

JAMES COOK MATHEMATICAL NOTES

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A handwritten signature in cursive script, reading "James Cook". The signature is written in black ink and features a large, decorative flourish at the end of the name.

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ANDREW PAUL GUINAND (1912-1987)

Dr Paul Guinand (M.D., Berne, 1890) migrated from Switzerland and in 1892 was registered as a medical practitioner in South Australia. He practised medicine in Renmark for many years, and for a period at Murray Bridge. He married Mabel Mary Louise Wooldridge, and they had three children, Andrew Paul was the youngest, born in Renmark on 3rd. March, 1912.

Another son, John Munro, later became a vine-grower, and a daughter, Mimi Helen, became a teacher.

Andrew Paul Guinand (often known as Andy) went to school in Adelaide at St. Peter's College from 1924 to 1929. He was a member of St. Mark's College, University of Adelaide, from 1930 to 1933, graduating with first-class honours in mathematics (J.R.Wilton was the Professor of Mathematics there at that time) Of his recreations, it is recorded that he was a proficient gymnast, he rowed with the Torrens Rowing Club, and he competed in bicycle races with the South Australian Amateur Cycling Association.

He was awarded a Rhodes Scholarship to go in 1934 to New College, Oxford, where he worked for his D.Phil. degree under the supervision of E.C.Titchmarsh. One of his examiners was G.H.Hardy, and Andy in later years treasured a note from Hardy asking him to postpone his oral examination because he (Hardy) had been asked to play in a cricket match for the Trinity College Servants' team. Leaving Oxford, Guinand went with a New College Senior Scholarship and the Jessie Theresa Rowden Scholarship to Göttingen in 1937-38, and to Princeton, U.S.A. as a Commonwealth Fund Fellow in 1939-40. In 1940 he enlisted in the Royal Canadian Air Force, becoming a navigator and eventually being promoted to Squadron Leader. At one time he was stationed at R.A.F. Upavon in Wiltshire, and in his spare time he would cycle to Oxford (fifty miles each way) for his mathematical research. He served at the Aeroplane and Armament Experimental Station, Boscombe Down, from late 1943 until the end of the war.

After a spell as an Assistant at the Cambridge University Mathematical Laboratory, Andy went, as a Principal Lecturer, to the Royal Military College of Science, Shrivenham, England, in 1947. He later became Associate Professor of Mathematics.

It was in 1947, while at Shrivenham, that he married Olive Scott Webster, and they lived for a time in a converted war surplus landing craft on the River Thames at Lechlade. D.G.Kendall recalls helping to bring the boat up the river from London, and how Andy once had to dive for the battery which had fallen overboard.

Leaving in 1955, he returned to Australia as the Foundation Professor of Mathematics and Head of Department at the University of New England, Armidale, N.S.W. until 1957. In that year he moved to Canada to the University of Alberta, Edmonton (1957-60), then the University of Saskatchewan, Saskatoon (1960-64), and finally Trent University, Peterborough, Ontario, where he was the founding Chairman of the Department of Mathematics (1964-72). He retired formally in 1977, but continued to teach at Trent, first on a full-time basis and later on a part-time basis, for the next four years.

During the lengthy illness that preceded his death on March 22, 1987, he maintained his research, and it was during that time that he wrote his final paper, "Another view of prime number theory."

He is survived by his wife, Olive, a son Paul Scott (also a mathematician), and a daughter, Anne Louise (a theatrical lightning designer).

BINOMIAL IDENTITY 26

Is  $\binom{2n}{n} \binom{n}{r}^2 / \binom{2n}{2r}$  always an even integer? (Where  $0 \leq r \leq n < \infty$ ) This formula arises in the expression of the Legendre polynomial  $P_n(\cos \theta)$  as a trigonometric polynomial, which, according to Whittaker and Watson, goes back to Legendre himself.

JUBILEE INEQUALITY (JCMN 46, p.5092)  
Paul Penning

$$\frac{16}{JCMN + (J+1)(C+1)(M+1)(N+1) + 1 - JM - CN} \leq \frac{1}{(J+1)(C+1)M} + \frac{1}{(N+1)(J+1)C} + \frac{1}{(M+1)(N+1)J} + \frac{1}{(C+1)(M+1)N}$$

for any positive J, C, M and N, with equality only if J=C=M=N=1.

Proof: Multiply by  $(JCMN)^{\frac{1}{2}}$ , and denote the left and right hand sides of the resulting inequality by L and R respectively.

Introduce new variables j, c, m and n by  $J = \exp(2j)$ ,

$C = \exp(2c)$ ,  $M = \exp(2m)$  and  $N = \exp(2n)$ .

$$16/L = (JM-1)(CN-1)(JCMN)^{-\frac{1}{2}} + (J+1)(C+1)(M+1)(N+1)(JCMN)^{-\frac{1}{2}}$$

$$= 4 \operatorname{sh}(j+m) \operatorname{sh}(c+n) + 16 \operatorname{cosh} j \operatorname{cosh} c \operatorname{cosh} m \operatorname{cosh} n$$

$$4/L = (\operatorname{sh} j \operatorname{cosh} m + \operatorname{cosh} j \operatorname{sh} m)(\operatorname{sh} c \operatorname{cosh} n + \operatorname{cosh} c \operatorname{sh} n)$$

$$+ 4 \operatorname{cosh} j \operatorname{cosh} c \operatorname{cosh} m \operatorname{cosh} n$$

$$= \operatorname{cosh} j \operatorname{cosh} c \operatorname{cosh}(m+n) + \text{terms from cyclic permutation} \quad (1)$$

$$4R = \exp(n-m) / (\operatorname{cosh} j \operatorname{cosh} c) + \text{terms from cyclic permutation} \quad (2)$$

Now apply the A.M.-G.M. inequality to each of the two equations

(1) and (2) above. For any four positive numbers the arithmetic mean is greater than or equal to the geometric mean, with equality only when all four numbers are equal.

$1/L \geq \text{Geometric mean} \geq (\operatorname{cosh} j \operatorname{cosh} c \operatorname{cosh} m \operatorname{cosh} n)^{\frac{1}{2}}$  because the factors  $\operatorname{cosh}(m+n)$  etc. are all  $\geq 1$ .

Similarly, from (2),  $R \geq (\operatorname{cosh} j \operatorname{cosh} c \operatorname{cosh} m \operatorname{cosh} n)^{-\frac{1}{2}}$

Therefore  $R/L \geq 1$ , and  $R \geq L$ , the required inequality.

In the case of equality we must have  $\operatorname{cosh}(m+n)$ , etc. all = 1, and so  $m+n = n+j = j+c = c+m = 0$ , and  $j = m = -n = -c$ .

From the equality of the four terms in (2) it follows that  $n-m = j-n = c-j = m-c$ . This shows that  $j=c=m=n$ , and since  $m+n=0$ , all these four variables must be zero, and  $J=C=M=N=1$ .

BINOMIAL IDENTITY 27

$$\sum_{r=0}^n \binom{2r}{r} \binom{2n-2r}{n-r} = 2^{2n}$$

THE FULHAM MYSTERY

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It had been a hard winter but one punctuated by brief intervals that gave promise that spring was on the way only to disappoint, but now at last we had a good two weeks of lovely weather. Holmes and I were sitting on opposite sides of the cheerful fireplace in our rooms, and after a morning filing his newspaper clippings it seemed Holmes was for once in a mood to talk. It had always been my custom to keep a look out for these rare moments, for he could be very abrupt if I intruded upon his thoughts with any simple remarks that ordinary folk regard as polite conversation. Yet I was ever anxious to have him supply details from his own lips that might prove useful for these memoirs. More particularly because his was not by any means an easy character to try to describe, and I believed that his spontaneous remarks if remembered and recorded would cast more light for the reader upon his remarkable mentality than anything my imperfect judgment and more imperfect memory, as I had found, might manage.

