane and if $r = r(x) > 0$ is such that $\partial/\partial x(r^2 y_x) = r^2 y_{tt}$ and if y vanishes outside a diagonal strip where $a < t - x < b$, prove or disprove that $\int_{-\infty}^{\infty} y_x dt = 0$.

The considerations above lead us to consider the case $r(x) = 1/x$. The general solution of the P.D.E. may be found by observing that

$$(\partial/\partial x^2 - \partial^2/\partial t^2)(y_x/x) = (1/x)(\partial/\partial x)(y_{xx} - 2y_x/x - y_{tt}) = 0$$

which leads to $y_x/x = F(x-t) + G(x+t)$ for any F and G. We may put $F = f''$ and $G = g''$ and integration gives

$$y = xf'(x-t) - f(x-t) + xg'(x+t) - g(x+t) + h(t).$$

Substitution in the original P.D.E. gives $h''=0$ and so h may be absorbed into the other two functions, and the general solution is

$$y = xf'(x-t) - f(x-t) + x\,g'(x+t) - g(x+t)$$

showing that $r(x) = 1/x$ makes the P.D.E. echo-free.

A little digging around will uncover the fact that for $r(x) = \tan x$ there is a solution $y = f(x-t) + \cot x\, f'(x-t)$ which of course is valid only for $0 < x < \pi/2$. To find the general solution in this case note that

$$(\partial^2/\partial x^2 - \partial^2/\partial y^2)(y + y_x \tan x) = (1 + \tan x\, \partial/\partial x)(y_{xx} + 4y_x \operatorname{cosec} 2x - y_{tt}) = 0$$

and so $y + y_x \tan x = F(x-t) + G(x+t)$. To integrate this take the F term first, any $F(u)$ may be expressed as $F(u) = f(u) + f''(u)$, so that we must solve:

$$y + y_x \tan x = f(x-t) + f''(x-t)$$

Changing the dependent variable from y to $z = y - f(x-t)$ leads to

$$z \cos x + z_x \sin x = f''(x-t) \cos x - f'(x-t) \sin x$$

and integration gives

$$z \sin x = f'(x-t) \cos x + h(t)$$

or $$y = f(x-t) + f'(x-t) \cot x + h(t) \operatorname{cosec} x$$

Now adding the solution of $y + y_x \tan x = G(x-t)$ we obtain the general solution

$$y = f(x-t) + f'(x-t) \cot x + g(x+t) + g'(x+t) \cot x + h(t) \operatorname{cosec} x$$

Substitution in the original shows that $h'' + h = 0$, so that there are terms $y \sin x = \cos t$ or $\sin t$, but these may be absorbed into the other terms by adding sines and cosines to f and g. The general solution for $r(x) = \tan x$ is therefore

$$y = f(x-t) + f'(x-t) \cot x + g(x+t) + g'(x+t) \cot x$$

which again gives the property of being echo-free.

Are there other functions $r(x)$ that make the P.D.E. echo-free? There are some that do not, for example $r(x)$ = step function or $r(x) = \exp mx$. The latter is of interest, it is discussed in elementary acoustics because it gives the theory of the "exponential horn" type of loud speaker. However let us not get side-tracked into the study of horns and trumpets, fascinating though it might be. The motivation for this investigation is strictly practical.

In studies of atmospheric pollution great importance is attached to temperature inversion layers, and more generally to the temperature as a function of height above ground level. The Physics Department of the JCUNQ has a loud speaker at the focus of a paraboloidal reflector to send a beep upwards every seven seconds. What could they find out about the variation of temperature with height by analysing the echoes?

