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Title: Radii of robust feasibility and robust optimality for uncertain convex programs

Abstract

In this talk we suppose that the data (the objective and the constraint functions) of a given *nominal* scalar convex program

$$\overline{P}: \ \min \left\{ \overline{f}(x) : \overline{g}_j(x) \leq 0, \, j \in 1, \ldots, m \right\},$$

may suffer linear and affine perturbations, respectively, giving rise to perturbed functions of the form

$$f(x) = \overline{f}(x) + c^\top x, \text{ with } c \in \gamma \mathbb{B}_n,$$

and

$$g_j(x) = \overline{g}_j(x) + a_j^\top x + b_j, \text{ with } (a_j, b_j) \in \alpha_j \mathbb{B}_{n+1}, j \in 1, \ldots, m,$$

where $\gamma, \alpha_1, ..., \alpha_m$ are non-negative parameters to be chosen by the decision maker (the closed balls $\gamma\mathbb{B}_n, \alpha_1\mathbb{B}_{n+1}, ..., \alpha_1\mathbb{B}_{n+1}$ are called *uncertainty sets*).

If $\alpha_1, ..., \alpha_m$ are taken too large, the *robust feasible set*

$$X(\alpha_1, ..., \alpha_m) := \left\{ x \in \mathbb{R}^n : \overline{g}_j(x) + a_j^\top x + b_j \leq 0, \forall (a_j, b_j) \in \alpha_j \mathbb{B}_{n+1}, j = 1, \ldots, m \right\}$$

could be empty. In order to guarantee the feasibility under any possible perturbation we define the *radius of robust feasibility* as the non-negative number

$$\sup \left\{ \min\{\alpha_1, ..., \alpha_m\} : X(\alpha_1, ..., \alpha_m) \neq \emptyset \right\}.$$

Now assume that $\alpha_1, ..., \alpha_m$ have been chosen so that $X(\alpha_1, ..., \alpha_m) \neq \emptyset$ and let $\gamma > 0$ be given. A robust feasible solution $\overline{x}$ is called *highly robust optimal solution* when it is an optimal solution of the parametric convex problem

$$P(c): \ \min \left\{ \overline{f}(x) + c^\top x : x \in X(\alpha_1, ..., \alpha_m) \right\}$$

for any $c \in \gamma\mathbb{B}_n$, which implies that $\overline{x}$ is the unique optimal solution to

$$\min \left\{ \overline{f}(x) : x \in X(\alpha_1, ..., \alpha_m) \right\}.$$

In order to guarantee the existence of highly robust optimal solution we introduce the *radius of optimality* as

$$\sup \left\{ \gamma \in \mathbb{R}_+ : \overline{x} \in \operatorname{argmin} P(c) \ \ \forall c \in \gamma \mathbb{B}_n \right\}.$$

In [1], we give exact formulas for the radii of robust feasibility and robust optimality of uncertain convex programs.

References

[1] M.A. Goberna, V. Jeyakumar, G. Li, N. Lin, Radius of Robust Feasibility and Optimality for Uncertain Convex, Preprint.