But now he pleased me by opening up in an analytical kind of mood about some of his cases. "You know, Watson," said he, "you have often portrayed me as consumed with energy and activity in my investigations, and on the other hand you have to some extent aspersed dear brother Mycroft for his indolent method of just sitting in his club and thinking. Yet, when I look back on some of my cases, I see only too well that if I had thought about them more clearly at the outset, there might have been much less need for some of those reckless outings and dangers that I rushed you into at all hours of the day and night."

"But my dear Holmes," I protested at once, "it was my greatest pleasure to accompany you, danger or no, to enjoy the thrill of seeing you at work, and to find that you see clearly things that are not at all apparent to others. And then there was the joy of being in on your solutions of problems that seemed entirely inexplicable, and that were destined sometimes to become the talk of the whole country."

"There you go, Watson," said he chidingly, "and that is one of your great weaknesses. Now admit it, you came along on those occasions because you enjoyed it; you neglected your practice; you were like a schoolboy opting for a field-march or playing truant to get away from the dreariness of everyday routine. That is a bad bad motive, my dear Watson, but I suppose a very human one in this very dull world."

I must confess that for some reason I felt distinctly nettled by this coolly accurate analysis of my motives. True as I see it now to have been, there immediately began to flood into my mind thoughts of the privations I had suffered, and of cases I had relinquished to be available on missions that I knew might prove dangerous. I was about to burst into a catalogue of them when he held up a silencing hand, evidently reading my thoughts, and surprised me by saying "But you do realise, Watson, that my own conduct has on occasions been just as bad or even worse than yours. And I really have less excuse, for

it has often been sheer mental incapacity or even laziness that has sent me on some of those premature and foolhardy errands."

"Mental laziness!" I echoed, "Holmes, you can't be serious. You are the last person in the world I would accuse of that."

"I know, I know, Watson, but these are questions of degree. And by my standards, when I look back over some of my efforts, that is the conclusion I have come to in judging myself. To the gullible public you have presented me always as a thinker par excellence, instead of ruthlessly showing up all my mistakes and stumblings and making less of my slight successes. That is the aspect of your narrations that seems to me so reprehensible, their pandering to the vulgar taste. — No, no, no." he went on, "Do not demur, I assure you that I appreciate the better side of your motives and intentions too. But look at the thing rationally. Many of my solutions have been like the work of a mathematician who after months of struggling constructs a proof of something that takes page and pages to set down, only to find years later that if he could have relaxed properly and not allowed emotion and hurry to cloud his judgment, the result might have been made obvious on half a sheet of paper!"

These sentiments ran so counter to my opinion of him that I tried hard to think up some objection, but could only say lamely that I did not consider emotion altogether so improper a thing. But I had long since found that it was wellnigh impossible to take Holmes onto any line of thought to which he could not at once find a rational reply. And now he said, with his usual calm assurance, "But never deride emotion and enthusiasm, my dear Watson, for it is they that turn on the engine of the mind in the first place and give it energy. Without them no one would ever attain to the higher mathematics or to anything else. But understand, Watson, some of my own results, which I have spent days and even weeks arriving at, probably deserve to rank no higher in the end than what our mathematical friends would term a 'remark', quite undeserving of the excesses of adulation you have expended upon them."

Rather abashed by this pronouncement, I sat in silence for a time, often the surest way to lead Holmes on once he had got started, and after an interval he resumed. "Consider now, Watson, how the existence of the universe seems so mysterious. It is this apparent mystery that has inspired many of the religions, but great thinkers though the prophets may have been, they certainly came to their conclusions on quite insufficient evidence. Before Newton, no one knew for certain whether the Earth was round or flat, but once he had hit upon the law of gravitation it became obvious not only that it must be round, but more importantly just why it is round. To my way of thinking, Watson, the universe itself may be analogous to this, and once the laws of nature are sufficiently understood it will become obvious why there must be a universe."

"But surely," I protested, "only a god of some kind can have made the universe: there can be no escape from a first cause of this sort." I had recently been reading up some metaphysics, and although most of it was quite over my head this was the main conclusion of the work.

"Not so fast, Watson. You and I will very likely not live to see the solution. But that is no excuse for being so naive as to suppose that these earnest men of former generations,

men that had never heard of gravitation, nor of electromagnetics, could nevertheless guess the riddle of the universe. Absurdly impossible, Watson! We admire their struggling efforts, and their literary skill in persuasion, but let us not mistake these for true understanding. No, no, no! Watson. We have a long long way to go yet admittedly. Science and its methods, — my methods, Watson," and he thumped the arm of his chair with emphasis — "have still to come into their own, and there can be no chance of guessing the secret of the universe after taking just a few superficial glances at a tiny part of it."

"Are you saying, Holmes, that all the great religions are wrong?" I exclaimed, trying to slow down what seemed to me Holmes's far too facile manner of discussing such sacred and baffling matters.

Had not the doorbell rung at that moment, I cannot conceive where things might have ended, for Holmes's calmly analytical attitude towards questions that I had been brought up to regard with awe and reverence, and settled by far greater intellects than mine, might well have irritated me beyond control, with what result can only be conjectured. Be that as it may, perhaps providentially the bell did ring, and a minute later none other than our good friend Lestrade came into the room and brought us back to realities.

Holmes greeted him cordially. "Let Watson take your coat, Lestrade," said he, "while I get you something from the decanter that may cheer your spirits", and he waved him to the vacant chair between us directly facing the fire.

Lestrade was soon established, glass in hand, with toes towards the fire. But he seemed in no hurry to start any conversation, and neither Holmes nor I pressed any upon him. As I contemplated the pair of them, I could not help thinking of two ill-assorted prizefighters: the one a fleet-footed Carpenter of lightning speed, the other a heavy-footed unimaginative slogger, but capable of standing any amount of punishment before admitting defeat. On these occasions there was always some silent sparring between them, Lestrade not wishing to disclose anything too official, and Holmes not wishing to show any anxiety to learn confidential news from headquarters. When Lestrade visited it was usually because he was in difficulties over a case, yet just because of this he was often reluctant to come to the point and explain his problem. Holmes's interest was always to hear of anything troublesome, the more puzzling the better, so he bided his time, pulling slowly and evenly on his pipe, while Lestrade stroked his bristly chin and reflected.

There the three of us sat a good five minutes, when quite suddenly Lestrade commenced, "I suppose you've read about this Fulham flat business, Holmes?"

"Only the mere fact, reported in the press one day last week, of a man being found shot. But there were few details, as I remember, nothing out of the common beyond mention that the dead man was found by his own son."

"Well, there's a bit more than that, but not much, I'm afraid. Although there is nothing to connect her with the

crime, a woman is involved somehow who knew both the father and the son, and it was just below her flat that the man was found."

"And the problem is to find the person that did the shooting, I suppose", interjected Holmes. "What is the official theory, Lestrade?"

"Well, that's where we run into difficulty. Some fourth person or more must have been responsible. As far as we can find, none of the three people ever owned a gun, and I doubt if the woman or the seventeen-year-old boy ever even set eyes on one outside of a shop-window. No weapon could be found at the scene of the crime. The only thing we can suppose is that the father surprised an armed burglar lurking in or near the flat below, and that the intruder was forced to use his revolver to make good his escape — the father was an unusually powerful man. He had been in the building trade all his life and had worked his way up from an apprentice as a boy to owning his own small business."

