An improved algorithm for solving profit-maximizing cattle diet problems

J. G. O. Marques [1], R. de O. Silva [1], L. G. Barioni [2], J. A. J. Hall [3], L. O. Tedeschi [4] and D. Moran [1]

[1] *Global Academy of Agriculture and Food Security, the University of Edinburgh, Edinburgh, UK.*

[2] *Embrapa Agricultural Informatics, Campinas, Brazil.*

[3] *School of Mathematics, the University of Edinburgh, Edinburgh, UK.*

[4] *Department of Animal Science, Texas A&M University, College Station, Texas, USA.*

Corresponding author: João Gabriel Marques. Email: j.g.o.marques@sms.ed.ac.uk

Short title: Diet optimization for beef cattle

Abstract

Feeding cattle with on-pasture supplementation or feedlot diets can increase animal efficiency and system profitability while minimizing environmental impacts. However, cattle system profit margins are relatively small and nutrient supply accounts for most of the costs. This paper introduces a nonlinear profit-maximizing diet formulation problem for beef cattle based on well-established predictive equations (NASEM, 2016). Non-linearity in predictive equations for nutrient requirements poses methodological challenges in the application of optimization techniques. In contrast to other widely used diet formulation methods, we develop a mathematical model that guarantees an exact solution for maximum profit diet formulations. Our method can efficiently solve an often-impractical nonlinear problem by solving a finite number of linear problems, i.e. linear time complexity is achieved through parametric linear programming. Results show the impacts of choosing different objective functions (minimizing cost, maximizing profit, maximizing profit per daily weight gain) may lead to different optimal solutions. In targeting improved ration formulation on feedlot systems, the paper demonstrates how profitability and nutritional constraints can be met as an important part of a sustainable intensification production strategy.

Keywords: linear programming, nonlinear programming, ration formulation, optimization, feedlot.

Implications

This paper introduces a nonlinear profit-maximizing diet formulation problem for beef cattle based on established predictive equations (NASEM, 2016). We develop a mathematical model that can guarantee an exact solution for maximum profit diet formulations. This contrasts with widely used but less robust least-cost diet formulation approaches. Our method can efficiently solve an often-impractical nonlinear problem by solving a finite number of linear problems. By optimizing ration formulation on feedlot systems, this work contributes to the sustainable intensification of livestock production.

Introduction

Cattle system profit margins are relatively small compared to other land uses and nutrient supply is the largest production cost element; e.g. for feedlot finishing systems in Brazil, feed can represent as much as 88% of variable costs, disregarding animal purchase (Sartorello et al., 2018). Feeding cattle with on-pasture supplementation or feedlot diets intensifies production by increasing animal efficiency and profitability, compared to extensive pasture-based systems (Kaimowitz and Angelsen, 2008). By shortening the animal production cycle and therefore reducing methane (CH_4) from ruminant enteric fermentation, balanced diets may also decrease the greenhouse gas (**GHG**) emissions intensity per unit product. Increasing the adoption of these measures is therefore desirable from different sustainability perspectives.

Ration formulation is a complex problem typically analyzed using mathematical optimization (Hertzler et al., 1988; Nicholson et al., 1994; Tedeschi et al., 2000; Soto and Reinoso, 2012; Garcia-Launay et al., 2018).

Ration formulation requires empirical and mechanistic equations to predict growth and nutrient requirements as functions of animal characteristics and the diet composition(Tedeschi et al., 2005; NASEM - National Academies of Sciences, Engineering, and Medicine, 2016). The nonlinear and dynamic nature of biological responses and lack of data hinder the construction of completely mechanistic models. Thus, animal nutrition models rely on a statistical fit of available data, and a mix of mechanistic and empirical equations to predict physiological function (Tedeschi et al., 2005). This nonlinear characteristic of biological systems is a complicating factor in diet optimization models.

The objective of this paper is to describe a framework to optimize maximum profit diets. We introduce and analyze a non-linear profit-maximizing diet model based on the latest version of the “Nutrient Requirements for Beef Cattle” by NASEM (2016). However, any cattle growth predictive model that can be parametrically linearized can be solved using this approach. We propose a new methodology to solve a nonlinear profit-maximizing diet efficiently. We further explore how performance may be improved between linear and logarithmic time complexity. This paper is structured in four sections. Firstly, the material and methods section provides background on diet formulation problems, describes the mathematical model for the nonlinear programming (**NLP**) problem of a profit-maximizing diet, and explores how to obtain an exact solution solving a finite amount of linear programming (**LP**) problems. The results section shows the solutions using the proposed algorithms, sensitivity analysis on key parameters, and convergence. We then discuss in more detail the implications of using the model, before a conclusion that summarizes our outcomes in terms of its applications and identifies future research.

