

# On the Role of External Constraints in a Spatially Extended Evolutionary Prisoner's Dilemma Game

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**Abstract.** We study the emergency of mutual cooperation in evolutionary prisoner's dilemma games when the players are located on a square lattice. The players can choose one of the three strategies: cooperation ( $C$ ), defection ( $D$ ) or "tit for tat" ( $T$ ), and their total payoffs come from games with the nearest neighbors. During the random sequential updates the players adopt one of their neighboring strategies if the chosen neighbor has higher payoff. We compare the effect of two types of external constraints added to the Darwinian evolutionary processes. In both cases the strategy of a randomly chosen player is replaced with probability  $P$  by another strategy. In the first case, the strategy is replaced by a randomly chosen one among the two others, while in the second case the new strategy is always  $C$ . Using generalized mean-field approximations and Monte Carlo simulations the strategy concentrations are evaluated in the stationary state for different strength of external constraints characterized by the probability  $P$ .

## INTRODUCTION

The successful applications of game theory in the area of economics and political decisions initiated its increasing development after the second world war [1]. Originally, the game theory is devoted to find the optimal strategy for a given game between two intelligent players. The straightforward developments involve the generalization toward the iterated games of  $n$  players with assuming local interactions among the spatially distributed players. The spatial evolutionary prisoner's dilemma games (SEPDG) has attracted a particular attention because of its applicability in the human and behavior sciences as well as in biology [2–6]. Nowadays the prisoner's dilemma game is considered to be the metaphor for studying the emergence of cooperation among selfish individuals. The emerging cooperation appears to be crucial at many transitions in evolution [7]. The first numerical

investigations have shown that the cooperation can be maintained by very simple strategies in the iterated games [5]. Very recently it is demonstrated that the players can be as simple as bacteriophages (viruses that infect bacteria) [8,9].

In these systems the players wish to maximize their individual income coming from games with other players. The prisoner's dilemma game is a simple version of the two-player matrix games where the players' income depend on their simultaneous choice between two options. Following the widely accepted expressions each player can choose defection or cooperation with the other player. The defector reaches the highest payoff  $t$  (called temptation to defect) against the cooperator, which receives then the lowest reward  $s$  (called sucker's payoff). For mutual cooperation [defection] each player receives the same payoff  $r$  (reward for mutual cooperation) [ $p$  (punishment)]. The game is symmetric in the sense that player's income is independent of the player itself, it depends only on their choice. The mentioned payoff values satisfy the inequalities  $t > r > p > s$  and  $2r > t + s$ . These assumptions provide the largest total payoff for the mutual cooperators. Comparing to this situation the defector reaches extra income against the cooperator whose loss exceeds the defector's benefit. Consequently, the choice of defection can be interpreted as an exploiting behavior. These are the main features for which the prisoner's dilemma games are used to study the emergence of mutual cooperation, altruism and ethic norms among selfish individuals [5,10].

The rational players should defect as this choice provides the larger income, independently of the partner's decision. However, this situation creates a dilemma for intelligent players as mutual cooperation would result in higher income for each of them than mutual defection does.

In the iterated round-robin prisoner's dilemma games we can introduce some simple evolutionary processes without assuming intelligent players (who are capable to find the best strategy if it exists). These games are started from an initial set of strategies, which defines the player's decision in the knowledge of their previous choices. The evolutionary process is devoted to model the Darwinian selection principle among  $n$  (selfish) players whose total income comes from  $n - 1$  games within a given round. In the simplest evolutionary models the worst player adopts the winner's strategy round by round.

The numerical simulations have demonstrated the crucial role of the so-called "tit for tat" strategy in the emergence of mutual cooperation. Despite of its simplicity the "tit for tat" strategy won the computer tournaments conducted by Axelrod [5]. The "tit for tat" strategy cooperates in the first step and then always repeats his co-player's previous decision. This strategy cooperates forever with all the other so-called nice strategies which never defect first. Furthermore, its defection and cooperation can be interpreted as a punishment and forgiveness when reacting to the previous decision for other strategies. The most remarkable feature of this strategy is that it is capable to sustain the mutual cooperation among themselves in the presence of defectors.

Early numerical investigations have also indicated the importance of local interactions because it favors the formation of cooperating colonies. In the simplest

models the players are distributed on a lattice and the interaction (the games between two player as well as the strategy adoption) is limited to a given neighborhood. Evidently, the short range interactions enhance the role of fluctuations at the same time. These models can be well investigated by sophisticated methods of non-equilibrium statistical physics.