"Had there been burglaries in that area?", asked Holmes, who was looking over his street-map of London as he spoke. "Several usually occur in quick succession and then they cease. But it doesn't seem an area that would attract burglars, the residents there are not of the wealthy class."

"None had been reported anywhere near that area up to the time. But there are some curious things that may or may not be significant. For instance, the father was wearing soft carpet-slippers, and had taken off his boots and left them in the flat below, which is in fact standing vacant at present."

"Very odd," agreed Holmes, "it may or may not be of importance, but it is a point to note. However, hadn't you better tell us the whole story from the beginning, and what you have done so far? I take it you haven't found the killer, or you would not be here?"

"No, we haven't. There were some possible suspects on our files, but none of them were anywhere near the district that night. But let me tell you what we know about it, little as it is."

He drew out a small notebook, and then after reflecting a moment he gave the following account of the matter, which I here reconstruct from my notes made at the time. It seemed that on the Wednesday evening just eight days before, a constable on his beat heard what he thought might be shots some distance away in the general direction of the river. He quickened his stride and took the first turn leading that way, and after only a minute or two he was met by a man who rushed up to him to say that two shots had rung out as he was passing by some flats near the other end of the road he had just left. The two of them then went together down this side street, the man indicated where the shots came from, and the constable began to examine the premises. There was a little dusk of evening light left, and there seemed nothing peculiar outside, but on trying the front door of a flat that was obviously unoccupied it was found to be unlocked, and by means of his torch he soon saw from equipment lying round the place that the place was being redecorated. Just inside the front door in an alcove were found a pair of heavy boots standing next to a small empty leather handbag.

It occurred to the constable that someone may have taken off the boots in order not to be heard, and that their owner might still be in the flat. But search soon made clear that no one was hidden anywhere, and the constable was about to leave by the front door when he thought he should check the back door. This too he found unlocked, and going out through it into a small paved area beyond he nearly tripped over the body of a man lying stretched at the foot of an outside iron stairway that led to the upper flats. By the light of his torch, the constable saw that the man had two serious wounds on the right side of his face, and although not dead was quite unconscious. The man was wearing soft slippers. As the victim was still alive, the constable hurried off to get a doctor and notify his superior officers.

Within a matter of fifteen minutes a doctor arrived and another officer returned with the constable. But the man was already past any aid, and had died without uttering a word. Further examination of the scene there and then revealed little in the failing light beyond the strange circumstance of the carpet-slippers on the feet of the deceased, except that he had a short length of lead pipe in the tail-pocket of his overcoat as if to serve as a possible weapon, and suggesting that he may even have expected some sort of attack on himself. But no gun could be found anywhere, either that night or next morning when the most intensive search of the whole area and premises was made. Whoever fired the gun had taken it with him, and everything pointed to a case of murder.

The occupants of the two adjacent flats happened not to be at home at the time of the shooting, but a light showed that the flat near the head of the outside stairway directly above the empty one was occupied, as it turned out by a middle-aged woman in her late forties named Ruth Brandon and a youth named Peter Graham just on seventeen. The boy told the officers that they had quite suddenly heard two loud explosions outside and below somewhere, and that it had terrified them so much that they had locked the door. But upon looking out of the window the boy had actually seen the dark figure of a man climbing quickly over the wall at the side of the garden. Beyond that he could tell them nothing more. The police then explained that a man had been shot and was now lying at the foot of the stairway, but they did not say that he was already dead. This news clearly shocked them both a great deal, particularly the woman, so much so that the officers thought it best that the woman remain in the flat while the boy alone accompanied them below to see if he recognised the man. He was unable to identify him by the light of the torch as anyone he knew, but the face was much damaged and disfigured by the shots.

"How soon did you yourself learn of the case?" interrupted Holmes.

"It was reported to my department routinely that same night, and I myself went down to the scene of the crime with our homicide men first thing next morning."

"So you must know all the details from then on first hand?"

"That is right. But they seem to throw no light on

things at all, and that's why I hoped they might suggest some line of investigation to you that we have not already tried."

"Do then go ahead, Lestrade," said Holmes inviting him to proceed. Lestrade explained that the woman was so upset at the thought of murder having been done at her very door that her doctor had to be sent for. He gave her a sedative, but as she showed the greatest fear of being left alone he sent for a nurse to spend the night at the flat. The boy stayed there too to help comfort her.

"When we got down there next morning, there was no difficulty in checking the boy's account of what he had seen. Along the side wall of the garden ran a flower-bed some eight feet wide, and there were footprints clearly visible crossing it to the point where the fugitive had been seen scaling the wall. In fact, as expected, there were two identical sets of prints, one going in each direction, showing that the man had gone by almost precisely the route he had come. Flower-beds in the adjacent garden over the wall showed the same double lines of footprints which ended finally at an outside wall bordering the main road that leads on down to the river bridge about a quarter of a mile distant.

"As no identification of the dead man could be made from the few items in his pockets, the boy was asked to come to the mortuary to see if in daylight he could help at all. It would have been useless taking the woman there, for she was in such a state of hysteria that we were not even able to question her. The boy, on the other hand, had shown commendable calm all along, until upon being shown the body again he broke down almost completely, and it was only with great difficulty that he was able to speak and tell us that the man was his own father.

"When we were able to resume questioning the lad, he said that he had no idea what his father could have been doing in the flat at such a time. It seems **the** man had been a widower for some years, and besides the boy there was a daughter quite recently married, now living in Wimbledon. The father had met Ruth Brandon a couple of years or so after his wife's death. It seems this Mrs Brandon had been on the stage in the earlier years of her life and had married. But she and her husband had separated a few years ago, some while before she met the elder Graham, but after the style of theatre people they had never troubled to obtain any formal divorce. By all accounts she had considerable musical talents and was well-read, and she had been taking an interest in both the girl's and the boy's studies for some years. The boy frequently visited her to receive music lessons, and the boy said that his father knew that he would be coming to see her that evening for a lesson.

"The Graham boy was straightforward and clear about everything we asked him, but he was unable to suggest any possible reason why his father should have been in the flat below that evening. Meanwhile, however, our officers had paid a visit to the deceased's house, and there they soon discovered that the dead man had had some sort of close friendship with Mrs Brandon over a period of several years. She had evidently welcomed his interest in her, and had shown considerable regard not only for him but also for his two children, which would be natural enough. Latterly, however, business difficulties had been so great as to unsettle Graham, and bankruptcy threatened him, with the result that he had

become morose and ill-humoured. Mrs Brandon on the other hand enjoyed gay and lively company, and for some time had been trying to discourage his visits, though she was quite willing to continue to see the boy and help him with his music and studies.

"The daughter, who visited her father from time to time, confirmed that this was to her knowledge the state of his relations with Mrs Brandon, and that it was causing him added unhappiness. As far as she knew, he had never thought of marrying Mrs Brandon. His business had been going downhill almost from the time he met Mrs Brandon if not before then, but she thought her father much too concerned with his many existing responsibilities to think of undertaking anything more. Some of the recent entries in his diary showed that the man was getting morbidly suspicious of the cause of Mrs Brandon's coolness towards him, and letters found at her flat also made this clear, for in some of these he had latterly begun accusing her indirectly of having some secret lover in her life. On this, however, we have questioned the neighbours closely, and there is not the slightest evidence that Mrs Brandon had any regular callers beyond Graham senior and the boy and his sister, though the girl only occasionally now. Nor did she go out a great deal, and nothing we came across at the flat gave any reason to think there was or is any other man in her life.

"Indeed, all the indications are that she still nurtured considerable affection for Graham despite his increasing gloominess, for when she was finally made to understand that it was the boy's father that had been killed, she threw herself down on the floor in a fit of convulsive sobbing, and eventually went into so dead a faint that the nurse had to send for the doctor again. He formed the opinion that her mind may have given way under the shock, and the woman has since been removed to hospital for observation. But it was already plain that she knew nothing of how the killing had come to be done and could in no way help us."