Material and methods

Background to diet formulation problems

Previous work with a nonlinear problem based on the nutrient requirements of beef cattle (NRC, 1984) explored the trade-offs between profit and cost when dealing with diet optimization problems (Hertzler et al., 1988). However, recent work on these equations (NASEM, 2016) hindered the viability of solving a nonlinear problem directly. Detailed descriptions of the evolution of nutrition models for cattle, sheep, and goats have been published recently (Tedeschi and Fox, 2018; Cannas et al., 2019; Tedeschi, 2019). As discussed by Tedeschi (2019), a significant advancement in the development of the Cornell Net Carbohydrate and Protein System was achieved in the 1990s (Fox et al., 1992; Russell et al., 1992; Sniffen et al., 1992), allowing researchers to apply heuristic approaches with linear programming models for least-cost diets (Tedeschi et al., 2000; Soto and Reinoso, 2012).

Deriving a profit-maximizing beef cattle diet implies dealing with non-linear animal weight gain equations associated with the variable energetic concentration of a diet. However, many cattle ration formulation studies are based on linear cost-minimizing diets (Oishi et al., 2011, 2013; Moraes et al., 2012, 2015; Cortez-Arriola et al., 2016; Mackenzie et al., 2016; Garcia-Launay et al., 2018). Linear cost-minimizing diet models based on NASEM (2016) assume a fixed daily shrunk weight gain (**SWG**) rather than a variable to be determined. Since SWG depends on the concentration of net energy for maintenance (**CNEm**), and gain (**CNEg**) in the diet, the least cost modeling approach works under the assumption that these are fixed. Profit maximization varies with growth rate, animal sales price, diet composition, and feed costs. Unless we know optimal CNEm and CNEg beforehand, fixing these parameters hinders the possibility of finding profit-maximizing diets.

Cattle growth model

This work is based on the NASEM (2016) model to predict nutrient requirements and growth in beef cattle, which is frequently reviewed and updated to increase accuracy. The model includes the Cornell Net Carbohydrate and Protein System (Fox et al., 1992; Russell et al., 1992; Sniffen et al., 1992; O'Connor et al., 1993) mechanistic equations, recommendations on possible fit adjustments and variable parameters for a broad range of biophysical conditions, including hormones, lactation, sex, breed, climate, heat loss, growing, finishing. Their predictive model for nutrient requirements is especially helpful to pinpoint possible shortfalls that hinder growth and metabolic efficiency. The process of defining a diet starts with empirical equations to predict approximate energy, protein and dry matter intake requirements. After determining the diet, nutrient utilization is refined using more sophisticated equations.

Based on animal weight, NASEM (2016) estimates the net energy (*NEm* [Mcal per day]) and metabolizable protein (*MPm* [g per day]) requirements for maintenance as a function of shrunk body weight (*SBW*), sex (*SEX*), breed (*BE*), lactation (*L*) and acclimatization factor (*a2*) as:

$$\mathrm{NEm} \ = \ \mathrm{SBW}^{0.75}(0.077 \ \mathrm{BE \ L} \ (\mathrm{a2} + 0.05 \ (\mathrm{BCS} - 1)\mathrm{SEX} + 0.8)) \tag{1}$$

$$\mathrm{MPm} \ = \ 3.8 \ \mathrm{SBW}^{0.75} \tag{2}$$

For a given *NEm*, the dry matter intake (*DMI* [kg per day]) required can be predicted by:

$$DMI \ = \ SBW \ (1.2425 \ + \ 1.9218 \ NEm - \ 0.7259 \ NEm^2) \tag{3}$$

DMI required for growing/finishing cattle must also hold:

$$\mathrm{DMI} \ = \ \mathrm{NEm/CNEm} \ + \ \mathrm{NEg/CNEg} \tag{4}$$

Where *CNEm* [Mcal/kg] is the concentration of net energy for maintenance; *NEg*[Mcal per day] is net energy available for gain and *CNEg*[Mcal/kg] is the concentration of net energy for gain (Anele et al., 2014).

The daily shrunk weight gain (*SWG*[kg per day]) for the given diet is given by:

$$\mathrm{SWG} \ = \ 13.91 \ \mathrm{NEg}^{0.9116} \ \mathrm{SBW}^{-0.6837} \tag{5}$$

Beef cattle profit-maximizing diet

Given animal attributes, e.g., shrunk body weight, breed, sex, and a set *J* of possible ingredients, we formulate a diet by defining its composition in terms of proportion of each ingredient $x_j \in [0; 1]$ and the respective cost c_j [US$/kg], $\forall \ j \in J$. In *T* days, total profit Z [US$] is defined as:

$$\mathrm{Z} \ = \ \mathrm{T} \left[\mathrm{s} * \mathrm{SWG} - \mathrm{DMI} \sum_{j \in J} c_j x_j \right] \tag{6}$$

where *s* is the animal sale's price in US$/kg of *SBW*. Note that *SWG* is a function of *NEg*, which can be written in terms of *CNEm* and *CNEg* by:

$$\mathrm{NEg} = \mathrm{CNEg}\,(\mathrm{DMI} - \mathrm{NEm}/\mathrm{CNEm}) \tag{7}$$

$$\mathrm{CNEm} = \sum_{\mathrm{j \in J}} \mathrm{cnem_j x_j} \tag{8}$$

$$\mathrm{CNEg} = \sum_{\mathrm{j \in J}} \mathrm{cneg_j x_j} \tag{9}$$

thus, combining equations (8) and (9) with (7), we can rewrite profit as:

$$Z = \mathrm{T} \left\{ 13.91 * \mathrm{s} * \mathrm{SBW}^{-0.6837} \left[\left(\sum_{\mathrm{j \in J}} \mathrm{cneg_j x_j} \right) \left(\mathrm{DMI} - \frac{\mathrm{NEm}}{\sum_{\mathrm{j \in J}} \mathrm{cnem_j x_j}} \right) \right]^{0.9116} - \mathrm{DMI} \sum_{\mathrm{j \in J}} \mathrm{c_j x_j} \right\}. \tag{10}$$

