BFGS preconditioners for the normal equations arising in the Interior Point solution of constrained optimization problems

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We propose a class of deflated preconditioners to address the problem of efficiently solving the systems of the normal equations (NE) arising at each step of the interior point method in constrained optimization. As well known, the condition number of these SPD linear systems grows asymptotically as the sequence of the Newton iterates approaches the solution. Iterative solution of such linear systems, which is mandatory for large scale problems, requires the design of *ad hoc* preconditioners [4].

To this aim we will apply to a given preconditioner P_0, computed for the NE matrix at the initial Newton point, the BFGS-like low rank update as analyzed and tested in [1] for a general nonlinear system of equations as well as in [2, 3] for solving the sequence of nearly-singular linear systems to compute the leftmost eigenpairs of large SPD matrices by the Inexact Newton's method. The bounded deterioration property of the sequence of preconditioned matrices, as proved in [3] for singular or ill-conditioned Jacobians, is expected to mitigate the ill-conditioning of the systems of the normal equations towards the interior point solution of the optimization problem.

References

[1] L. Bergamaschi, R. Bru, and A. Martínez, *Low-rank update of preconditioners for the inexact Newton method with SPD jacobian*, Mathematical and Computer Modelling, 54 (2011), pp. 1863–1873.

[2] L. Bergamaschi and A. Martínez, *Parallel RFSAI-BFGS preconditioners for large symmetric eigenproblems*, J. Applied Mathematics, 2013, Article ID 767042, 10 pages (2013).

[3] ——, *Efficiently preconditioned inexact Newton methods for large symmetric eigenvalue problems*, Optimization Methods & Software, 30 (2015), pp. 301–322.

[4] J. Gondzio, *Matrix-free interior point method*, Computational Optimization and Applications, 51 (2012), pp. 457–480.

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