

# Optimal refinery operations

With the option of exceeding  
demands at discounted price

Methodology Modelling and Consultancy Sills  
Xpress-MP Assignment 1

## Introduction

This report is the result of a study to the optimal production plan for a refinery. This report gives a short description of the problem considered and the results obtained after solving the problem. Indications of what happens if important factors change in the model are given in a sensitivity analysis of the model. It is then studied what the effect will be if it becomes possible to sell more than demand for a lower price. Finally some recommendations for further studies are given to improve the operations of the refinery even further.

## Problem definition

The problem we have solved involved finding the optimal operating policy for a refinery plant; the maximization of the profit. Income is generated from selling final products, butane, petrol, kerosene, diesel, and tar. Butane and tar are direct outputs of the distillation process but petrol, kerosene and diesel are blended according to different combinations of outputs from the distillation process.

Distillation happens in three independent distillation columns that transform crude oil into butane, tar and a range of intermediate fractions that have to be blended into final products. The distillation process also generates an amount of waste. Each of the three distillation columns creates a different combination of outputs and each have a different running cost.

Waste from the distillation process and any distillation output that is not blended into final products or not sold, has to be disposed of at a cost.

Beside the constraints that model the flow of products from distillation to the final products the solution is also constrained by the fact that we can not use a distillation column more than 24 hours a day. At the moment it is company policy that sales of final products may not exceed the demand for these products.

The objective is to maximize the profit from selling the output minus the running cost of the columns and any costs incurred for having to dispose of unused distillation outputs.

This problem is formulated as a linear problem and solved with Xpress-MP. Details on the model formulation with comments for each constraint can be obtained in the accompanied Mosel file.

## Results obtained

The optimal policy for the company is to sell the following amounts of each output:

	Butane	Petrol	Kerosene	Diesel	Tar
Production (tons)	3000	2000	1168.37	1750	500

The total income from the sales of these outputs is £1,351,071.8.

In order to produce these products we have to use distillation columns A and B for 24 hours and column C for 4.295 hours. The total running costs for operating these columns are £582,195.

The total output of these distillation columns and the amount of waste (the output that could not be used in final products) is shown in the following table:

	Butane	Fraction 1	Fraction 2	Fraction 3	Tar	Waste
Output (tons)	3000	1761.09	1721.68	2014.74	1187.37	2507.37
Waste (tons)	0	60.58	0	518.56	687.37	2507.37

The total cost for disposing of waste is £3,478.8.

This makes the total optimal profit £795,398 per day.

## Sensitivity analysis of the solution

### Increasing kerosene sales

All demands are completely satisfied except for kerosene for which a total demand of 1500 exists. An extra ton of kerosene would give an extra income of £140 but requires column C to operate for an extra 1/120 hours which gives a running cost of £175 and would also increase the disposal cost by £1.01. So only if the price of kerosene is higher than £176.01 higher production levels for kerosene become profitable.

Later in this report it will show that for higher multiplier values it even becomes economical to sell excess petrol instead of kerosene. These are clear signs that the price of kerosene could be reconsidered.

### Effects of demand increases

If the demand of the products could be increased, for instance by marketing efforts, it is important to know how much the profit will increase in order to obtain a maximum price for those marketing efforts.

The following table shows the fair prices for an increase in demand of one ton:

	Butane	Petrol	Kerosene	Diesel	Tar
Fair price (£/tons)	155.01	62.15	0	80.25	11

Since the demand for kerosene is not completely utilized yet, there is no improvement if the demand for kerosene is increased.

These results show that marketing efforts to increase the demand for especially butane can be very profitable. This is because butane can be produced together with kerosene, which makes the use of the production capacity of distillation column C very profitable.