The objective function in Equation (10) must be constrained by nutritional requirements and feasibility constraints ($\sum_{j \in J} x_j = 1$). The key nutritional requirements are the metabolizable protein for maintenance (*MPm*) and gain (*MPg*). Protein for maintenance and gain [g per day] are straightforwardly obtained by:

$$\mathrm{MPm} = 3.8\,\mathrm{SBW}^{0.75} \tag{11}$$

$$\mathrm{MPg} = 268\,\mathrm{SWG} - 29.4\,\mathrm{NEg} \tag{12}$$

but the metabolizable protein contribution of each feed j∈J (mp_j) is a function of ruminal microbial growth, which depends on the total digestible nutrients (*TDN*) and fat composition (*FAT*), rumen-undegradable protein (*RUP*), crude protein (*CP*) and forage content. We adopted the equation developed by (Galyean and Tedeschi, 2014) to estimate microbial growth without adjustment for dietary fat (Eq. (14)) rather than the previously adopted fixed coefficient of 13% of TDN.

$$\mathrm{mp_j} \;=\; 0.64\,\alpha_j \;+\; \mathrm{RUP_j}\,\mathrm{CP_j}\,\beta_j \tag{13}$$

where:

$$\alpha_j = \begin{cases} (42.73 + 0.087(\mathrm{TDN_j})(x_j))/1000 & \mathrm{FAT} < 3.9\% \\ (53.33 + 0.096(\mathrm{TDN_j} - 2.55(\mathrm{FAT_j}))(x_j))/1000 & \mathrm{FAT} \geq 3.9\% \end{cases}$$

$$\beta_j = \begin{cases} 0.8 & \mathrm{Forage} < 100\% \\ 0.6 & \mathrm{Forage} = 100\% \end{cases}. \tag{14}$$

A further constraint determines the presence of non-detergent fiber in the diet to prevent acidosis on the animal. We can calculate physically effective non-detergent fiber (*peNDF*) [%DMI] requirement based on expected pH in the rumen (rearranging NASEM (2016)'s equation to predict pH based on peNDF content):

$$\mathrm{peNDF} = \begin{cases} 0.01\;(\mathrm{pH} - 5.46)/0.038 & \mathrm{pH} < 6.46 \\ 26.3\% & \mathrm{pH} \geq 6.46 \end{cases}. \tag{15}$$

Thus, it is possible either to constraint peNDF content to be higher than 26.3% or to constrain it based on a limit pH desired below 6.46.

Further constraints to guarantee rumen microorganism efficiency in fiber digestion include the fat content, which should be lower than 6% of DMI, and the presence of rumen-degradable protein (*RDP*) to sustain bacterial yield, that should be greater than 12.5% of *DMI* (NASEM, 2016). We assume supplementation of vitamins and minerals to the diet, thus we do not include constraints with requirements for those ingredients.

Using the parameters defined on the Equations (10) to (15) and replacing (5) and (7) in (12), we can write the nonlinear programming (NLP) model of daily profit as:

Max

$$\begin{aligned} \mathrm{Z} \\ &= \mathrm{T} * 13.91 * \mathrm{s} \\ &* \;\mathrm{SBW}^{-0.6837}\left[\left(\sum_{j\in J} \mathrm{cneg_j}x_j\right)\left(\mathrm{DMI} - \frac{\mathrm{NEm}}{\sum_{j\in J}\mathrm{cnem_j}x_j}\right)\right]^{0.9116} \end{aligned} \tag{16}$$

$$- \mathrm{T} * \mathrm{DMI} \,.\, \sum_{j \in J} c_j x_j$$

s.t.

