Memory-efficient interior point method for solving a time-dependent scheduling problem

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We propose a new algorithm, based on the interior point method [3], for approximate solving the following open time-dependent scheduling problem (P), [1,4]. A set of n independent non-preemptable jobs, $J_1, J_2, \dots, J_n$, has to be scheduled on a single machine. The processing time p_j of J_j linearly deteriorates in time, i.e. $p_j = 1 + \alpha_j t$, where deterioration rate $\alpha_j > 0$ and the job starting time $t \geq 0$ for $1 \leq j \leq n$. The aim is to find a schedule with minimal sum $\sum_{j=1}^{n} C_{[j]}$, where $C_{[j]}$ is the completion time of the jth job in the schedule.

The main idea of our algorithm is as follows. Let $C(a)$ be the vector of job completion times, $A(a)$ be an $n \times n$ square matrix with $1's$ on the main diagonal, components $a_j = 1+\alpha_j$ of the sequence a multiplied by -1 below the main diagonal and equal to 0 otherwise, and $d = (1, 1, \dots, 1)^{\intercal}$, [1]. Then problem (P) is min $W_P(a) := \|C(a)\|_1$ subject to $A(a)C(a) = d$, where the minimization of $W_P(a)$ is taken over all a from the set $\mathcal{P}(a^\circ)$ of all permutations of an initial sequence a° of deterioration coefficients $a_j = 1 + \alpha_j$ for $1 \leq j \leq n$.

Since any optimal solution to (P) must be a V-shaped sequence, we consider the polyhedron of all 2^n such V-sequences for a given a°. Next, we attach to each vertex of this polyhedron a permutation matrix of a special kind and consider the convex hull of these permutation matrices, which coincides with n-dimensional polyhedron of all doubly stochastic matrices of a special kind. The convex polyhedron, in turn, we use in a new formulation of problem (P), to which we apply the primal-dual interior point method.

In order to make our algorithm computationally more efficient compared to its predecessor [2], we propose to replace in interior point method the Newton method by an another method, preserving the same size of matrices but without using the Hessian inverse. We also propose to reduce memory usage by applying a steepest descent method to the goal function with the barrier and the barrier and penalty components added, respectively. We illustrate the presentation of the proposed algorithm by results of extensive numerical experiments.

References

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