Coercive polynomials and their Newton polytopes

Tomáš Bajbar[1] and Oliver Stein[2]

Many interesting properties of polynomials are closely related to the geometry of their Newton polytopes. In this talk we analyze the coercivity on $\mathbb{R}^n$ of multivariate polynomials $f \in \mathbb{R}[x]$ in terms of their so-called Newton polytopes at infinity. In fact, we introduce the broad class of so-called gem regular polynomials and characterize their coercivity via conditions solely containing information about the geometry of the vertex set of the Newton polytope at infinity, as well as sign conditions on the corresponding polynomial coefficients.

For all other polynomials, the so-called gem irregular polynomials, we introduce sufficient conditions for coercivity based on those from the regular case. For some special cases of gem irregular polynomials we establish necessary conditions for coercivity, too. Using our techniques, the problem of deciding the coercivity of a polynomial can be reduced to the analysis of its Newton polytope at infinity. We relate our results to the context of the polynomial optimization theory and the existing literature therein, and we illustrate our results with several examples.

[1]Institute of Operations Research, Karlsruhe Institute of Technology (KIT), Germany, bajbar@kit.edu

[2]Institute of Operations Research, Karlsruhe Institute of Technology (KIT), Germany, stein@kit.edu