$$\sum_{j \in J} mp_j x_j \geq \mathrm{DMI}^{-1} \Bigg\{ 3.8\,\mathrm{SBW}^{0.75}$$

$$+3727.88 \left[\left(\sum_{j \in J} cneg_j x_j \right) \left(\mathrm{DMI} - \frac{\mathrm{NEm}}{\sum_{j \in J} cnem_j x_j} \right) \right]^{0.9116} \mathrm{SBW}^{-0.6837}$$

$$- 29.4 \left[\left(\sum_{j \in J} cneg_j x_j \right) \left(\mathrm{DMI} - \frac{\mathrm{NEm}}{\sum_{j \in J} cnem_j x_j} \right) \right] \Bigg\}$$

$$\frac{\mathrm{NEg}}{\sum_{j \in J} cneg_j x_j} + \frac{\mathrm{NEm}}{\sum_{j \in J} cnem_j x_j} = DMI$$

$$\sum_{j \in J} peNDF_j x_j \geq \mathrm{peNDF}$$

$$\sum_{j \in J} FAT_j x_j \leq 0.06$$

$$\sum_{j \in J} RDP_j\, CP_j\, x_j \leq 0.125$$

$$\sum_{j \in J} x_j = 1$$

$$x_j \in [0;\ 1]\ \forall j \in J$$

It is important to note that the parameters of each feed j∈J: $cneg_j$, $cnem_j$, $peNDF_j$, FAT_j, RDP_j, and CP_j, are available for over 200 feedstuff on NASEM (2016)'s feed library. Furthermore, the constraint for minimum and maximum values of specific ingredients on the diet can be easily added without changing the complexity of the model. Such constraints simply change the domain of x_j from $x_j \in [0;\ 1]$ to $x_j \in [lb_j;\ ub_j]$, where lb_j and ub_j are the minimum and maximum concentration of the feed j.

The parametric linear programming model for profit-maximizing diets

The proposed model contains nonlinearities in the objective function and the first constraint. We can remove the complicating factor $NEg^{0.9116}$ via a linear function.

NASEM (2016) uses the exponential term only for fine adjustments based on the R^2 value.

We use the linear function f(NEg) = 13.91 $SBW^{-0.6837}$ (0.86 NEg) as an alternative to NASEM (2016)'s SWG = 13.91 $NEg^{0.9116}$ $SBW^{-0.6837}$. This approximation presents, with the original equation, an R^2=0.999 for NEg values between 0 and 8 Mcal per day, which is the practical viable range of NEg.

In general, NLP models can not be solved exactly. Thus, we aim to find a point in the solution space that is guaranteed to be within a tolerance ε of the exact solution. Considering the suggested linear approximation we can solve the nonlinear programming model via parametric linear programming (Dantzig, 1998). The profit function Z in (16) is given by the nonlinear animal weight gain function Ω(CNEm, **x**) multiplied by the selling price *s* (US$/kg), with the diet costs C(CNEm, **x**) subtracted:

$$\mathrm{Z}(\mathrm{CNEm}, \mathbf{x}) = \mathrm{s}.\Omega(\mathrm{CNEm}, \mathbf{x}) - \mathrm{C}(\mathrm{CNEm}, \mathbf{x}) \tag{17}$$

where CNEm ∈ K = {[lb, ub], lb, ub ∈ $\mathbb{R}_0^+$} is the net energy for maintenance available in the diet; *lb* and *ub* represent respectively the lower and upper bounds of CNEm; and **x** ∈ $\mathbb{R}^n$ is a vector variable representing the daily feed intake proportion of each diet ingredient. Profit *Z* is subject to a set of nonlinear nutritional constraints Φ(CNEm, **x**) and linear constraints F(CNEm, **x**). For a given animal and a fixed CNEm = k_i, the nonlinear function Ω and constraints Φ become linear. Thus maximizing the NPL {Z(CNEm, **x**): Φ(CNEm, **x**), F(CNEm, **x**)} is equivalent to solving the LP{Z(k_i, **x**): Φ(k_i, **x**), F(k_i, **x**)} for k_i=CNEm. Thus, the optimal solution for Z(CNEm, **x**) is given by:

$$\begin{aligned}\mathrm{Z}^* = \max\{\mathrm{Z}_\mathrm{i}^* \mid \mathrm{Z}_\mathrm{i}^* = \max\{\mathrm{s}.\Omega(\mathrm{k_i}, \mathbf{x}) - \mathrm{C}(\mathrm{k_i}, \mathbf{x}) \mid \Phi(\mathrm{k_i}, \mathbf{x}) = 0, \mathrm{F}(\mathrm{k_i}, \mathbf{x}) \\ = 0, \mathbf{x} \in (\mathbb{R}_0^+)^\mathrm{n}\}, \forall\, \mathrm{k_i} \in \mathrm{K}\}.\end{aligned} \tag{18}$$

For a fixed value of $CNEm_i$, the model in (16) is equivalent to the linear problem:

$$
\begin{aligned}
\text{Max}\quad & Z = \sum_{j\in J} x_j \left(13.91 * s * SBW^{-0.6837} 0.86\left(DMI - \frac{NEm}{CNEm_i}\right) cneg_j - DMI.c_j\right) \\
\text{s.t.}\quad & \sum_{j}\left(mp_j - \left(DMI - \frac{NEm}{CNEm_i}\right)(3205.97\, SWB^{-0.6837} - 29.4) cneg_j\, x_j\right. \\
& \qquad \geq DMI^{-1}\, 3.8\, SBW^{0.75} \\
& \sum_{j\in J} cnem_j x_j \leq CNEm_i + \delta \\
& \sum_{j\in J} cnem_j x_j \geq CNEm_i - \delta \\
& \sum_{j\in J} peNDF_j x_j \geq peNDF \\
& \sum_{j\in J} FAT_j x_j \leq 0.06 \\
& \sum_{j\in J} RDP_j\, CP_j\, x_j \leq 0.125 \\
& \sum_{j\in J} x_j = 1 \\
& x_j \in [0;\ 1]\ \forall\, j \in J, \delta \to 0
\end{aligned}
\tag{19}
$$

Figure 1 shows a “brute force” algorithm for parametric linear programming. This approach solves (*ub*-*lb*)/ε linear programming models and compares the obtained solutions for each $CNEm_i$.