"What then was your next move?" enquired Holmes.

"Well, our standard procedure in such a case," said Lestrade. "With a thing like this happening at round nine in the evening, somebody somewhere here in the neighbourhood must have seen something. So we put on every plain-clothes man we could muster to question everyone in the area, but I regret to say with unbelievably little luck."

"You mean you found no one that heard or saw anything?"

"Well, not quite. A lot of people heard the shots. Some said they thought it was a vehicle backfiring, others said they heard something but didn't give it a second thought. No one came forward with any information. It's amazing how people seem to fear getting involved in anything."

"But the escaping man, Lestrade, did no one see him?"

"Well, at one point we did think we were getting a line on that, Mr Holmes, when we found two workmen who are regular patrons of the local inn, which is set back a bit on the main road about two hundred yards from the north end of the bridge. But I'm afraid that petered out into nothing in the end."

"Why, what did they see if it came to nothing?", Holmes pressed.

"It appeared that after leaving the inn they stood outside for a few minutes arguing about something. We questioned them separately, and one told us that he remembered seeing a man running rapidly and silently towards the bridge, but he thought nothing of it at the time, and he could not say to within half an hour just when this was, nor did he get any clear view of the person in the fading light.

"When we questioned his friend, he recalled nothing unusual at first, but when asked if any passers-by had attracted his attention, he said he did think he remembered seeing someone running swiftly away from the direction of the river. He said that he was quite sure of that, but he could give us no description of the man as the light was very poor and he really didn't pay any attention to the incident.

"So they did not even agree on the direction the man was making for. But as you know, Mr Holmes, it's almost impossible getting any worthwhile information from that class of person. Besides, on their own admission, they had been inside drinking ale for about two hours, and their accounts were so contradictory as to be quite useless. They may have seen one of the youths and boys that play in the streets in that area, but none we spoke to could help in the least. The man we want seems to have got clear away with no trace whatever for us to follow."

"Well, what about the burglar idea," queried Holmes, "have you tried to trace the man following up that theory?"

"Yes, we've done absolutely everything we can there. But as I said, none of our suspects were anywhere within miles that evening. We have information that an armed gang from the Midlands have recently been operating in wealthy suburbs of London, but always together, at least three there seem to be, but this isn't a district that would interest them. The man we are looking for could only be a petty thief operating alone, not a real professional."

"But would you expect such a one to carry a revolver?", put in Holmes. "They usually rely on flight if disturbed, they don't want to be seen. And they know armed robbery is regarded far more seriously by the law than mere petty theft."

"That's all very true. Mr Holmes, but gun there must have been. We can't get away from that, and whoever fired it obviously took it with him. My theory is that Graham was waiting there quietly expecting to intercept some secret admirer of the woman, and just by the merest accident this petty thief creeps stealthily up. Or it could even have been some quite innocent person at that stage exploring the empty flat and bumping into the waiting Graham. Can you suggest any other possible explanation?"

"Certainly not at the moment, Lestrade," confessed Holmes, "but innocent people looking over empty property, even at so unsuitable a time of day, would scarcely be likely to carry a gun, don't you think?"

I could see that Lestrade did not care for Holmes's seeming unhelpfulness, and he was almost testy in his reply. "Well, innocent or not, there was a gun, and a man was killed with it. That we do know. I think Graham attacked the man at once, and the only means of escape for him was to shoot."

"But the weapon Graham carried, the length of piping?

You say that was found in his coat-pocket. Don't you think that odd, if he had it to attack someone? Dying men do not put things back in their pockets."

"I know, I know," said Lestrade irritably, "that point had not escaped me. But he might just have attacked the intruder hand to hand, forgetting his weapon. Graham was a big strong man, and might easily have overpowered him without any blow being struck. Certainly nothing was heard by the boy other than the sudden shots. He was quite definite about that."

"It's very possible, Lestrade. I suppose in such circumstances a man would act more by instinct than by any thought-out plan. And if this marauder you seek were a weakling, Graham might simply have seized him with such strength that he in turn had to use his gun. That might well explain things, I suppose."

We sat in silence for a span, and then Holmes asked, "Well, what do you propose to do next, Lestrade, to get your man?"

"That's exactly it, Mr Holmes, just what can we do next?", and he waved his open hand towards Holmes in despair. "You see, at first we felt it must prove a simple case, but now we've reached a stage when we seem to have nothing whatever left to go on. We might catch the fellow by pure luck if it was a burglar and he does any more jobs. But he'll see from the papers that the man has died, and he's bound to lie low for months."

"I suppose none of your unofficial sources can offer any suggestions?"

"No, we've tried there of course, Mr Holmes. But they aren't even cagey about it, which is their usual style if they know anything. We've made it clear that we'll pay well for anything that helps to break this one, but even that hasn't produced a ripple anywhere. No one knows anything about it, that's clear."

All this while I had been occupied making brief notes. This had become my habit in recent years after many times regretting that I had trusted too much to memory for most of Holmes's earliest cases. It was Holmes himself who had more or less driven me to this practice, for he was highly critical of remarks attributed to him that were in any detail inaccurate or even ill-expressed. But I noticed now that, despite Lestrade's lengthy account of the matter, Holmes unlike his usual self had put very few questions, although I who knew him so well could see that he was keenly interested and absorbing every detail. It is because of this lack of Holmes's usual interrogations that I have seen fit to give most of Lestrade's narrative in the form of reported speech.

But now having listened attentively and made sure that Lestrade had told everything he had to tell, Holmes came directly to the point and asked him "Well, what would you like me to do?"

"What we obviously need, Mr Holmes, is some way to find the killer. But how? Is there some clue that we have overlooked?"

That certainly seemed to be the problem in a nutshell, and I began to think of the teeming millions of London as against the little information that had been collected after a whole week of work by officials with every resource and facility at their disposal. Was there the slightest possibility that my friend, much as I admired and have extolled his superlative



skill, could really do anything to help this time? How does one go about discovering a solitary individual, quite unknown, intent on effacing himself, possibly engaged on some daily routine as if nothing had happened to him, with those around him having no suspicion of his connection with the murder?

All three of us sat in silence thinking our various thoughts, Lestrade worriedly and expectantly glancing at Holmes every now and then for some feasible suggestion. But Holmes sat deep in thought, and seemed by no means in any hurry to say anything. I myself looked through my notes and went over the events in my mind, but I found it hopeless to think of anything that could be done. So many cases of killing and murder had been investigated down the years by the authorities, that the standard steps for solving them were all known, but here was a case where an entire stranger had suddenly come upon a private scene, as it were, and, perhaps mistaken by this frenzied man for an imaginary rival instead of a would-be criminal, was forced to commit a far greater crime than mere theft in order to make good his escape. Had he left the weapon behind, standard procedure would probably already have got him behind bars, but all he had left were some footprints, and to match those it would first be necessary to find the man. No wonder Lestrade and his men were baffled.

Suddenly Holmes spoke, "Watson, you have the address in your notes? The most I can do, Lestrade, is to go down there first thing tomorrow and look over the scene. Mind you," he said warningly, "I cannot promise anything of the least help. A whole week has gone by, and you come to me only when the trail has run cold."

"Yes I know, Mr Holmes, you're only too right" apologised Lestrade regretfully, "But no one thought it would be any trouble once we'd identified the dead man. How I wish I had called you in right away. One thing however you can rely on; apart from moving the body of course nothing has been touched inside or outside the flat. We've had a man on duty ever since, and no sightseers have been allowed beyond the pavement. So everything is still there for you to see for yourself."

