Maths Education in Scotland

A Rough Guide

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Year Groups

		Age	England (US?)
PI	1	5	Reception/Year 1
	2	6	Year 1/2
	3	7	Year 2/3
	4	8	Year 3/4
	5	9	Year 4/5
	6	10	Year 5/6
	7	11	Year 6/7
SI	1	12	Year 7/8
	2	13	Year 8/9
	3	14	Year 9/10
	4	15	Year 10/11
	5	16	Year 11/12
	6	17	Year 12/13

Secondary School – Overview

		Age	What you do	Subjects
S	1	12	Nothing much!	Lots, no choice
	2	13		
	3	14	*In my day:* Standard Grade *Recently:* Intermediate 1 & 2 *Soon:* National 4 & 5	Choice of 8
	4	15		
	5	16	Higher	Choice of 5
	6	17	Advanced Higher	About 3

English Equivalents

Scotland	**≈ England**
Standard Grade, Intermediate 1 & 2, National 4 & 5	GCSE
Higher	AS Level
Advanced Higher	A Level

Maths Syllabi

Standard Grade

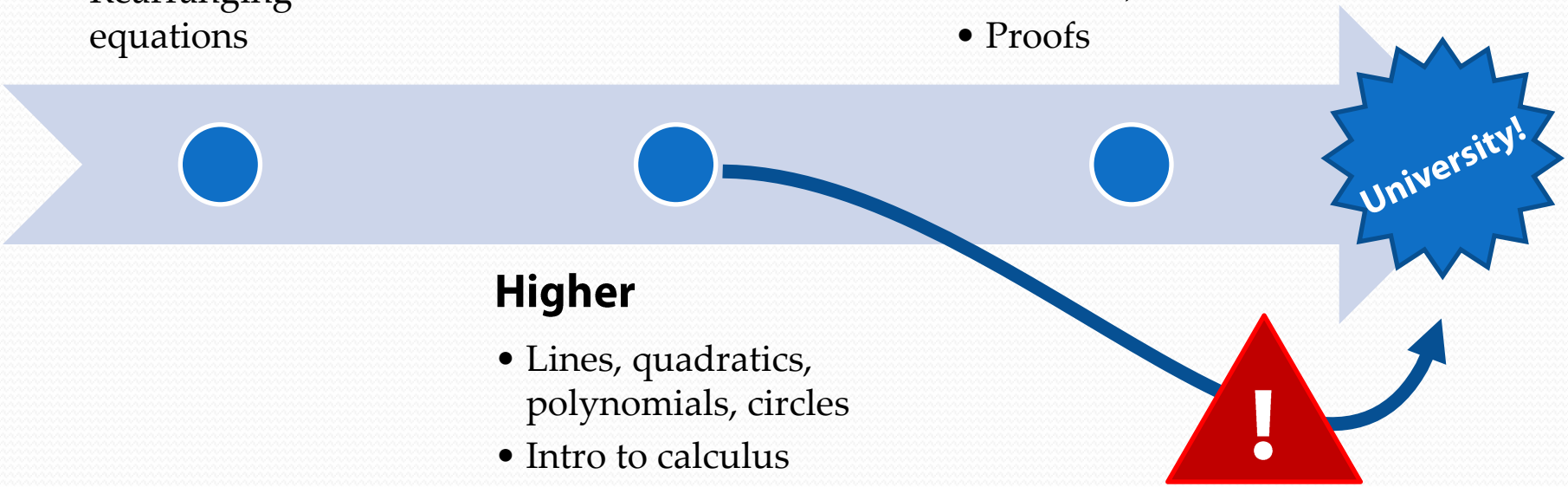
- Pythagoras
- Trigonometry
- Rearranging equations

Higher

- Lines, quadratics, polynomials, circles
- Intro to calculus
- Logs, exponentials
- Properties of graphs

Advanced Higher

- Lots more calculus
- Matrices, vectors
- Proofs



Standard Grade

- Number and money
- Relationships
- Measure and shape
- Statistics

SG: Number & Money

- Basic operations
- Fractions
- Square roots
- Indices & surds
- Percentages, e.g.
  - Interest
  - VAT

Evaluate

$(846 \div 30) - 1{\cdot}09.$

Evaluate

$4\frac{1}{3} - 1\frac{1}{2}$.

This year, Ben paid £260 for his car insurance.

This is an increase of 30% on last year's payment.

How much did Ben pay last year?

SG: Relationships

- Using formulae
- Solving equations
- Basic algebra
  - Factorising
  - Expanding brackets
  - Rearranging
- Graphs & tables
  - pictograms, bar charts, line graphs, pie charts and scattergraphs

Given that

$$f(x) = x^2 + 3,$$

(*a*) evaluate $f(-4)$

(*b*) find t when $f(t) = 52$.

