A New Proposal for the Approximate Solution of the Normal Equations in Primal-Dual Interior Point Methods

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ABSTRACT

The main computational operation of an Primal-Dual interior-point method (IPM) for linear programming problem is the solution of a Newton method system [1]. In real applications this system often has large dimension and a high degree of sparsity and its solution is the most expensive method step. For this reason, much effort has been employed by researchers in the attempt to find methods for solving this system more efficiently.

This system can be reduced to other system which is symmetric positive definite, called normal equations system. The normal equations solution using Cholesky factorization is preferable whenever this factorization is not very expensive. When the Cholesky factorization is very expensive, iterative methods for solving the normal system becomes more appropriate. In this context, an approach could be used is the preconditioned conjugate gradient method with the hybrid preconditioner composed by the controlled Cholesky factorization (CCF) in early iterations and the splitting preconditioner in the later iterations [2].

The CCF is an incomplete Cholesky factorization, where we obtain a lower triangular matrix L less dense than the Cholesky factor, such that $ADA^t = LL^t - R$, where R is the remainder matrix. The fill-in at each L column is controlled by a parameter η in the following way: given L_j column, only the $k_j = \eta + n_j$ largest nonzeros entries will be kept in the column, where n_j is the column j nonzero entries of the normal equation system matrix [3].

The aim of our work is to improve the Primal-Dual IPM reducing processing time and/or the storage required in each iteration. This will be done by replacing the Cholesky factorization by CCF. In early iterations, we can obtain an approximate direction of the original one, adopting a CCF parameter value such that the matrix obtained in the factorization is very sparse, speeding up the linear system solution. In later iterations we compute CCF factorizations closer to the complete Cholesky factorization, in such a way that the method achieve convergence. Numerical experiments show that this approach obtains competitive results.

References

[1] S. J. Wright, *Primal Dual Interior Point Methods*, SIAM Publications, SIAM, Philadelphia, PA, USA, 1997.

[2] S. Bocanegra, F. F. Campos, and A. R. L. Oliveira, *Using a Hybrid Preconditioner for Solving Large-Scale Linear Systems arising from Interior Point Methods*, Computational Optimization and Applications, v. 36, p. 149-164, 2007

[3] F. F. Campos, *Analysis of Conjugate Gradients - type methods for solving linear equations*, Tese de Doutorado, Oxford University Computing Laboratory, Oxford, 1995.