"That's as may be," said Holmes rather icily. "Much can change in a week, and how do you know your man isn't on his way to Australia or Canada by now, and you expect me to scour London for him!"

"The ports are one of our specialties, Mr Holmes, and we hadn't overlooked that. All I want from you really", he went on imploringly, "is some brainwave that gives us something to work upon. The papers are beginning to ask questions. If you have any ideas at all, we'll do the necessary leg-work, believe me."

"I'm not concerned with questions in the press," said Holmes sourly. "Their only interest in crime is not to help to end it but to make use of it to sell their papers by sensationalism. First they want crimes to be committed, the more horrific the better, and after the chase they want the public exhibition of an Old Bailey trial with a man fighting for his life against a pack of legal dogs — all the ingredients of a human fox-hunt. By failing to catch your man, Lestrade, you are depriving them of this second round. That's what

they're really up in arms about."

"But Watson and I will get down there in the morning," went on Holmes, "so make sure your man expects us."

"Not today, Mr Holmes? There's no time to be lost."

"No, it will have to be tomorrow first thing, Lestrade. You need not trouble to be there yourself, I just want to look around undisturbed. And I don't want to take your time with a case that I see little or no possibility of my being able to help with now." Holmes was peremptory, almost bordering on incivility.

"Well, I can only apologise for not coming before, Mr Holmes. But if you were in the force you'd see my difficulty. If every case that came along I ran to you, what would my superiors say? And what would you say? There isn't one case in forty we can't solve by routine methods, but we can never tell that a real snorter has come along till we've failed."

"I'm not reproving you, Lestrade," said Holmes much more kindly, "What you say is very right, and I do understand your position. On the whole I do not think you have acted unwisely — there is a lot of experienced wisdom underlying the methods of officialdom."

"I'm relieved to hear you say that, Mr Holmes, though I often wish we could use your free-lance methods. We could take a lot of short-cuts."

"Very likely, but we mustn't start that. I have no one to answer to, and no job to lose. If I fail on a case, it is usually only after others have failed before me, but when I succeed, well ...". Holmes trailed off at this point because, as I well knew, it was only cases that others could not solve that really interested him, and as often as not he would hand his solutions over to the official force free, gratis and for nothing if he felt that failure would reflect adversely on their records.

Nevertheless it had often struck me that if Holmes's wish had really been for honours and celebrity and worldwide reputation he could scarcely have acted more effectively and judiciously to these ends than the way he did act. Who was it said that 'Modesty is the only sure bait when angling for praise'? By never claiming any merit for himself, he found others more than willing to accord him credit even beyond what he may have achieved, much as that was on most occasions, and there was almost no honour that Holmes could not have received had he shown the slightest inclination for such recognition. But the truth was that in his inner self he felt no need for the praise and approval of others. He himself was the most competent to appreciate his powers and skill, and to recognise his own failures, and he reserved the strongest criticisms for himself when he did fail, though this was now seldom indeed, perhaps for these very reasons. But his greatest joy was to succeed in unravelling a problem, and it added nothing to receive congratulations and commendations from lesser thinkers.

At last Lestrade took his leave, and Holmes and I sat in silence. Then with unexpected energy Holmes said, "Come Watson, we will not leave this thing till tomorrow after all. Let us make use of what is left of the afternoon". So saying, we got into our outdoor coats, and within minutes were bowling

westward in a cab. Holmes had told the driver to go to Fulham, and I assumed we were making for the scene of the crime, but as we neared this area with the river on our immediate left, Holmes began to peer ahead and suddenly called out, "This will do, right here, driver." We alighted, and I saw only a few hundred yards ahead the river bridge referred to by Lestrade, so the flat itself could only be a quarter of a mile further on to the north of the river. To my surprise, however, Holmes turned left onto the bridge itself, and about a third the way across we stopped to look down between the girders at the dark water of the Thames flowing steadily below us. Looking down the river, we could see the great mass of London to the northeast, and despite the years that had passed I recalled our chase through the Pool of London in the vain pursuit of Jonathan Small and his devilish companion.

Holmes then crossed the road, and again we looked down at the water. There were few craft this far up to be seen. If Holmes was looking for something, he did not confide to me what it was, but after a minute or so he looked carefully at his watch, and then said, "Right, Watson, let us go now and see exactly where this affair took place." Again Holmes acted somewhat unexpectedly, and instead of making for the flat turned left up the main street which I would have judged to run parallel to the one the flat must be in. We passed by an inn on the right side, which Holmes remarked must be where Lestrade's two workmen had sojourned, and after a few old houses a wall over six feet high flanked the footpath, just such a wall as he and I had scrambled over after our mission to the premises of the sinister Milverton years ago. "Four and a quarter minutes, Watson, from the bridge to here at fair walking pace. So a sprinter might well do it in under two minutes, don't you think?" As I had run the quarter myself in under a minute in my youth, I readily assented, though I could not see any relevance to the actual problem. Holmes examined the wall carefully inspecting parts of it with his lens before eventually turning on his heel and making back the way we had come. I felt sure that at last we were going to the actual flat, and two left turns quickly brought us to it. There at the front was Lestrade's man, who eyed us doubtfully for a moment, but then, recognising Holmes, seemed proud to be able to address him by name.

"Surely it's Mr Sherlock Holmes?", said he.

"Yes, officer, and this is my good friend Dr Watson. As I am sure you know, we have come down at Mr Lestrade's request."

"Well, I'm afraid there isn't much to see. But the place is open to you, and you're welcome to look round inside and in the back."

We entered the downstairs flat by the front door, and saw at once the decorators' equipment standing round. Holmes paid little attention to this, but stood looking at the small recess a few feet beyond the entrance. There were chalk marks

on the floor in the shape of feet, and also a rectangular area about eighteen inches by ten marked next to these. "Where the boots and hand-case were found, evidently," explained Holmes. We then passed on out through the rear door, and only

a few feet ahead of us was the iron stairway which led to a partially covered balcony running alongside the two upper flats. We halted at its foot for a time, and then Holmes stood in underneath it for a few moments at a point where it spiralled back on itself to ascend to the balcony. Anyone standing there in the dark would have been quite well concealed, it seemed to me. Next we ascended to the top of the stairway, and with the aid of his glass Holmes first examined the ribbed metal floorway, and then the door and windows of the flat straight ahead. "Nothing to be seen there," said he finally, clicking his lips and shaking his head as if to suggest there might have been a week ago.

"Now, Watson, just a look at the garden wall on this side," and we went down the stairway and across the small bit of lawn. Holmes paused at the edge of this, opposite a place where the wall sloped down from about six feet high near the flats to about four and a half feet running along the garden. "This low point, you see," said he "is obviously where anyone escaping would make for. And there you see the two lines of footprints, just as Lestrade described, running from the wall and towards it."

Even from where I stood on the lawn, the prints were as plain as could be in the soft clayey ground, and might have been made a few moments before. Some planks had been placed alongside them. Holmes stepped onto one of these delicately, and taking out his lens again squatted down and examined them very closely, looking at practically every one in turn. Then coming back on the lawn, he made towards the foot of the little garden, and after peering over the wall quite suddenly laid his two hands on it and vaulted easily over. "Come on, Watson," he bade me impishly, "you can still manage a wall this high, surely?" I had some doubts, but piqued by his words I did join him on his side mainly by rolling over the top and dusting up my coat. Already Holmes was down intently studying the footprints on this side, where the two sets were just as clearly to be seen. Then he moved over to the opposite wall, on the other side of which we had in fact stood in the street not long before. There again were the tell-tale tracks, ending of course at the wall, but this one was a good six feet high all along and topped off with a kind of curved coping.