For the precision of $\varepsilon=10^{-2}$, the proposed method will need to solve $O(\varepsilon^{-1})$ LP models. The values *lb* and *ub* can be calculated by solving: NEg = CNEg (DMI – NEm/CNEm) ≥ 0, thus CNEm ≥ NEm (SBW (1.2425 + 1.9218 NEm – 0.7259 NEm^2))$^{-1}$.

The well-behaved characteristic of the problem suggests that we can obtain a faster solution with numerical optimization methods. We use the golden-section search method (**GSS**) (Press et al., 2007) which breaks the interval using the golden-ratio proportion $\varphi = (1-\sqrt{5})/2$. This method requires solving $O(\log \varepsilon^{-1})$ LPs, a considerable reduction in comparison with the brute force approach.

We chose a precision of $\varepsilon=10^{-2}$ since, in reality, it is unlikely that beef producers can achieve a higher precision when mixing feedstuff to prepare the ration. Furthermore, one can easily incorporate other predictive equations or constraints in the proposed model, provided they are linear within each feedstuff x_j. In section "Discussion" we comment on the possibility of implementing predictive equations for CH_4 emission suggested by NASEM (2016).

The model developed in Python 3 (Marques) uses the HiGHS solver (Huangfu and Hall, 2018) to optimize LP models in the algorithm. All results from this work can be obtained by replicating the execution with the same input data.

Bioeconomic Data

We use bioeconomic data based on a representative feedlot finishing system in Brazil (ANUALPEC, 2017) consisting of Nellore steers with average body condition score (*BCS*) 5 and the initial shrunk body weight of 300 kg under a finishing time of 60 days. Table 1 shows the used ingredients and costs (CEPEA, 2018). Nellore selling price *s* was assumed 1.44 [US$/kg] (CEPEA, 2018). We obtained the ingredient's properties from the NASEM (2016) feed library, presented in Supplementary Table S1.

Results

Figure 2 shows the profit, diet cost and shrunk weight gain as a function of CNEm. It is clear from the results that the global optimum for the least-cost diet is not the same for maximum profit. We also notice that there are changes in the curve inflection

($\partial f/\partial CNEm$) in similar positions for all three curves. Figure 3 shows diet profiles for different values of CNEm. The chosen values are the neighborhood where the subtle change in inflection occurs. We spot those occurrences by the dual values, i.e. indicators of activity of each constraint, and reduced costs, i.e., an indicator of minimal change in each variable coefficient to change optimal solution.

Figure 2 also shows that for CNEm < 1.26 the diet cost varies greatly. This is because of the change in urea concentration in the diet as we see in Figure 3. Urea is used as a protein source and is one of the most expensive ingredients available. The limited allowance of energetic concentration pushes the use of urea as a protein source. As CNEm increases, cheaper feedstuff replaces increase contribution to protein requirements.

Figure 2 shows that for 1.26 ≤ CNEm < 1.67 diet costs remain considerably steady. Thus, the increase in profit derives from an increase in shrunk weight gain up until the point diet costs start to increase again. From Figure 3 we notice that as CNEm increase in this range, the diet drifts the use of sugarcane silage to sorghum grain. This shift is a response to the increase in required protein to achieve higher SWG since sorghum grain has more than twice the amount of proteins of sugarcane silage. The change in the diet composition from CNEm = 1.66 to CNEm = 1.67 is imperceptible in Figure 2. However, there is a shift in the dual variable related to peNDF, indicating that this constraint is active (slack = 0) from this solution and beyond. This can be noticed in Figure 3 by the insertion of new ingredients previously unused, cottonseed whole. Analogously, the fat content becomes an active constraint for CNEm ≥ 1.76, from this point and beyond slack on the maximum fat content constraint is 0 and soybean meal high in CP is introduced in the diet.

For CNEm ≥ 1.94, dual values are 0 for metabolized protein constraint and for $\sum_{j \in J} cnem_j \, x_j \leq CNEm_i + \delta$, so at this point, protein requirements are easily met but the increase in shrunk weight gain per CNEm does not compensate anymore and the daily profit significantly decreases beyond this point.

The optimal diet has CNEm = 1.92 Mcal/kg and DMI of 6.78 kg per day containing 21.35% of cottonseed whole, 0.11% of distillers grain plus soluble, 24.21% of sorghum grain, 40.13% of soybean meal high CP, 12.63% of sugarcane silage and 1.57% of urea. The daily profit of 0.76 US$ per day is associated with a shrunk weight gain of 1.21 kg per day and a diet cost of 0.99 US$ per day.