For the numerical investigation of the spatial effects Nowak and May [11] have introduced an SEPDG model, which is equivalent to a two-state cellular automaton. Each lattice site can be in one of the two states  $C$  and  $D$ , representing the two simple strategy “always cooperate” and “always defect” respectively. The income for a given player (site) comes from games with its neighbors (and also with itself in some version of the model). According to the cellular automaton rule the players modify their strategy simultaneously in discrete time steps. Namely, each player adopts the best strategy found in its neighborhood. The step by step visualization of the strategy distribution on a two-dimensional lattice exhibits different spatio-temporal patterns (homogeneous and coexisting strategies, transitions between these states, competing interfacial invasions, etc.) depending on the payoff matrix, which is characterized by a single parameter. In these models the randomness is restricted to the initial states. In a subsequent work Nowak *et al.* [12] have extended the former models by allowing irrational strategy adoptions with some probability. The simulations indicated that the randomness favors the spreading of  $D$  strategies. These results have initiated systematic numerical investigations of many stochastic cellular automata [13–16].

The study of spatio-temporal patterns observed in nature, however, requires continuous time description [17,11]. Moreover, it is difficult to analyze the above mentioned stochastic cellular automata in the framework of generalized mean-field approximation, which is often used in non-equilibrium physics. To reduce the technical difficulties Szabó and Tóke have suggested a simplified dynamics [18]. The systematic investigations of this model have justified that when tuning the model parameters the stationary state undergoes two consecutive phase transitions which belong to the directed percolation (DP) universality class [18,16]. Very recently this SEPDG model has been extended by allowing three strategies for the players [19]. In the present work this three-strategy model will be compared with its simplified version. During the model descriptions and discussion, our attention will be focused on the motivations, the elementary processes and their consequences as well as on the universal features relating the SEPDGs to the area of complex systems.

## SPATIAL EVOLUTIONARY MODEL WITH THREE STRATEGIES

In the present spatial evolutionary prisoner’s dilemma game the players are located on the sites  $\mathbf{x} = (i, j)$  of a square lattice, where  $i, j = 1, \dots, L$ . To avoid the undesired boundary effects we assume periodic boundary conditions. Each player follows one of the three strategies:  $D$  defects always;  $C$  cooperates unconditionally;

$T$  accommodating to the partner's strategy chooses defection against  $D$  and cooperation with  $C$  and  $T$ . In fact the name  $T$  refers to the strategy "tit for tat" which first cooperates and later repeats the partner's previous decision. Consequently, after the first step the decisions of these two strategies are equivalent against  $C$ ,  $D$  and themselves. The consequences of the different first decisions become irrelevant if the strategy changes (defined below) are rare comparing to the frequency of games. At the site  $\mathbf{x}$  the player's strategy is denoted by a three-component unit vector whose possible values are

$$\mathbf{s}(\mathbf{x}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (1)$$

corresponding to the  $D$ ,  $C$ , and  $T$  strategies respectively. At a given time the state of the whole system is described by the variables  $\mathbf{s}(\mathbf{x})$ .

For each player the total payoff comes from the games with its four nearest neighbors. Using the above formalism the total payoff  $m(\mathbf{x})$  for the player at site  $\mathbf{x}$  is given as

$$m(\mathbf{x}) = \sum_{\delta\mathbf{x}} \mathbf{s}^*(\mathbf{x}) \mathbf{M} \mathbf{s}(\mathbf{x} + \delta\mathbf{x}) \quad (2)$$

where  $\mathbf{s}^*(\mathbf{x})$  is the transpose matrix of  $\mathbf{s}(\mathbf{x})$  and the summation runs over the four nearest neighbors ( $\delta\mathbf{x}$ ). Accepting the simplified payoff matrix suggested by Nowak and May [11]  $\mathbf{M}$  is given by the following expression:

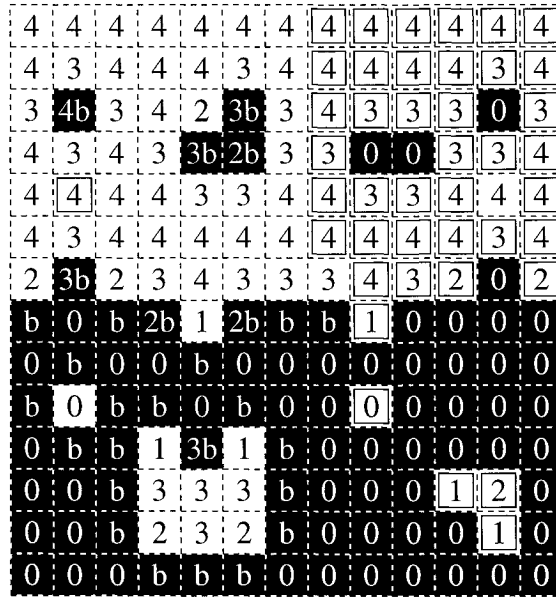
$$\mathbf{M} = \begin{pmatrix} 0 & b & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (3)$$

where the only free parameter  $b$  ( $1 < b < 2$ ) measures the temptation to defect. In the above mentioned notation the present payoff matrix corresponds to the choices:  $r = 1$ ,  $p = 0$ ,  $t = b$ , and  $s = -\varepsilon$  in the limit  $\varepsilon \rightarrow 0$ .

