Abstract:

Copositivity tests based on the Linear Complementarity Problem

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A symmetric $n \times n$ matrix is called copositive if it generates a quadratic form which takes no negative values on the nonnegative orthant. The problem of minimizing a linear form over the copositive cone, i. e. over the cone of copositive matrices, is called a copositive optimization problem. This type of optimization problems, and thus the possibility to evaluate whether a matric is copositive, is of interest due to its relation to combinatorial optimization. For instance the Maximum Clique Problem can be re-formulated as a copositive optimization problem. It was even shown that every quadratic program with linear constraints can be formulated as a copositive program, also if some of the variables are binary.

Various authors have proposed copositivity tests in the literature, but there are only a few implemented numerical algorithms which apply to general symmetric matrices without any structural assumptions or dimensional restrictions and which are not just recursive, i.e., do not rely on information taken from all principal submatrices. In this talk we present new copositivity tests which are of this type.

The test are based on new necessary and sufficient conditions requiring the solution of linear complementarity problems (LCP). Methodologies involving Lemke's method, an enumerative algorithm and a linear mixed-integer programming formulation are proposed to solve the required LCPs. A new necessary condition for (strict) copositivity based on solving a Linear Program (LP) is also discussed, which can be used as a preprocessing step. The algorithms are applied to test matrices from the literature and to max-clique instances with matrices up to dimension 496×496. We compare the results with those from three other copositivity tests from the literature as well as with a general global optimization solver. The numerical results are very promising and equally good and in many cases better than the results reported elsewhere.

This is a joined work with Carmo Brás (Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Portugal) and Joaquim Júdice (Instituto de Telecomunicações, Portugal).

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