Figure 4 shows the sensitivity analysis of CNEm vs. profit. For unused ingredients, the decrease in price [US$/kg] necessary to change diet composition is US$0.09 for citrus pulp, corn grain and wheat middlings, US$0.10 for corn silage, US$0.16 for cottonseed meal, and US$0.13 for soybean hulls. On the other hand, due to different SWG factors for each feed, a change in cattle's price higher than +US$0.42/kg would also change the optimal diet composition. Figure 4 also shows that max {daily profit/SWG} ≠ max {daily profit}. Daily profit is 5% lower for the alternative objective function. This means that optimal direction is different for those objective functions, implying that they are not proportional to each other.

We show in **Figure 5** convergence results using numerical optimization with precision $\varepsilon=10^{-2}$. The golden-section search takes $O(\log \varepsilon^{-1})$ iterations to complete. Daily profit, daily cost, and CNEm converged for the same values found using the brute force search: 0.76 US$ per day, 0.9864 US$ per day and 1.92 Mcal/kg. While the brute force algorithm took 96 iterations to find the optimal solution with $\varepsilon=10^{-2}$, the golden-section search took 10 for the same ε. Since the number of iterations to find a solution through brute force in a range D is $n = \lceil D^2/\varepsilon \rceil$, for $\varepsilon=10^{-6}$ and D = 1.15, it would require 1,322,500 iterations. On the other hand, for the same range D, the GSS would need $n = \lceil(\log \varepsilon - \log D)/\log \varphi\rceil$, i.e. 29 iterations for $\varepsilon=10^{-6}$.

Discussion

Transforming the nonlinear programming problem into a parametric linear programming equivalent has practical and computational advantages. Since LPs can be solved in pseudo-polynomial time, we have a good estimation of computational time based solely on a few characteristics of the data and model. The brute force search algorithm is the slowest possible approach, having to solve $O(\varepsilon^{-1})$ LPs to find the ε-solution for the NLP model. However, it is unrealistic to work with precisions greater than 10^{-2} in diet formulation, which means a solution can usually be found with no more than 200 LPs resolutions, depending on the feasible range of CNEm. Since this basic diet optimization model is small, i.e. 7 constraints and no more than 300 feed ingredients in NASEM (2016)'s library, it can be further developed for more complex scenarios, e.g. environmental constraints.

The golden-section search showed a significant increase in performance compared with the brute force search. For $\varepsilon=10^{-2}$ it took 10 iterations to complete, roughly 1/10 of the initial approach. Since the golden-section search has to solve only $O(\log \varepsilon^{-1})$ LPs, it can be useful for more complex diet optimization models. Empirical evidence shows that the function Z(CNEm, **x**) on Equation (17) is unimodular, thus both brute force and golden-section search are guaranteed to obtain a minimizer within ε of that exact minimizer.

Further model limitations are related to the assumed linear growth of SBW over time. Since the predicted requirements change with cattle bodyweight, ideally the model should revaluate diet periodically. Thus, the results only hold for small periods of feeding. For long periods of feeding the result will accumulate great inaccuracy from the dynamic weight of the animal. To overcome this, the feeding time *T* can be discretized by a micro-period *p* to maximize profit along the whole feeding time. Thus, a new index $t \in \{1, \ldots, T/p\}$ must be introduced into the model. This approach is equivalent to approximate nonlinear growth by linear segments.

Mathematical models to optimize diet are always bounded to limitations on the accuracy of predictive equations for nutrient requirements and absorption. Regardless, under the infinite set of possible diets, optimization techniques deliver a "close to optimal" solution facing variation proportional to the uncertainty in the process. Some suggest adjustments on predictive equations based on each particular application (NASEM, 2016) which may also correct the assertiveness of our model's solution. Still, it is key to have a proper understanding of the premises and scope of both the predictive equations and the mathematical optimization models when applying them to cattle feeding operations.

Furthermore, in response to concerns over livestock emissions researchers have been focusing on the environmental impacts of rations. Optimization models to evaluate economic and environmental (impact) trade-offs usually modify the traditional least-cost algorithm objective function (Wang et al., 2000a; b; Tedeschi et al., 2000; Pomar et al., 2007; Oishi et al., 2011; Moraes et al., 2012), use multi-criteria analysis (Hadrich et al., 2005; Moraes and Fadel, 2013; Moraes et al., 2015), develop multi-objective models (Garcia-Launay et al., 2018) or integrate life-cycle assessment analysis exogenously (Oishi et al., 2013; Mackenzie et al., 2016). As our results show, the choice of the objective function will impact the optimal solution and thus on related economic analysis.

The NASEM (2016) model suggests a variety of equations to predict CH_4 emissions. Such equations (e.g. those based on the International Panel on Climate Change guidelines (Penman et al., 2006)) can be also introduced into our model either in the constraints or in the objective function.

Conclusion

Our model improves optimal diet formulation by considering the interaction between CNEm and CNEg and diet profit. As the choice of maximum profit, profit/shrunk weight gain or minimum diet cost leads to different solutions, general results from one perspective cannot always be extrapolated.

