Tropical Optimization Problems: Solution Methods and Application Examples

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Optimization problems constitute a notable research and application area in tropical mathematics focused on the development of new solutions to known and emerging problems in operations research and other fields. Tropical mathematics, which is concerned with the theory and applications of semirings with idempotent addition, dates back to the pioneering studies of R. A. Cuninghame-Green, B. Giffler, A. J. Hoffman, S. N. N. Pandit, N. N. Vorob'ev, and I. V. Romanovskiĭ published in early 1960s.

In the next decades, optimization problems that can be formulated and solved in the framework of tropical mathematics were investigated in many publications, among which are monographs by R. A. Cuninghame-Green, U. Zimmermann , P. Butkovič, and a number of research papers. For some problems, complete direct solutions were obtained in a closed form under fairly general assumptions. Other problems are known to have only algorithmic solutions in the form of an iterative computational procedure to provide a particular solution, if any, or to show that no solution exists, otherwise.

We consider optimization problems in the general setting of tropical mathematics to minimize nonlinear functions that are defined on vectors over an idempotent semifield (semiring with multiplicative inverses) by using a multiplicative conjugate transposition operator. Both unconstrained problems and problems with constraints given by vector equations and inequalities are examined. To solve the problems, we use techniques based on the derivation of lower bounds on the objective function and on the solution of linear vector equations and inequalities. Under fairly general conditions, new direct solutions are given in a unified closed vector form. For many problems, the solutions obtained are complete solutions. The calculation of the solutions involves simple matrix-vector computations, which offers ease of implementation and provides low computational complexity. We discuss applications to real-world problems in various fields, including location analysis, activity scheduling, and decision making, and present examples.