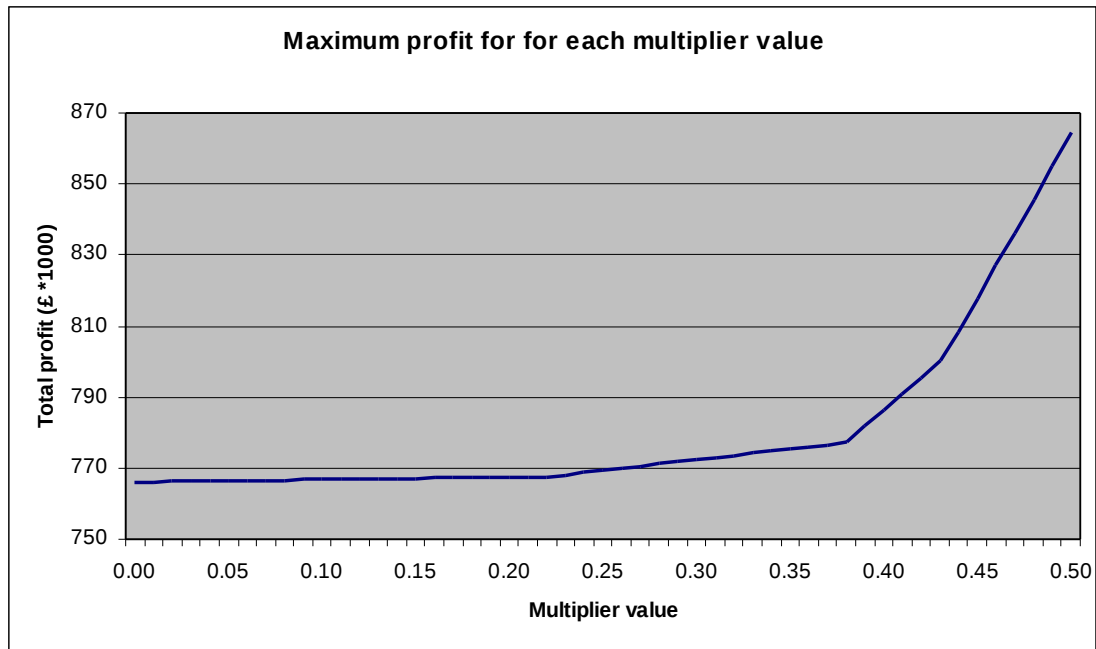
### Effects of decreases in capacity

An increase in the capacity of a column to over 24 hours a day is impossible, but it is very well possible that the full theoretical capacity of 24 hours will not be achieved due to maintenance work or other activities that would decrease the column utilization. A decrease in availability of one hour for column A would decrease the profit with £374.53 and a same decrease in availability for column B would decrease profits by £880.84. Of course a decrease in availability of column C has no negative effects because its capacity is not fully utilized at the moment.

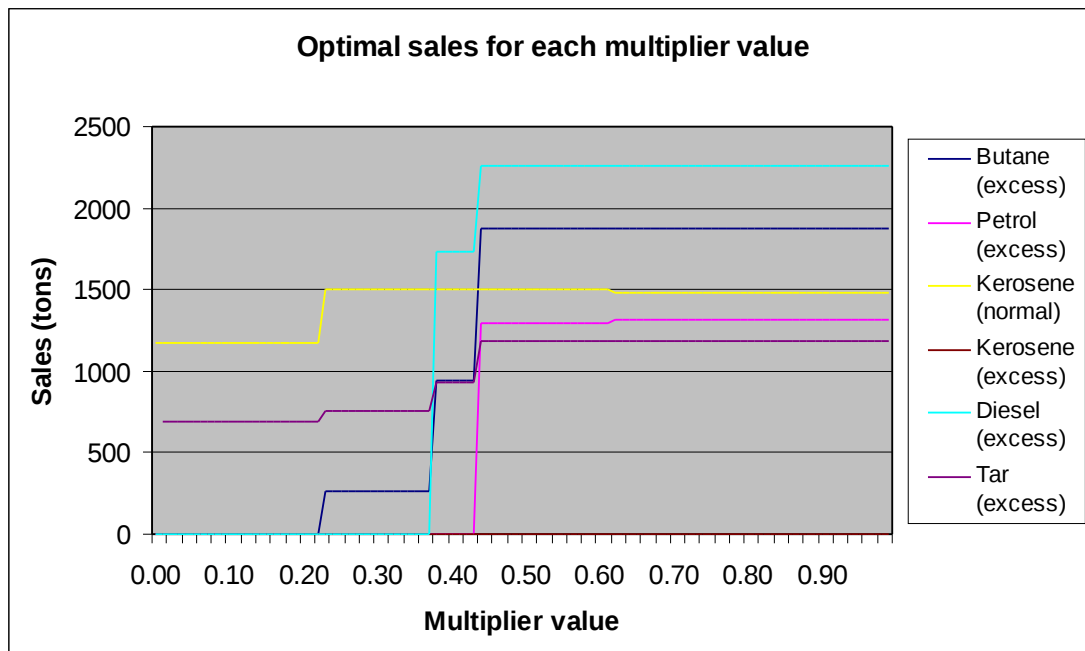
In spite of the higher running cost of column B its contribution to the total profit is much higher and maintenance schedules should be focussed on guaranteeing maximum availability of this distillation column.

