HiGHS: A High-performance Linear Optimizer

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THE UNIVERSITY *of* EDINBURGH

HiGHS

HiGHS: High performance linear optimization

- Linear optimization
  - Linear programming (LP)

$$\text{minimize} \quad \mathbf{c}^T\mathbf{x} \qquad \text{subject to} \quad A\mathbf{x} = \mathbf{b} \qquad \mathbf{x} \geq \mathbf{0}$$

  - Convex quadratic programming (QP)

$$\text{minimize} \quad \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \mathbf{c}^T\mathbf{x} \qquad \text{subject to} \quad A\mathbf{x} = \mathbf{b} \qquad \mathbf{x} \geq \mathbf{0}$$

    Q positive semi-definite
- High performance
  - Serial techniques exploiting sparsity in A
  - Parallel techniques exploiting multicore architectures

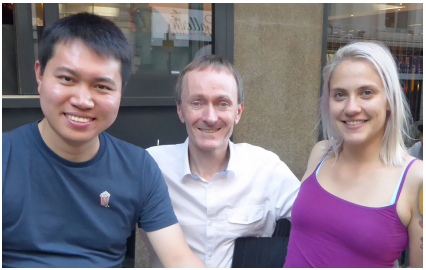
HiGHS: The team

What's in a name?

HiGHS: **H**all, **i**vet **G**alabova, **H**uangfu and **S**chork

Team HiGHS

- Julian Hall: Reader (1990–date)
- Ivet Galabova: PhD (2016–date)
- Qi Huangfu
  - PhD (2009–2013)
  - FICO Xpress (2013–2018)
- Lukas Schork: PhD (2015–2018)
- Michael Feldmeier: PhD (2018–date)
- Joshua Fogg: PhD (2019–date)



HiGHS: Dual simplex algorithm

Assume $\widehat{\mathbf{c}}_N \geq \mathbf{0}$ Seek $\widehat{\mathbf{b}} \geq \mathbf{0}$

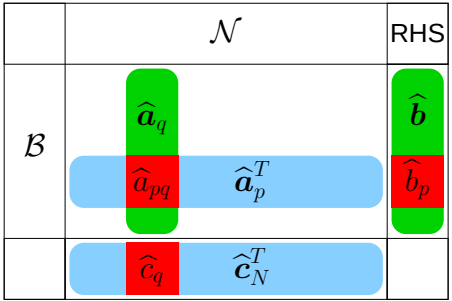
Scan $\widehat{b}_i < 0$ for p to leave $\mathcal{B}$

Scan $\widehat{c}_j/\widehat{a}_{pj} < 0$ for q to leave $\mathcal{N}$

Update: Exchange p and q between $\mathcal{B}$ and $\mathcal{N}$

Update $\widehat{\mathbf{b}} := \widehat{\mathbf{b}} - \alpha_P \widehat{\mathbf{a}}_q \qquad \alpha_P = \widehat{b}_p/\widehat{a}_{pq}$

Update $\widehat{\mathbf{c}}_N^T := \widehat{\mathbf{c}}_N^T + \alpha_D \widehat{\mathbf{a}}_p^T \qquad \alpha_D = -\widehat{c}_q/\widehat{a}_{pq}$



Computation

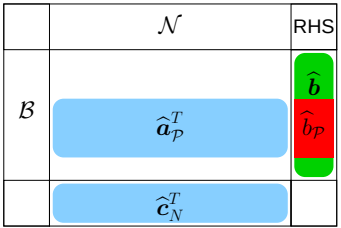
Pivotal row via $B^T\boldsymbol{\pi}_p = \mathbf{e}_p$ BTRAN and $\widehat{\mathbf{a}}_p^T = \boldsymbol{\pi}_p^T N$ PRICE

Pivotal column via $B\widehat{\mathbf{a}}_q = \mathbf{a}_q$ FTRAN Represent B^{-1} INVERT

Update B^{-1} exploiting $\bar{B} = B + (\mathbf{a}_q - B\mathbf{e}_p)\mathbf{e}_p^T$ UPDATE

HiGHS: Multiple iteration parallelism

- Perform standard dual simplex minor iterations for rows in set $\mathcal{P}$ ($|\mathcal{P}| \ll m$)
- Suggested by Rosander (1975) but never implemented efficiently *in serial*



- Task-parallel multiple `BTRAN` to form $\boldsymbol{\pi}_{\mathcal{P}} = B^{-T}\mathbf{e}_{\mathcal{P}}$
- Data-parallel `PRICE` to form $\widehat{\mathbf{a}}_{p}^{T}$ (as required)
- Task-parallel multiple `FTRAN` for primal, dual and weight updates

Huangfu and H (2011–2014)
COAP best paper prize (2015)
MPC best paper prize (2018)

HiGHS: Present (2016–date)

Developments

- Model management: Load/add/delete/modify problem data
  Feldmeier, Galabova, H
- Interfaces
  Feldmeier, Galabova and Vigerske
- Presolve
  Galabova
- Crash
  Galabova and H
- Primal simplex solver
  Huangfu
- Interior point method
  Schork

