Augmented Lagrangians, non-negative QP and extensions

Roger Fletcher *(University of Dundee)*

Augmented Lagrangians are not commonly used to solve quadratic programming (QP) problems with inequality constraints due to second derivative discontinuities in both the inner and outer iterations. For non-negative QP (that is the only constraints on the variables are $x \geq 0$) a transformation of variables is described that removes these discontinuities from the inner iteration which can then be solved simply as a system of linear equations. The outer iteration objective function retains second derivative discontinuities but under mild assumptions is strictly convex, and can, for example, be minimized quickly and efficiently by Newton's method with a suitable line search. The method avoids the combinatorial difficulties associated with active set methods and convergence is guaranteed. Extension of these ideas to include m equality constraints on the variables is described. These are particularly effective in the case $m = 1$. Inverse problems provide a rich source of these problem types, and numerical examples of these are described.