H_∞ control synthesis under structural constraints based on Global Optimization

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Controlling an autonomous vehicle or a robot requires the synthesis of control laws for steering and guiding. To generate efficient control laws, a lot of specifications, constraints and requirements have been translated into norm constraints and then into an constraint feasibility problem. This problem has been solved, sometimes with relaxations, using numerical methods based on LMI (Linear Matrix Inequalities) or SDP (Semi Definite Program). The main limitation of these approaches are the complexity of the controller for implementation in an embedded system. But, if a physical structure is imposed to the law control in order to make easier the implementation, the synthesis of this robust control law is much more complex. And this complexity has been identified as a key issue for several years. A efficient first approach was given by Apkarian and Noll based on local non-smooth optimization.

In this talk, we will present a new approach based on **global optimization** in order to generate **robust control laws**. This new global optimization algorithm is based on interval arithmetic and contractor programming. Contractor Programming is a methodology which allows to enclose each algorithm in a unify framework, in order to interact heterogeneous formulations or techniques. It is a set-membership method, considering sets in place of floating points. Using Contractor Programming, we will show a user-friendly way to solve problems with non-smooth functions and constraints with quantifiers. The main idea consists in constructing a contractor based on the feasible region of the optimization problem and its complementary. This contractor is then used to accelerate the convergence of a Branch and Bound algorithm.

We will illustrate this new approach on a example on the control of a periodic second order system G with a PID controller K subject to two frequency constraints on the error e and on the command u of the closed loop system. The objective is to find $k = (k_p, k_i, k_d)$ minimizing the H_∞ norm of the controlled system.

$$G(s) = \frac{k\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad K(s) = k_p + \frac{k_i}{s} + k_d s.$$

The feasible region $\mathbb{K}_{in}$ of our global optimization problem have the following form, with Re_1 and Im_1 the real and imaginary part of the transfer function C_1 corresponding to the first constraint on the error, and Re_2 and Im_2 of C_2 corresponding to the second constraint on the command. The objective function consists to minimizing γ.

$$\begin{gathered}\mathbb{K}_1 = \{(k,\gamma)\ :\ \|C_1\,(G\star K)\|_\infty \leq \gamma\} = \{(k,\gamma)\ :\ \forall\omega, \sqrt{Re_1^2(k,\omega) + Im_1^2(k,\omega)} \leq \gamma\},\\ \mathbb{K}_2 = \{(k,\gamma)\ :\ \|C_2\,(G\star K)\|_\infty \leq \gamma\} = \{(k,\gamma)\ :\ \forall\omega, \sqrt{Re_2^2(k,\omega) + Im_2^2(k,\omega)} \leq \gamma\},\\ \mathbb{K}_{in} = \mathbb{K}_1 \cap \mathbb{K}_2.\end{gathered}$$

This algorithm is implemented in the library IBEX (http ://www.ibex-lib.org) which is free available.