From first to second-order quality measures in direct-search methods

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Direct-search methods are one of the main classes in derivative-free optimization, due to their simplicity and well established convergence results. They proceed by iteratively looking for improvement along some vectors or directions. In presence of smoothness, a convergence proof can be derived by studying at each iteration the cosine of the minimum angle between those vectors and the direction of steepest descent. Provided this (cosine) measure remains bounded away from zero and the algorithm enforces a sufficient decrease condition to move towards a better point, a first-order global convergence property is obtained as well as a worst-case complexity bound.

In this talk, we develop a second-order study of a general class of direct-search methods. A measure of second-order criticality inspired by the Rayleigh quotient is presented. Using this measure, a weak second-order global convergence result is derived for any method in the class. Extensions of this result to ensure true second-order global convergence are discussed, among which a method using approximate Hessian eigenvectors as directions. The means to achieve such a global convergence are costly to ensure (a fact well reflected in the worst-case complexity analysis) but seem necessary as some pathological numerical instances indicate.