It looked a formidable obstacle to me, but Holmes said, "Come, watch, Watson," and handing me his coat he backed off some ten paces from the wall, and then dashed at it at speed. Then he leapt, the sole of his left foot met the wall halfway up, his arms went over the top, followed by his right leg, and there he was in a moment sitting astride the top of the wall. Enjoying my astonishment, he smiled down at me and said, "It's not really difficult, Watson. Have you never seen a cat climb a wall like this? He hits it halfway up, and uses the foothold to do the rest — he 'jumps it in two jumps', Watson, like the man trying to catch the ferry. Animals, humans included, do it instinctively if they have to, just like a cat chased by a dog."

So saying, he dropped down lightly to the ground, and pointed to the grazed mark his foot had made halfway up the wall where it had slipped slightly as he had heaved himself up. "Now come and look here, Watson," and he led me back to where

the prints stopped at the wall. "There, do you not see the same sort of mark? Not so obvious as mine, so probably made by a lighter more athletic person, and perhaps more suitably shod for climbing."

"Well, come along Watson," he went on, "I think we have seen all there is to be seen." and we made our way back to the street by a path alongside the adjacent flat. Holmes said a brief word of departure to the constable, who seemed half to expect some sort of confidential statement from him, but Holmes vouchsafed no comment on his visit either to him or to me. We walked in silence until, nearing the bridge again, he said "Watson, suppose you go on back to Baker Street and hold the fort. I have a few things to do elsewhere, but should be back in time for supper." As he spoke, he waved down a cab and saw me into it and away. Looking back, I saw his tall spare figure cross the road and turn once more onto the bridge till soon a curve in the road took him out of my sight.

As the cab carried me eastward again, I found myself overcome with a feeling of futility about the whole outing. We had seen absolutely nothing beyond what Lestrade had already told us was to be seen, and Holmes's feat of climbing the wall seemed quite pointless, since we already knew that was the way the murderer had left. Being alone with my thoughts and away from my imperturbable companion made the absence of any progress seem even more disappointing. I was scarcely conscious of the journey back, and once indoors again felt almost impelled to toss my notes into the fire, so unlikely did I think it that I could ever recount this case as one of Holmes's successes. All we had managed to do was to verify the complete correctness of Lestrade's account of the matter, and as to this there never seemed any element of doubt anyway.

For the next two or three hours I moped round our rooms unable to settle to anything, glancing continuously at the clock which several times I thought had stopped, so frustrated and bored did I feel. After what seemed an age I heard the front door, and at last Holmes was back. In he came, his energetic self, but I could read nothing from his manner. Pouring himself a drink from the sideboard he said, "Won't you join me, Watson, and let us have a talk about this affair. I will tell you at the outset that I'm afraid there is no chance of my solving Lestrade's problem for him. But the difficulty is that I do not wish to tell him so in as many words."

"Holmes," I exclaimed in my astonishment, "this is quite unlike you. There have been few such occasions, it is true, but never before have I known you to have shown reluctance to admit defeat."

"Defeat, Watson? I said nothing of defeat! Lestrade has just not seen into this matter, and I suspect that you may be as perplexed as he and his staff are."

"I would not say I am perplexed, Holmes, for it is abundantly clear what must have occurred," I protested. "But surely you admit that it will be difficult now to catch up with this villain so ready with his gun?"

"Yes, that's exactly Lestrade's problem," he cried, "and not only difficult, but quite impossible I would say," he added with great emphasis. And then he went on as if changing the subject entirely, "Tell me, Watson," he asked, "did you ever

hear of Ockham's razor?"

The irrelevance of the question astounded me, and the seemingly gruesome reference suggested nothing beyond memories of Jack the Ripper, another case of some years before on which Holmes had not been able to give any help to the official force. So I replied rather coldly, "No, I cannot say I ever have, Holmes."

Holmes laughed lightly, reading my thoughts, "Oh, it has nothing to do with any crime, Watson. But then philosophy was never one of your interests. Let me tell you, Watson, Ockham is an obscure Surrey village, whence hailed a fourteenth century divine now known as William of Ockham. His fame is limited, I regret to say, mainly to students of philosophy and scientific method, when in fact he should rank in eminence with the greatest thinkers."

Had I not known Holmes so well, I might have begun to have qualms about his mental state, but experience had taught me that he had seemingly devious ways of coming round to his point. So I bore with him and looked at him for enlightenment.

"No, Watson, this William of Ockham did not achieve any notoriety. His razor was not the implement of a cut-throat, but an idealistic principle of scientific method, Watson, a dictum we should never lose sight of, certainly not in my profession." At this point, Holmes recited some Latin phrase as if regaling a multitude, and then in English he added "We must never introduce unnecessary hypotheses!" That, my dear Watson, was the precious pearl of wisdom perceived by Ockham, and handed down by him for the benefit of mankind. And I may add, Watson, that if I myself had observed it more I might have saved myself a lot of wild-goose chases, believe me."

"But Holmes," I asked, still struggling to surface, "how does all this relate to finding our man in the present problem?"

"Well, Watson, I've kept you in the dark long enough. I am sorry I have had to do so, but the truth is I regard this as a very ticklish case for reasons that you will soon come to understand."

"The unquestioned facts, as you know, are that a man has been shot to death outside premises where his own son and a close woman-friend happened to be. Beyond that, everything is pure conjecture as far as any explanation goes."

"But we know that a fourth person was involved," I interrupted, "did not the boy actually see him scaling the wall?"

"No, Watson, we do not know that," said he, accentuating the words. "If you think carefully, all we really know reliably is that the boy told the police that. And that is not at all the same thing, you must see."

"Tut, Holmes, would the boy lie on such an occasion? He did not even know it was his father who had been killed. Besides, there are the footprints coming and going," I added, feeling confident that Holmes could not get away from that.

"Watson," said he, waving a reproving finger at me, "are you arguing with me or with yourself about this? You see, I know what happened, and have the further advantage of having this knowledge confirmed by someone that was present at the shooting. Now what do you say?"

"Holmes, the more you say, the more bewildering it seems to become. Do you mean you have actually found the thief or

whoever it was?"

"There was no thief, Watson. There was no fourth person at all. That is exactly where Lestrade, and I suppose you too, went astray. That was the unnecessary hypothesis which the use of Ockham's razor would have cut out for you. As it is, Lestrade is at his wits' end because he is searching for a non-existent person."

A faint glimmering of light began to dawn in my mind. Could the woman and the boy have done the killing, I began to wonder. But where could they have got the gun? Then again, how could they know the man was even there that night? Why should they want to kill him? And there were still the inescapable footprints. These ideas and others coursed through my mind in rapid succession, till in sheer desperation I blurted out "But the footprints, Holmes, we saw them ourselves. Surely the person that made those did the killing?"

"Correct to some extent, Watson, or at any rate nearly so. You see it is this unnecessary notion of an additional unknown person that keeps on confusing your thinking. So long as you try to fit him into the picture, everything is difficult, contradictory and insoluble. Do you recall Lestrade's account of the business?"

"But of course, every word, Holmes."

"Then you will remember the two men outside the inn. The one thought he saw someone running towards the bridge, and the other was sure he saw someone running away from the bridge. Lestrade rejects this testimony because of its contradictory nature, and the men unreliable. But it is only contradictory if a fourth person is introduced. These men had no reason to conceal anything, so suppose both were right. Could they not at slightly different times have seen one and the same person running first towards the bridge and then back away from it? If they were standing about arguing, then it could happen that one chanced to see him as he was going one way, and the other as he was coming back. Not only very probable, Watson, but a certainty, once the matter is understood."

