Simplicial branch and bound based on the upper fitting, longest edge bisection *

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Abstract. Simplicial partitions to divide a bounded area in branch and bound makes the use of an upper fitting appropriate for finding the bounds on the subsets. Bisecting the longest edge avoiding needle-shaped simplices leads to a choice of which longest edge to bisect in higher dimensions. We investigate the behaviour of the search and the resulting binary search tree depending on the selection rule for the longest edge.
Keywords: branching rule, upper fitting, simplices.

1 Problem formulation

We focus on the multidimensional box-constrained GO problem. The goal is to find at least one global minimum point x^* of

$$f^* = f(x^*) = \min_{x \in X} f(x), \tag{1}$$

where the feasible area $X \subset \mathbb{R}^n$ is a nonempty box-constrained area. Given a global minimum point x^*, let scalar K be such that

$$K \geq \max_{x \in X} \frac{|f(x) - f^*|}{\|x - x^*\|}. \tag{2}$$

The function $f^* + K\|x - x^*\|$ is an upper fitting according to [1] for an arbitrary $x \in X$. Consider a set of evaluated points $x_i \in X$ with function values $f_i = f(x_i)$, then the area below $\varphi(x) = \max_i\{f_i - K\|x - x_i\|\}$ cannot contain the global minimum (x^*, f^*). Let $f^U = \min_i f_i$ be the best function value of all evaluated points, i.e., an upper bound of f^*. Then the area $\{x \in X : \varphi(x) > f^U\}$ cannot contain the global minimum point x^*.

We investigate the resulting binary tree for several selection rules for the longest edge in simplicial branch and bound. Notice that only in dimensions higher than three, there is a choice.

References

1. Baritompa, W.: Customizing methods for global optimization, a geometric viewpoint. Journal of Global Optimization 3(2), 193–212 (1993)

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