HiGHS: Access

Source

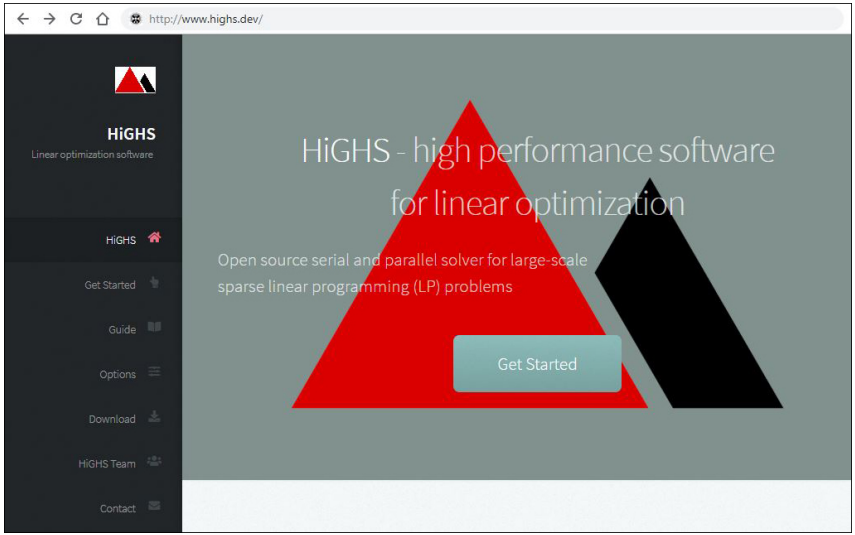
- Open source (MIT license)
- No third party code
- GitHub: ERGO-Code/HiGHS
- COIN-OR: Replacement for `Clp`?

Interfaces

- Existing
  - C++ `HiGHS` class
  - Load from `.mps`
  - Load from `.lp`
  - `C`
  - `C#`
  - `Julia`
  - `FORTRAN`
  - `Python`
  - `OSI` (almost!)
- Prototypes
  - `GAMS`
  - `SCIP`
- Planned
  - `AMPL`
  - `MATLAB`
  - `Mosel`
  - `PuLp`
  - `R`

  Suggestions?

HiGHS: http://www.highs.dev/



HiGHS: Benchmarks (9 October 2019)

Commercial

- Xpress
- Gurobi
- Cplex
- Mosek
- COPT
- Matlab
- QSopt
- SAS

Open-source

- Clp (COIN-OR)
- Glop (Google)
- Soplex (ZIB)
- Glpk (GNU)
- Lpsolve

Solver	COPT	Clp	SAS	Mosek	HiGHS	Glop	Matlab	Soplex
Time	1	1.4	3.2	3.7	5.4	7.2	8.1	10
Solved	40	40	37	38	37	35	32	36

Solver	QSopt	Glpk	Lpsolve
Time	26	28	108
Solved	34	31	23

HiGHS: Comparison with Clp

Solver	COPT	Clp	SAS	Mosek	HiGHS	Glop	Matlab	Soplex
Time	1	1.4	3.2	3.7	5.4	7.2	8.1	10
Solved	40	40	37	38	37	35	32	36

Why is the HiGHS score so bad?

- HiGHS triangular crash not used
- HiGHS parallel code not used

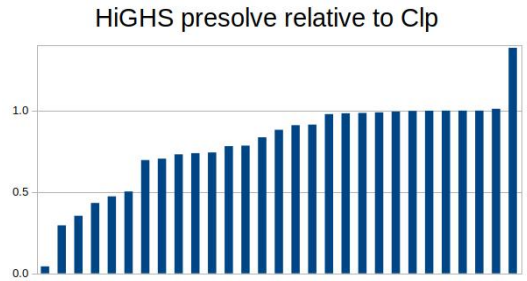
- Clp has a better presolve
- Clp has the Idiot crash
- Clp has a primal simplex solver

HiGHS: Presolve

Aim: eliminate rows, columns and nonzeros

Wide range of techniques

- Simple: interpret singleton rows as bounds on variables
- Complex: LP folding



Presolve measure

Product of

- Relative number of rows
- Relative number of columns
- Relative number of nonzeros

Presolve measure relative to Clp

- Better than Clp for 2/29 LPs!
- Within a factor 0.9 for 14/29 LPs
- Within a factor 0.7 for 23/29 LPs
- Within a factor 0.3 for 28/29 LPs
- Poor for one LP!

HiGHS: Tuning for the test problems

Everybody does it...

QAP problems

- Matrix entries $+1$ and -1, but $B = LU$ fills in
- Dual steepest edge weights become very inaccurate
- Monitor errors and switch to Devex
- Improves solution time for QAP15/NUG15 by a factor of 15!
- Contributes to reliability on general LPs?

HiGHS score now improved from 5.4 to ~4.4

HiGHS: Improvements

Solve all the problems!

- CONT11: Recognise when Markowitz tolerance ($u = 0.1$) needs to be increased
- IRISH-E: Add numerical pivot test to postsolve
- STP3D: Improve primal simplex "clean-up"

Immediate enhancements

- Add primal simplex solver (Huangfu)
- Resurrect `pami`
- Add triangular crash

HiGHS: The future

LP

- Improve presolve (Galabova)
- Add Idiot crash (Galabova)
- Add crossover (Feldmeier)
- Improved Idiot crash (Galabova)
- Direct linear system solver for IPM (Fogg)

MIP

- Publicise existing solver
- Improve existing solver

QP

- Active set QP solver (Feldmeier)
- IPM QP solver

Further interfaces

- `AMPL`
- `MATLAB`
- `Mosel`
- `PuLp`
- `R`

- High performance LP solver
- Reads: `.mps` and `.lp`
- Interfaces: C++ (native) `C`, `C#`, `Julia`, `FORTRAN`, `Python`
- Permissive licence and no third-party code
- Research and consultancy

Slides: http://www.maths.ed.ac.uk/hall/INFORMS-HiGHS19

HiGHS: http://www.highs.dev/

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