"But why should anyone run both towards the bridge and away from the bridge? Why not walk if he were on some wrongdoing? And what could it be to do with this tragedy?"

"The gun, Watson. Remember the missing gun. We have that for a real fact too. Where better to conceal the weapon than in the depths of that river?"

"The boy, Holmes?" I was simply flabbergasted. "You mean it was he that they saw?"

"Of course, Watson. Running swiftly, as he had to, to get back before anything was discovered."

"So you mean it was he that shot his father, and not some prowler at all?" I asked fearfully, as the awful possibility began to come to me.

"Not so fast, Watson, not so fast. If that were so, the boy would be in custody by now, I would have seen to that. Murder was in the air that night, but the killing that actually took place was all a terrible accident. You see, Watson, if you can once bring yourself to forget about any intruder, then immediately everything falls into place and has a natural explanation. Here is what actually happened." Holmes then at last opened out and gave the following account of the

affair.

"The father, you see," said he, "had become a dejected unsociable man through the ill-usage of the world. First he loses his wife, then his business gradually fails, then his sole friend in this woman begins to lose interest in him. Enough to upset most men. Possibly he had watched her flat on previous occasions, and seeing no stranger visit her the suspicion forms in his more or less crazed mind that perhaps his own son is the rival and has supplanted him in her affections. He determines to find out, and takes advantage of the empty flat to lurk there listening. When the light has failed sufficiently, he creeps stealthily up the staircase. Ordinary boots would be heard on the metal stairs, so he brings carpet-slippers in the small handbag, and a gun just in case. He peers in at the window, just about the time the youth is getting ready to leave. The woman makes some demonstration of affection, possibly as a farewell gesture, and seeing this so enrages the father as confirming all his worst suspicions that he bursts in on them. The outer door was not locked, for they had nothing to hide. Nevertheless he thinks quite otherwise, and angered to the point of madness hurls the wildest accusations at them both, but especially at the woman, and finally ends up screaming that he will kill her there and then. So saying, the father turns and rushes down to get the gun so purposefully that the boy realises what he has gone for, and quickly understands that he alone can possibly prevent murder being done. He tells the woman to lock and bolt the door, and then himself dashes down to try to reason with his father. They meet at the foot of the stair, but the father is quite beyond any reasoning, and gun in hand says he will shoot the boy too first if he has to. The boy bravely bars his way, for it is plain the father will carry out his threat, and by seizing the man's right arm in his two hands the youth succeeds in forcing the gun away from himself. In the struggle the trigger is pulled. The shot strikes the man's head but makes only a grazing wound, infuriating him still further if that were possible. The struggle continues, but a moment later a second shot enters much more deeply into the man's head, and he drops senseless to the ground."

"Now, Watson, what does this courageous boy do? With remarkable quickness of mind he sees a way out that may save everyone's good name in the matter. Pocketing the gun, he jumps the low wall, crosses the next garden, and scales the high wall into the road. Running at speed in light indoor shoes he makes for the bridge, hurls the gun into the centre of the river, and dashes back exactly the way he had gone. The whole trip would have taken three or four minutes at the very most, and there he was safely back in the upper flat by the time the constable found the body. And with great presence of mind he was ready with his story of seeing a fleeing suspect, realising that his own footprints would be there to confirm his story."

Holmes's account took my breath away. I was amazed at the utter simplicity of it now as he told it. "Incredible, Holmes," I cried with admiration, "and you saw all this as we went over the scene of the incident?"

"Certainly not, Watson, give me some credit. I perceived it as the most likely explanation while Lestrade was first telling us about it all. Ockham's razor was the key,

you understand. The apparently contradictory evidence of the two workmen, the few minutes available between the shots and the discovery of the body, the half-demented woman, even the footprints. Looked at from the right angle, everything fitted together without the need for another person, and left no loose ends."

"Why then," I asked, "did you trouble to go down to the flat at all? And why did you not tell Lestrade your solution, and let him take it from there?"

"Let me take your queries one at a time, Watson. The sole reason I went down was to examine the footprints. You recall that crude mistake you attributed to me in your account of the Priory School affair, when you wrote that I inferred the direction that the bicycle was going from the fact that the track of the rear wheel must cross that of the front wheel? No such inference can be made, Watson, if you will think about it. That was not how I settled the direction, your memory, Watson, played you quite false. But your idea works in the present case though, and I realised that almost certainly the two lines of footprints might cross, and one footprint going one way somewhere partially overlay another going the opposite way. Shoes are pointed, you see, Watson, so given any luck at all, a single coincidence of this kind would determine which line of footprints was made first, the ones coming or the ones going. This was obvious to me as soon as Lestrade mentioned the prints, and this was the only thing that needed to be checked to confirm my theory of the matter. Fortunately there had been no rain, the prints were almost as fresh as when made, and when I came to examine them there were no fewer than four or five instances where one footprint lay partly on top of another. And sure enough, Watson, they all showed that the return track back to the stairway was made second. I found this simply because I expected it. Lestrade and his men saw the lines of prints, but having fully accepted the boy's story they had no reason to think that anything more could be read from them. You see, Watson, it is only when you have a theory to guide you that you know what evidence to look for."

"Holmes, it's nothing short of amazing. And to think that I should have thought for a single moment that the problem baffled you! But you certainly gave every air of being completely perplexed."

"That wasn't difficult, Watson. I was perplexed, but not by the problem itself. Which brings me to your second point, 'Why didn't I tell Lestrade straightaway and save everyone trouble?'

"Well, as you now know, Watson, my solution, which I still had to check, meant that only these three people were involved. But the mere fact of the shooting did not enable me to decide their exact relationships and thence their motives, and until I understood these there was no reason for me to reveal anything, indeed quite the contrary. The boy dare not run away, and the woman was already in hospital.

"When I left you this afternoon, I went on down to see the boy himself in Wimbledon where he is staying with his sister, a natural enough thing after his father's death. As I expected, he turned out to be an exceptionally intelligent youth, Watson. The moment I mentioned my name, I could read

in his face that he knew the secret was no longer solely his own. I wasted no time, but put it to him squarely that I knew there was no fourth person involved, and that he must now for the first time tell exactly what had occurred without keeping anything whatever back. Otherwise, I made plain to him, I would take my leave and report my evidence to the authorities.

"He reflected only for a few moments, with my eyes looking straight into his, and then said very earnestly, 'But Mr Holmes, I did not shoot my father. It was an accident. I believe he was going to shoot me and then shoot Mrs Brandon.'

"'Very probably' said I severely 'but I do not want your pleadings or opinions. What I want to hear from you is a truthful account of what happened, with no trimmings.'

"He understood me thoroughly, Watson, and could see it was no use temporizing about anything now. And at that he launched into his story, and told it with the complete ring of truth exactly as I have repeated it to you.

"The father was evidently like a man possessed with rage, and the whole episode took place very quickly. Realising the dreadful situation created for them all, and the scandalous publicity that would be bound to result, the boy acted with extraordinary ingenuity, do you not agree, Watson? He immediately saw that there was just this one way that he might save everybody, even his poor father's name, such as it was."

"Save everybody, Holmes?" I cried, "but he has concealed evidence and lied to the police, and now that you know the truth, surely the law must take its course!"

"Good old Watson, splendid! Yes the law must take its course all right. But what course is it going to take? That is precisely what Lestrade came here to ask me, but you and I are not going to tell him, Watson, understand that."

"Not tell him, Holmes? But dare we do that?"

