JAMES COOK MATHEMATICAL NOTES

Issue Number 5, October 1976: to celebrate the imminent return of Prof. B. C. Rennie from overseas study leave.

1. ANECDOTE ONE from Dame Mary Cartwright F.R.S.

"In my Final examination at Oxford in 1923 I answered a question on contour integration. After calculating residues, and presumably showing that the integral on a sequence of circles tending to infinity converged, I obtained the series

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

but the question said that it should be $\pi^2/8$. I knew (probably by old fashioned trigonometry) that $\Sigma(1/n^2) = \pi^2/6$, and so I thought that I was supposed to know that $\Sigma 1/(2n-1)^2 = \pi^2/8$. I checked that it was reasonable by calculating a few terms, and stated that it was. In fact it is easy to deduce from the first result, but in the stress of the examination room I saw no obvious connection. Did I deceive the examiners? What can have been the function to be integrated?"

2. ANECDOTE TWO from Dr. C. F. Moppert.

"In teaching second year economics students I dealt with the function $z = ax^2 + bxy + cy^2 + dx + ey + f$. I pointed out that this function has 6 parameters and that therefore one can give any 6 points P_ν (x_ν, y_ν, z_ν) $(\nu = 1,2,\ldots,6)$ and then determine the parameters in such a way that the above function goes through the points. Of course the situation $x_\nu = x_\mu, y_\nu = y_\mu$, $z_\nu \neq z_\mu$ is not allowed.

It came as a nasty surprise to me that further conditions are necessary for the solvability of the system $z_\nu = ax_\nu^2 + bx_\nu y_\nu + cy_\nu^2 + dx_\nu + ey_\nu + f$ in $a,b,\ldots,f$. My problem is then:

What is the geometric condition pertaining to the 6 points (x_ν, y_ν) equivalent to the statement that

$$\begin{vmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ \vdots & & & & & \\ x_6^2 & x_6y_6 & y_6^2 & x_6 & y_6 & 1 \end{vmatrix} = 0?$$

3. GUINAND'S THEOREM

An interesting comment from J. B. Parker, but for definiteness I can now quote the abstract from:

Guinand, A.P., Graves triads in the geometry of the triangle, J. of Geometry, _6_, 131-142(1975)

"A Graves triad is a cyclic triad of triangles, each circumscribing the next, thus forming a Pappus configuration. In the Euclidean geometry of the triangle the points of contact of the inscribed and escribed circles, the intersections of internal and external angle bisectors with opposite sides, and the feet of altitudes give rise to sixteen Graves triads, in four sets of four each. Each set of four is associated with a rectangular hyperbola through the vertices, orthocentre, in-centre, Gergonne point, and Nagel point, or their external analogues, and having its centre at a Feuerbach point."

And to put the reader right up-to-date, here is the abstract from a second paper submitted by Guinand:

"A Graves triad is a cyclic triad of triangles, each circumscribing the next, forming a Pappus configuration. If two of the triangles are in perspective, then so are the other two pairs. In a non-cyclic triad of such circumscribing triangles, if two pairs are in perspective, then so is the third pair. These results correspond to certain properties of 3×3 zero-diagonal matrices. There is also an analogous result for Möbius pairs of mutually circumscribing tetrahedra and 4×4 zero-diagonal matrices.

No-one has been able to generalization the theorem given in JCMN 3 to higher order matrices.

4. ANOTHER MATRIX PROBLEM

From JCMN 3:

If B is a square matrix and I the unit matrix, show that it is impossible to find a positive integer m and a non-zero vector $\underline{v}$ such that $B^m\underline{v} = (B+I)^m\underline{v} = 0$.

B.C.R. solution: For any positive integer r we have $B^r\,(B+I)^m\,\underline{v} = 0$ and omitting the mth and higher powers of B,

$$\sum_{s=0}^{m-r-1} \binom{m}{s} B^{r+s}\underline{v} = 0.$$

Putting $r = m-1$ gives $B^{m-1}\underline{v} = 0$, then putting $r = m-2$ gives $B^{m-2}\underline{v} = 0$ etc. Finally $\underline{v} = 0$.

5. GEOMETRY WITHOUT COMPASSES.
HAVE A GUESS.
THE AIRPORT WAITING ROOM.

All from JCMN 3 and all wanting comments.

6. Question VI from the University of Adelaide Pure Mathematics I examination in 1876 (see JCMN 4) provoked little reaction from JCMN readers but a howl from readers of the Adelaide Advertiser. The problem crept into this daily paper one Monday morning. Alas! the meticulous Editor modernized it - £28 became \$56, 24 (reputed) quarts became 27.3 litres and ale became beer! What's worse, the problem became insoluble!

The correct answer is that the bibulous character buys 8 dozen quarts of claret and 20 dozen quarts of ale, and if the price of ale rises, he drinks more ale. Full marks to D. B. C. Richards for a solution with simplex tableaus and pivots - bonus marks required the application of duality theory.

7. NEW THEOREM

Almost every convex polyhedron is a tetrahedron.

8. NEW QUESTION

If a real function $f(x)$ in an interval is such that the derivative $f'(x)$ exists and is bounded, does it follow that $(f'(x))^2$ is a derivative?

9. 100 STATEMENTS

On one page are written 100 statements:

"One of these statements is incorrect.
Two of these statements are incorrect.
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.
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One hundred of these statements are incorrect."

Which (if any) of the 100 statements is correct?

10. FINAL STATEMENT

Your guest editor signs off - all correspondence and contributions should again be sent to:

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8th October 1976 R. B. POTTS