Advances in some fractional variational problems with Caputo derivatives

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Derivatives and integrals of fractional order have recently gained more attention due to their successful application to non local phenomena. Motivated by numerous applications in physics and other scientific areas, fractional calculus of variations finds itself in fast development.
In this work we consider variational problems of the following kind: find $y \in {}_a^\alpha E_b^\beta$, so that it maximizes or minimizes the functional

$$J(y) = \int_a^b L(x, y, {}_a^C D_x^\alpha y, {}_x^C D_b^\beta y)\, dx$$

where L is a Langrangian function, ${}_a^C D_x^\alpha y$, ${}_x^C D_b^\beta y$ denote the left and right fractional derivatives in Caputo's sense of order $\alpha, \beta \in (0,1)$ and

$${}_a^\alpha E_b^\beta = \left\{ y : [a,b] \to \mathbb{R} : {}_a^C D_x^\alpha y\,,\ {}_x^C D_b^\beta y \text{ exist and are continuous on } [a,b] \right\}$$

Different necessary and sufficient conditions of optimality are considered, particularly some of fractional Euler-Lagrange equation type:

$$\frac{\partial L}{\partial y} + {}_x D_b^\alpha \frac{\partial L}{\partial\, {}_a^C D_x^\alpha y} + {}_a D_x^\beta \frac{\partial L}{\partial\, {}_x^C D_b^\beta y} = 0.$$

Several fractional variational problems are presented: with fixed or free boundary conditions, in presence of integral constraints that also depend on Caputo derivatives and some examples are given.

References

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