From a nutritional perspective, our model presents a straightforward approach to define a baseline diet solely based on animal characteristics. Since it does not require net energy concentration beforehand, it has a broader solution space than a traditional least-cost diet. Furthermore, the relation with the NASEM (2016)'s prediction system allows the same approximations to be made in our model.

It is unlikely that in future the NASEM (2016)'s equations will evolve in a way that compromises our model since it would concurrently jeopardize the process by which the nutritionist formulates the baseline diet. Thus, as their system continues to evolve, our model should be able to accommodate changes in the equations for the nutrient requirement prediction.

The parametric linear programming approach makes it easier to implement further developments to the model to asses a more complex situation. Furthermore, the possibility of solving the profit-maximizing diet problem with the golden-section search suggests that our model could be extended or integrated with others, and still be solvable in a reasonable time.

References

Anele UY, Domby EM and Galyean ML 2014. Predicting dry matter intake by growing and finishing beef cattle: Evaluation of current methods and equation development12. Journal of Animal Science 92, 2660–2667.

ANUALPEC 2017. Anuário da pecuária brasileira. FNP Consultoria/Agros Comunicação, São Paulo, SP, Brasil.

Cannas A, Tedeschi LO, Atzori AS and Lunesu MF 2019. How can nutrition models increase the production efficiency of sheep and goat operations? Animal Frontiers 9, 33–44.

CEPEA 2018. PIB do Agronegócio Brasileiro. Piracicaba, SP, Brasil.

Cortez-Arriola J, Groot JCJ, Rossing WAH, Scholberg JMS, Améndola Massiotti RD and Tittonell P 2016. Alternative options for sustainable intensification of smallholder dairy farms in North-West Michoacán, Mexico. Agricultural Systems 144, 22–32.

Dantzig GB 1998. Linear programming and extensions. Princeton university press.

Fox DG, Sniffen CJ, O'Connor JD, Russell JB and Van Soest PJ 1992. A net carbohydrate and protein system for evaluating cattle diets: III. Cattle requirements and diet adequacy. Journal of Animal Science 70, 3578–3596.

Galyean ML and Tedeschi LO 2014. Predicting microbial protein synthesis in beef cattle: Relationship to intakes of total digestible nutrients and crude protein1. Journal of Animal Science 92, 5099–5111.

Garcia-Launay F, Dusart L, Espagnol S, Laisse-Redoux S, Gaudré D, Méda B and Wilfart A 2018. Multiobjective formulation is an effective method to reduce environmental impacts of livestock feeds. British Journal of Nutrition 120, 1298–1309.

Hadrich JC, Wolf CA and Harsh SB 2005. Optimal livestock diet formulation with farm environmental compliance consequences.

Hertzler G, Wilson DE, Loy DD and Rouse GH 1988. Optimal Beef Cattle Diets Formulated by Nonlinear Programming. Journal of Animal Science 66, 1115.

Huangfu Q and Hall JAJ 2018. Parallelizing the dual revised simplex method. Mathematical Programming Computation 10, 119–142.

Kaimowitz D and Angelsen A 2008. Will Livestock Intensification Help Save Latin America's Tropical Forests? Journal of Sustainable Forestry 27, 6–24.

Mackenzie SG, Leinonen I, Ferguson N and Kyriazakis I 2016. Towards a methodology to formulate sustainable diets for livestock: Accounting for environmental impact in diet formulation. British Journal of Nutrition 115, 1860–1874.

Marques JGO. MaxProfitFeeding/README.md at master · BlackNellore/MaxProfitFeeding. Retrieved on 1 December 2019, from *https://github.com/BlackNellore/MaxProfitFeeding/blob/master/README.md*.

Moraes LE and Fadel JG 2013. Diet Optimization Models.

Moraes LE, Fadel JG, Castillo AR, Casper DP, Tricarico JM and Kebreab E 2015. Modeling the trade-off between diet costs and methane emissions: A goal programming approach. Journal of Dairy Science 98, 5557–5571.

Moraes LE, Wilen JE, Robinson PH and Fadel JG 2012. A linear programming model to optimize diets in environmental policy scenarios. Journal of Dairy Science 95, 1267–1282.

NASEM - National Academies of Sciences, Engineering, and Medicine 2016. Nutrient Requirements of Beef Cattle, 8th Revised Edition. National Academies Press, Washington, D.C.

Nicholson CF, Lee DR, Boisvert RN, Blake RW and Urbina CI 1994. An optimization model of the dual-purpose cattle production system in the humid lowlands of Venezuela. Agricultural Systems 46, 311–334.

NRC 1984. Nutrient Requirements of Beef Cattle. National Academies Press, Washington, D.C., USA.

O'Connor JD, Sniffen CJ, Fox DG and Chalupa W 1993. A net carbohydrate and protein system for evaluating cattle diets: IV. Predicting amino acid adequacy. Journal of Animal Science 71, 1298–1311.