### Allowing for sales exceeding the demands

If it would be possible to sell more products than the demand if this excess production would be sold for a discounted price, this could improve the profitability of the refinery. It should at least be possible to give final products away for free and the best situation would be if we could sell any excess demands for almost the same price as we would be able for sales up to the demand. The price that is earned from excess sales is the normal selling price multiplied with a multiplier value between 0 and 1. The following graph shows how profit changes as the multiplier value increases:



This graph shows a piecewise linear increase in profit if the discount factor increases. The points where the slope of the line changes, are at multiplier values of 0.23, 0.38, 0.44 and a marginal change outside the graph at 0.62. At each of these points it becomes efficient to produce more and sell this in excess demand. The following graph shows how much of each product is sold above the demand for each value of the discount factor:



Tar is being sold even with a multiplier value of 0 since it was previously being disposed of for £1 per ton and thus it is better to give it away for free if possible. If the multiplier value is greater than 0.22 it also becomes profitable to produce kerosene to its demand limit since now the extra butane and tar income compensates for the original loss of producing more kerosene. For multiplier values from 0.38 it becomes profitable to produce diesel, butane and tar only for excess sales and at values from 0.44 also butane, petrol, diesel and tar are

produced for excess sales only. At this point all three columns are used for 24 hours per day.

For multiplier values higher than of 0.62 the normal sales of kerosene drops slightly again plus a fractional decrease in the excess sales of diesel to make room for a small increase in excess petrol sales.

From 0.62 upwards the objective function increases linearly.

The following table gives the exact sales values for each of the multiplier value ranges:

	0 – 0.22	0.23 – 0.37	0.38 – 0.43	0.44 – 0.61	0.62 - 0.99
Normal sales of kerosene (tons)	1168.37	1500	1500	1500	1482.46
Excess sales of butane (tons)	0	262.54	946.56	1872	1872
Excess sales of petrol (tons)	0	0	0	1288.39	1313.85
Excess sales of kerosene (tons)	0	0	0	0	0
Excess sales of diesel (tons)	0	0	1728.05	2262.26	2261.69
Excess sales of tar (tons)	687.368	756.46	936.46	1180	1180
Increase in profit (£ / 0.01 multiplier)	68.74	600.73	4578.84	9316.81	9356.69
Total use of columns (hours)	52.295	55.058	62.259	72	72

## Further research directions

The cost of disposing unused fractions is currently very low compared to the running cost of the distillation column, only 0.59% of the total cost. The effect on the optimal solution is unknown, but can be significant if these disposal costs increase dramatically. Further research has to be done to see what the effects are on dramatic changes in the disposal cost.

For environmental reasons the refinery might also wish to see what the effects will be of constraints on the total tons of fractions disposed of. This can also be part of further research.

Instead of the strict separation between the price of products sold up to the demand and exceeding the demand; reality might be better represented by a piecewise linear relation between the price of the products and the total demand. The problem would then be to find the optimal price for each product and it is likely that this way the refinery will be able to increase its profitability even further.