A formula used to calculate the flow of water in a pipe is

$$f = \frac{kd^2}{20}.$$

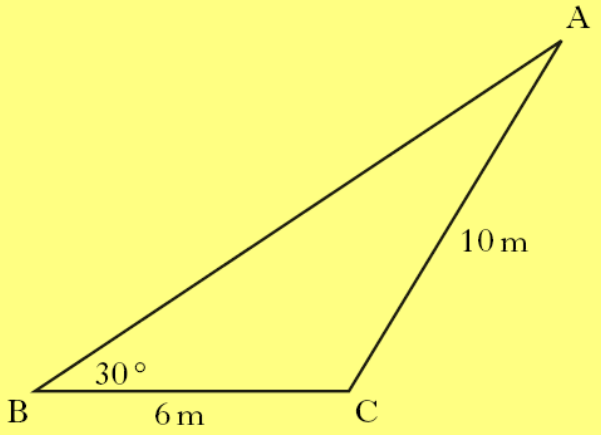
Change the subject of the formula to d.

SG: Measure and shape

- Scale drawings
- Similarity
- Find area & volume
- Angles
- Pythagoras
- Trigonometry
  - SOHCAHTOA (!)
  - Sine/cosine rule
  - Solve simple equations

In triangle ABC:

- BC = 6 metres
- AC = 10 metres
- angle ABC = 30 °.



Given that sin 30 ° = 0·5, show that sin A = 0·3.

SG: Statistics

- Calculate:
  - Mean
  - Mode
  - Median
  - Quartiles
  - Standard deviation
- Basic probability

Tom looked at the cost of 10 different flights to New York.

He calculated that the mean cost was £360 and the standard deviation was £74.

A tax of £12 is then added to each flight

Write down the new mean and standard deviation.

There are 4 girls and 14 boys in a class.

A child is chosen at random and is asked to roll a die, numbered 1 to 6.

Which of these is more likely?

A: the child is female.

OR

B: the child rolls a 5.

Justify your answer.

Higher

- Geometry
- Trigonometry
- Algebra
- Calculus

H: Geometry

- Straight lines
  - Gradient (from points/eqn/angle)
  - Find equation of line
  - Median, altitude, perpendicular bisector
- Circles
  - Equation to centre and radius (and vice versa)
  - Find intersections of line and circle
  - Find tangent / determine if a given line is tangent
- Vectors (2d and 3d)
  - Add, subtract, multiply by scalar
  - Dividing lines in a ratio / collinearity
  - Compute and work with scalar product (e.g. find angle)

H: Geometry (Examples)

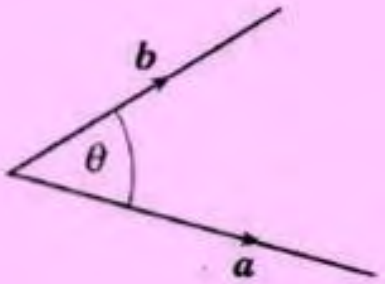
Find the equation of the line which passes through the point $(-1, 3)$ and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

(*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

(*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



H: Trigonometry

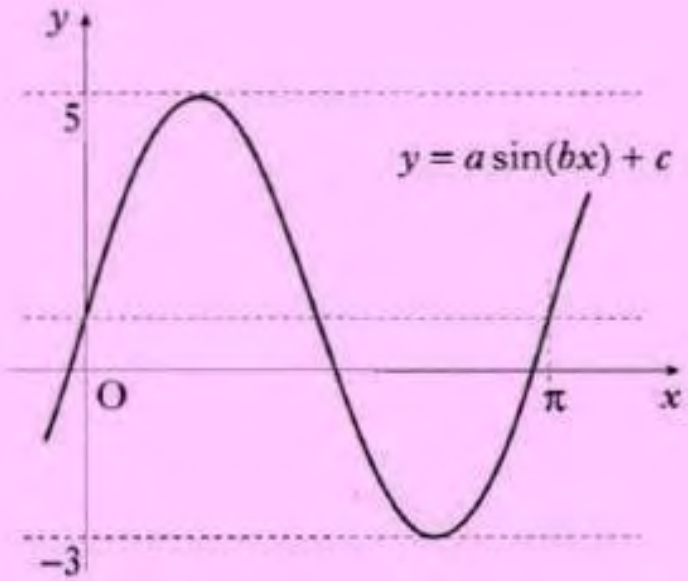
- Graphs
  - Recognise function from graph (and vice versa)
- Radians
  - Convert between degrees and radians
  - Mostly expected to work in radians
- Compound and double-angle formulae
  - Use to simplify equations
  - Use in geometrical situations
- Wave functions
  - Express $p\cos(x)+q\sin(x)$ in the form $k\cos(x-a)$ etc.