Oishi K, Kato Y, Ogino A and Hirooka H 2013. Economic and environmental impacts of changes in culling parity of cows and diet composition in Japanese beef cow-calf production systems. Agricultural Systems 115, 95–103.

Oishi K, Kumagai H and Hirooka H 2011. Application of the modified feed formulation to optimize economic and environmental criteria in beef cattle fattening systems with food by-products. Animal Feed Science and Technology 165, 38–50.

Penman J, Gytarsky M, Hiraishi T, Irving W and Krug T 2006. 2006 IPCC - Guidelines for National Greenhouse Gas Inventories.

Pomar C, Dubeau F, Létourneau-Montminy M-P, Boucher C and Julien P-O 2007. Reducing phosphorus concentration in pig diets by adding an environmental objective to the traditional feed formulation algorithm. Livestock Science 111, 16–27.

Press WH, Teukolsky SA, Vetterling WT and Flannery BP 2007. Numerical recipes : the art of scientific computing.

Russell JB, O'Connor JD, Fox DG, Van Soest PJ and Sniffen CJ 1992. A net carbohydrate and protein system for evaluating cattle diets: I. Ruminal fermentation. Journal of Animal Science 70, 3551–3561.

Sartorello GL, Bastos JPST and Gameiro AH 2018. Development of a calculation model and production cost index for feedlot beef cattle. Revista Brasileira de Zootecnia 47.

Sniffen CJ, O'Connor JD, Van Soest PJ, Fox DG and Russell JB 1992. A net carbohydrate and protein system for evaluating cattle diets: II. Carbohydrate and protein availability. Journal of Animal Science 70, 3562–3577.

Soto C and Reinoso V 2012. Modelo de formulación de raciones al mínimo costo para ganado de carne basado en el sistema NRC 2000. Archivos de Zootecnia 61, 255–266.

Tedeschi LO 2019. ASN-ASAS SYMPOSIUM: FUTURE OF DATA ANALYTICS IN NUTRITION: Mathematical modeling in ruminant nutrition: approaches and paradigms, extant models, and thoughts for upcoming predictive analytics1,2. Journal of Animal Science 97, 1921–1944.

Tedeschi LO and Fox DG 2018. The ruminant nutrition system : an applied model for predicting nutrient requirements and feed utilization in ruminants. XanDu.

Tedeschi LO, Fox DG, Chase LE and Wang SJ 2000. Whole-Herd Optimization with the Cornell Net Carbohydrate and Protein System. I. Predicting Feed Biological Values for Diet Optimization with Linear Programming. Journal of Dairy Science 83, 2139–2148.

Tedeschi LO, Fox DG, Sainz RD, Barioni LG, Medeiros SR de and Boin C 2005. Mathematical models in ruminant nutrition. Scientia Agricola 62, 76–91.

Wang SJ, Fox DG, Cherney DJR, Chase LE and Tedeschi LO 2000a. Whole-herd optimization with the cornell net carbohydrate and protein system. III. Application of an optimization model to evaluate alternatives to reduce nitrogen and phosphorus mass balance. Journal of Dairy Science 83, 2160–2169.

Wang SJ, Fox DG, Cherney DJR, Chase LE and Tedeschi LO 2000b. Whole-herd optimization with the cornell net carbohydrate and protein system. II. Allocating homegrown feeds across the herd for optimum nutrient use. Journal of Dairy Science 83, 2149–2159.

Table 1 Common Brazilian ingredients to ration formulation (CEPEA, 2018).

Ingredient	Cost [US$/kg]
Citrus pulp, dry	0.14
Corn grain	0.18
Corn silage	0.19
Cottonseed meal	0.33
Cottonseed whole	0.11
Distillers grain plus soluble, dry	0.14
Grain sorghum grain	0.10
Soybean hulls	0.16
Soybean meal high CP	0.20
Sugarcane silage	0.09
Wheat middlings	0.15
Urea	0.40

Figure captions

Figure 1 Parametric linear programming algorithm for solving the nonlinear programming model. The concentration of net energy for maintenance (CNEm) varies inside the feasible range (LB – lower bound to UB – upper bound) with a step ε. Each solution is stored and the one with maximum objective function (z_i) is retrieved at the end.

Figure 2 Results from the parametric linear programming algorithm. The green dots represent maximum daily profit for that concentration of net energy for maintenance (CNEm), calculated as shrunk-weight gain (SWG – blue triangle) times animal's sale price, minus daily costs (Daily cost – yellow rhombus). The white markers represent the optimal solution, i.e maximum daily profit.

Figure 3 Diet profiles for frontier points on the objective function. The star (*) represents a diet profile of the NLP model's optimal solution.

Figure 4 Comparison of daily profit and profit per bodyweight gain. The daily profit divided by the shrunk-weight gain (blue rhombus) and daily profit (green dots) for each value of concentration of net energy for maintenance (CNEm). For each of the objective functions the optimal solution is highlighted (white rhombus and dot, respectively).

Figure 5 Golden-section search convergence for daily profit (blue dot), diet cost (red triangle), and concentration of net energy for maintenance (CNEm – white rhombus).