Linear programming on graphs through star sets and star complements

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Consider a graph G with vertex set $V(G)$ of cardinality n and adjacency matrix A_G. Let λ be an eigenvalue of A_G (simply called an eigenvalue of G) with multiplicity q. A vertex subset X with cardinality q such that λ is not an eigenvalue of $G - X$ is called a λ-*star set* of G, while $G - X$ is called a *star complement* forλ in G [3]. A (κ, τ)-regular set of G is a vertex subset S inducing a k-regular subgraph such that every vertex not in S has τ neighbors in it. A graph G has a (κ, τ)-regular set iff the linear system

$$(A_G - (\kappa - \tau)I)\, x = \tau \hat{e}, \tag{1}$$

where I is the identity matrix of order n and $\hat{e}$ is the all one n-vector, has a $0-1$ solution x. In such a case, this solution is the characteristic vector of a (κ, τ)-regular set. There are many combinatorial problems in graphs which are equivalent to the recognition of a (κ, τ)-regular set. For instance, a p-regular graph of order n is strongly regular with parameters (n, p, e, f) iff for every vertex v its neighborhood is (e, f)-regular in $G - v$ [2], a graph is Hamiltonian iff its line graph has a $(2, 4)$-regular set inducing a connected graph [2], etc. When $\kappa - \tau$ is not an eigenvalue of G to decide whether G has a (κ, τ)-regular set is easy, otherwise this problem could be very hard. However, assuming that $\kappa - \tau$ is an eigenvalue of G and $X \subset V(G)$ is a $(\kappa - \tau)$-star set, the linear system (1) can be replaced by the reduced system formed by the $n - |X|$ equations corresponding to the vertices in $V(G) \setminus X$. Furthermore, the subset of columns of this reduced system indexed by the vertex subset Y defines a basic matrix iff $V(G) \setminus Y$ is a $(\kappa - \tau)$-star set [1]. Based on these results, we are able to produce a sequence of $(\kappa - \tau)$-star sets towards to a $(\kappa - \tau)$-star set Y^* such that the corresponding star complement has a (κ, τ)-regular set. A combinatorial simplex like technique is then introduced for deciding whether a (κ, τ)-regular set there exists and several related results are presented.

References

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