Minimization in Banach spaces by Conjugate Gradient method

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In many inverse problems, the "geometry" of the L^p Banach spaces, for $1 < p < 2$, can substantially reduce the over-smoothness effects with respect to any conventional iterative regularization algorithm in the L^2 Hilbert space [1].
In this work we introduce a generalization of the conjugate gradient method for the minimization of the p-norm cost functional $\Phi(x) = ||Ax - y||_Y^p$, related to the solution of the ill-posed operator equation $Ax = y$, where $A : X \to Y$ is a linear operator between two Banach spaces. We first prove the convergence of the iterations to a solution of the operator equation in both noise-free and noisy data cases [2]. Then we show that the high convergence speed of conventional conjugate gradient in L^2 Hilbert space gives rise to a fast iterative method in L^p spaces too.
The minimization algorithm is applied to enhance the spatial resolution of microwave radiometer data [3]. The problem which describes the relationship between the coarse but partially correlated measurements and the brightness temperature is ill-posed problem since it belongs to the class of Fredholm integral equation of the first kind.

References

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