The first question is how to set up a good mathematical model, and for the sake of simplicity we would like to replace the three-dimensional atmosphere by a one-dimensional continuum. The beam is certainly not well enough focussed to be treated as parallel, but we may suppose that a vertical cylinder can be chosen fat enough to contain almost all the beam, then by averaging over horizontal sections of this cylinder we reduce the 3-dimensional equations of motion and continuity to one dimension. This leads to the wave equation

$$c^2 \frac{\partial^2 y}{\partial x^2} + \left(\frac{dc^2}{dx} + \frac{c^2}{\rho} \frac{d\rho}{dx}\right) \frac{\partial y}{\partial x} = \frac{\partial^2 y}{\partial t^2}$$

where y is either vertical displacement or velocity, ρ is the density and c is the speed of sound at height x. This equation may be transformed by using a new variable

$X = \int dx/c$ instead of x, and then we have

$$\frac{\partial^2 y}{\partial X^2} + \left(\frac{\partial c}{\partial x} + \frac{c}{\rho}\frac{\partial \rho}{\partial x}\right)\frac{\partial y}{\partial X} = \frac{\partial^2 y}{\partial t^2}$$

The coefficient in round brackets is a function of x or of X. If the air is assumed a perfect gas with $c^2\rho = \gamma p$ then the coefficient may be expressed as

$$-\frac{\gamma g}{c} - \frac{\partial c}{\partial x}$$

The mathematical problem is to find out something about the r(x) (or about r'/r) in the P.D.E. $y_{xx} + 2r'y_x/r = y_{tt}$ from a study of the echoes that come back to the origin. If this can be done then there will be no essential difficulty in measuring atmospheric temperature gradients from ground level.

Any comments or suggestions would be welcome, even if they are not for publication, and so if you have any ideas do please write straight away.

BOOK REVIEW

Logic by Wilfred Hodges, a Penguin paperback, published in 1977.
ISBN 0 14 02 1985 4.

This book is intended for people who want to learn some elementary logic. It is written as a conversation between the reader and the author, and is easily read. There are many exercises with answers and the book could be used as a class text. Starting from the first chapter "Consistent Sets of Beliefs", the book progresses to "Propositional Calculus", and "Predicate Logic".

The author's sense of humour adds a pleasant touch to many of the examples.

For example: A case where *if* should not be translated by '→' is:
The choir was sensitive, if a little strained.

The logic of likelihood, analysing sentences such as
"It'll probably be a girl. It'll be a boy",
is elegant and convincing, and meshes well with the mathematical theory of probability.

In any book containing a large amount of symbolism, special care needs to be taken not to overwork the symbols. Usually this is well handled but occasionally one has to reread a sentence, for example, page 99.

Note the third line, which distinguihes '↔' from '→'.'↔' is pronounced 'if and only if'.

B.B. Newman

THE PANCAKE PROBLEM

This has been around for some time. John Mack writing in the Australian Mathematical Society Gazette (December 1977) mentions it as having been contributed by Ivan Rose to the Sydney University Mathematical Society Competition, and as being in the American Mathematical Monthly (Vol. 84, page 296).

A stack of $n \geq 4$ pancakes sits on a plate. They are to be rearranged so that the smallest is at the top, the next smallest second, and so on down to the largest at the bottom. The only permitted move is to insert a slice in the stack, and to invert the pile above the slice, so that the pancake just above the slice now sits on top. What is the minimum number $f(n)$ of moves that is needed to achieve the final arrangement, irrespective of the initial configuration?

As the explicit answer seems elusive, we are asking readers for any improvement of the elementary inequality $n \leq f(n) \leq 2n - 4$.

A SPHERE IN A VISCOUS LIQUID

Suppose that a sphere moves at a small Reynolds number in a viscous liquid which is otherwise undisturbed. It was suggested (JCMN 12, the problem "Can you solve a quadratic equation?") that the external force on it must be $B + A\dot{x} + M\ddot{x}$ where x is the position vector. The term B was established by Archimedes and the term $A\dot{x}$ by Stokes, but has anyone ever found the right coefficient M for the third term of the series?

HISTORICAL NOTE

The 18th of January, when with luck this number of JCMN will be in the mail, is the 200th anniversay of Captain Cook's discovery of what he called the Sandwich Islands, now known as the Hawaiian Group. The fact that these islanders were related to the Tongans confirmed Cook's high opinion of Polynesian boats and seamanship. In fact we know now that Tongan double canoes used to make the 2000 mile journey between Tonga and Hawaii.

Your editor would like to hear from you anything connected with mathematics or with James Cook.

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