H: Trigonometry (Examples)

Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c. **3**



H: Algebra

- Functions
  - Domain/range
  - Finding composite functions
  - Graph transformations
- Sequences
  - Linear recurrence relations
- Quadratics & polynomials
  - Quadratic inequalities
  - Factor & remainder theorems for polynomials
- Logs & exponentials
  - Definition; working with graphs
  - Solving equations using log laws

H: Algebra (Examples)

Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

$f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

"Scottish division"

(*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

H: Calculus

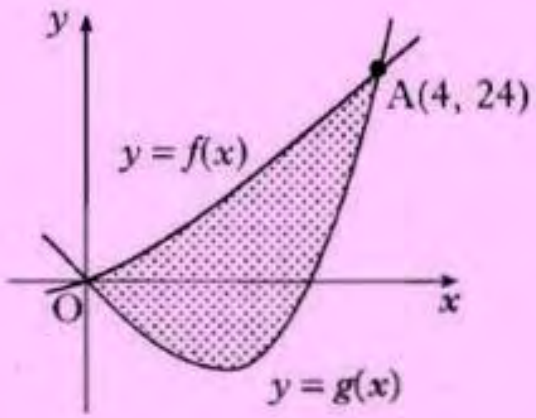
- Basic differentiation
  - Differentiating ax^b and sums/differences
  - Finding stationary points and their nature
  - Equations of tangents
- Basic integration
  - Indefinite integration (anti-diff)
  - Definite integrals; application to finding areas
  - Very basic DEs
- Further calculus
  - Diff/int with sine and cosine
  - Chain rule

H: Calculus (Examples)

Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves. **5**



If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

Advanced Higher

- Algebra
- Geometry
- Differentiation
- Integration

AH: Algebra

- Binomial theorem
- Partial fractions
- Gaussian elimination
- Matrices: algebra, determinants, inverses, as plane transformations
- Arithmetic/geometric sequences/series
- Complex numbers: basic operations, polar form, deMoivre
- Proof – induction, contradiction

AH: Geometry

- Vector product
- Equation of line:
  - Vector
  - Parametric
  - Symmetric
- Equation of plane:
  - Vector
  - Parametric
  - Cartesian
- Intersections of planes and lines

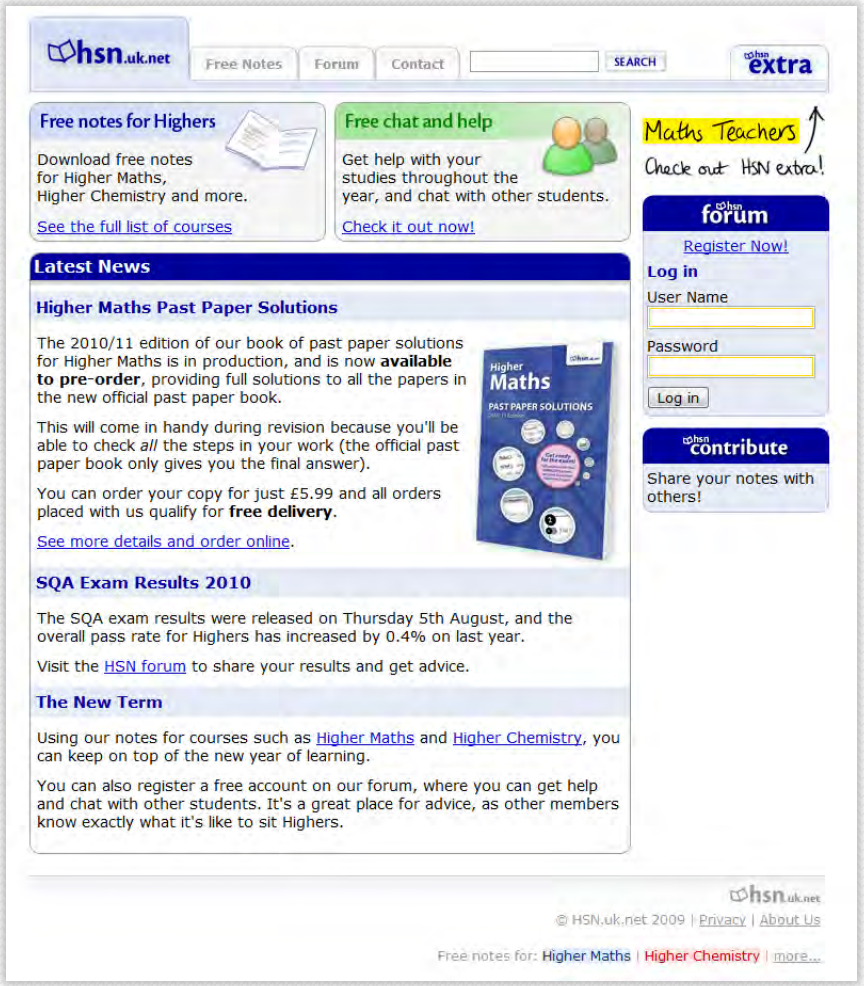
AH: Differentiation

- Curve sketching (e.g. rational functions, with asymptotes and critical points)
- Product/quotient/chain rule
- Derivatives of exp, log, arcsin, arccos, arctan
- Maclaurin series
- Implicit differentiation
- Parametric differentiation

AH: Integration

- Integrals corresponding to the known derivatives
- Substitution (generally given)
- Volumes of solids of revolution
- Integration by parts
- Separable DEs
- First & Second order linear DEs

Useful reference



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- Full course notes for Higher Maths
- Course summary for Advanced Higher

Thank you!