"Just ask yourself, Watson," said Holmes, lowering his voice with solemnity, "can we in fairness and decency do otherwise? Think what would happen if Lestrade were told. He must report it, the whole matter would become public knowledge, the newspapers would have the time of their lives wallowing in the possibilities conjured up. A father, a former actress, an attractive son; love, jealousy, what not, leading on to violence; all written up as luridly as possible. The boy might have to stand trial for murder despite being guiltless. And if such considerations as these do not influence you, Watson, just think of the poor ratepayer's bill that such a trial would involve. As far as I am concerned, the boy is not guilty, but the pitiful victim of a tragedy that might have happened to anyone. He is suffering enough, believe me, without my setting in train such further useless horrors as these."

"But what then will you tell Lestrade, Holmes?"

"I don't have to tell him anything. But I shall call in on him tomorrow and say that I am despondent about the whole case, and can see no special means beyond routine methods for seeking his man. And I shall mildly reprove him once more for delaying so long before referring to me. If he ever does have a flash of inspiration and see the answer, it will not be

because of anything I shall say."  
 "And what is to become of the boy, Holmes?" I asked, thoroughly persuaded of Holmes's wisdom in the matter.  
 "That has not escaped me, Watson. By not admitting everything at the outset, he broke the law of course. But otherwise he seems to have shown the highest character and selflessness, not to mention remarkable mental ingenuity. By acting as he did to try to spare others, he has placed himself in the greatest possible danger and jeopardy if it is ever discovered. Quite possibly, just because of these high qualities, the woman may have begun to regard him more as a man than as a boy, whether she realised it or not. At all events, it was the danger to him that really finished her, if you recall the successive stages, Watson. The shooting of a man who had grown irksome to her by some unknown, though very disturbing, was a shock she might have withstood, but when the man died and she realised later that the boy she cared for so much might have to stand trial for his life, it was that that destroyed her mental fibre completely. But I am afraid there is nothing we can do to help her but preserve our silence.

"I have told the boy that he should clear out of this country altogether in due time. You see, Watson, the associations here might well prove too much for him, and his only hope can be to start a new life in entirely fresh surroundings, and let time efface these unfortunate occurrences from his mind. He is young enough, and there are plenty of opportunities in Canada and Australia. After three or four months, the whole matter will have dropped into oblivion, and then he can quietly leave.

"But one final word to you, Watson. Dismiss the whole matter from your mind. Put your notes away in that strong-box of yours under lock and key, and do not think of using them for any of those memoirs of yours for a quarter of a century at very least. Those are my instructions, Watson.

"And come now, some supper." With that, Holmes sat down eagerly at the table and began looking under the lids of the various dishes. Thirty years have gone by since that day, but I have never heard him refer to the Fulham flat mystery again in any shape or form.

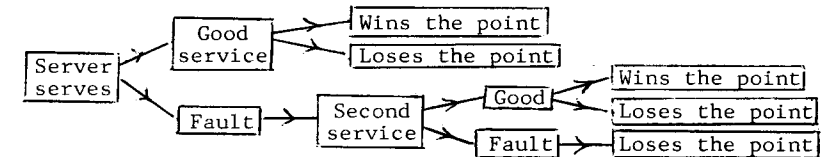
SHERLOCK HOLMES PROBLEM

R. A. Lyttleton

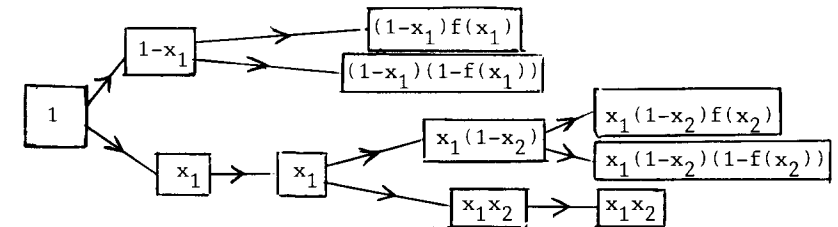
Recall "The Adventure of the Priory School", a short story first published in the Strand Magazine in 1904. In it is recounted how Sherlock Holmes examined the tracks made by a bicycle on Lower Gill Moor in the Peak District. He said to Dr Watson "This track, as you perceive, was made by a rider who was going from the direction of the school." How was Holmes able to tell the direction?

HOW TO SERVE AT TENNIS

In lawn tennis (and in some other games) the typical element of play, the point, can be represented as a path in the directed graph below.



A player does not have a single optimal way of serving. The server chooses from a one-parameter family of ways, for it is possible to trade accuracy (which prevents faults) against power (which if the service is not a fault makes the ultimate winning of the point more likely). Suppose that when the probability of the service being a fault is  $x$ , then the probability is  $f(x)$  of winning the point if the service is not a fault. The function  $f$  depends of course on the server, the opponent and various other circumstances. Clearly  $f(x)$  is an increasing function of  $x$  in the open unit interval. Suppose that the player uses a first service which has probability  $x_1$  of giving a fault, and a second service which gives a fault with probability  $x_2$ . Then the probabilities associated with the nodes of the graph above are as follows.

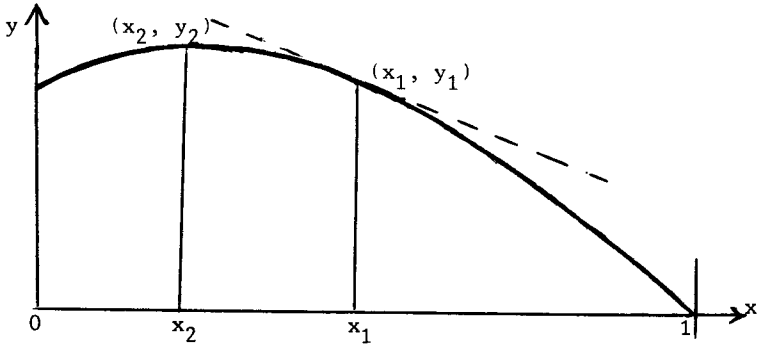


The probability that the server will win the point is therefore  $(1-x_1)f(x_1) + x_1(1-x_2)f(x_2)$ . How can the server choose  $x_1$  and  $x_2$  to maximize this expression? Consider the graph (now using the word in its other sense) of the function  $y = (1-x)f(x)$ . See the sketch on the next page. The parameter  $x_2$  for the second service must be the  $x$  that maximizes  $y$ . Now, having fixed  $x_2$  and  $y_2$ , how should one choose the parameter  $x_1$  for the

first service to maximize  $(1-x_1)f(x_1)+x_1(1-x_2)f(x_2) = y_1+x_1y_2$ ?

This is a simple case of linear programming, finding the support line of slope =  $-y_2$ . Clearly  $x_1 > x_2$ . This is the observed phenomenon that the first service of a good player is fiercer than the second.

By collecting statistics from a long match or series of matches between two players, it should be possible to estimate the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  and the tangent at the former, on the otherwise unknown curve.



For a numerical example suppose that  $f(x) = (2+5x-2x^2)/6$ . Then  $y = (2+3x-7x^2+2x^3)/6$  as shown in the graph above, with the maximum at  $x_2 = .2387, y_2 = .3907$ . The point where  $dy/dx = -y_2$  is at  $x_1 = .4808, y_1 = .3411$ . On this assumption the probability of the server winning the point is  $y_1+x_1y_2 = .5290$ , and the probability of a double fault is  $x_1x_2 = .1448$ . When the first service goes in, the probability of the server winning the point is  $f(x_1) = .6570$ .

### GEOMETRICAL IDENTITY

Mark Kisin

Let  $A_1, A_2, \dots, A_n$  be uniformly spaced round the unit circle, and let  $P$  be any point on the circle. Then, if  $m < n$

$$\sum_{r=1}^n (PA_r)^{2m} = \binom{2m}{m} n.$$