To model the Darwinian selection rule the players are allowed to modify their strategy. In the simplest case the system evolution is governed by random sequential updates. It means that a randomly chosen player (e.g. at site  $\mathbf{x}$ ) adopts one of its neighboring strategy,  $\mathbf{s}(\mathbf{x} + \delta\mathbf{x})$ , if  $m(\mathbf{x} + \delta\mathbf{x}) > m(\mathbf{x})$  and this elementary process is iterated many times.

Here it is worth mentioning that a state consisting only of  $C$  and  $T$  strategies leads to a uniform payoff distribution [ $m(\mathbf{x}) = 4$ ] and the above dynamics leaves this state unchanged. An example of a more complicated situation is given in Figure 1. The payoffs associated with the three different strategies,  $D$ ,  $C$  and  $T$ , are explicitly given.

The reader can easily check that inside a  $D$  region the defectors receive zero payoffs. The same is true for a solitary  $T$  surrounded by defectors. In the absence

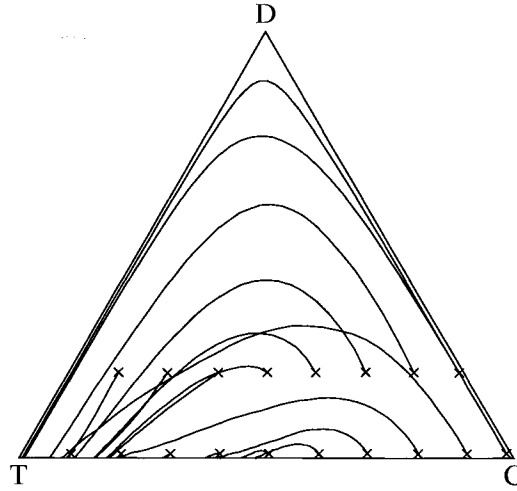


**FIGURE 1.** The payoff distribution is indicated by figures on the square lattice for a given configuration of  $D$  (black box),  $C$  (empty), and  $T$  (open box) strategies.

of  $C$  strategies, however, two (or more) neighboring  $T$  strategies will invade the  $D$  territories because their mutual cooperation gives them some incomes, while the defectors' payoff remain zero.

In the presence of  $C$  strategies, however, the above situation becomes quite different as the exploitation provides large incomes for the defectors. As a result, the defectors can invade the neighboring  $C$  or  $T$  sites for some configurations. This process dominates the time evolution for small  $T$  and large  $C$  concentrations as illustrated in a ternary diagram (see Figure 2). Note that the trajectories are two dimensional projections of a many dimensional space. Accordingly, there can be crossing of trajectories. As the average defector's payoff decreases with the  $C$  concentration, sooner or later the  $T \rightarrow D$  invasion processes will govern the system evolution and, finally, all the  $D$  strategies extinct. Figure 2 shows clearly that the ratio of  $C$  and  $T$  strategies in the final (frozen) state depends on the initial state.

It is emphasized that in the absence of  $T$  strategies the defectors will dominate the present system in the final state. It is not evident as in Figure 1 one can find many  $D$ - $C$  pairs where  $C$  beats  $D$ . In general, these pairs are located along the horizontal and vertical straight fronts separating the  $D$  and  $C$  domains. The random sequential invasions, however, makes the smooth fronts irregular and this situation generally prefers the  $D \rightarrow C$  invasion to the opposite one. As a result, the "sharp"  $D$  fronts cut the  $C$ 's domains into small pieces and finally all the  $C$ s



**FIGURE 2.** Monte Carlo results for the time evolutions in the absence of mutation if the system is started from different (uncorrelated) initial states indicated by X symbols.

will be eliminated.

The reader can easily recognize that in most of the  $C$ - $D$  (or  $T$ - $D$ ) confrontations the direction of dominance is not affected by the value of  $b$  within the prescribed region ( $1 < b < 2$ ). The systematic analysis shows that there is only one situation when the value of  $b$  becomes important. Namely, if a defector has a payoff of  $2b$  while its  $C$  (or  $T$ ) counterpart has 3. In this case,  $D$  wins if  $b > 3/2$ , otherwise  $D$  will be invaded. These types of elementary processes, however, do not modify the system behavior drastically [19], therefore the subsequent investigations will be focused on the case  $b > 3/2$ .

The above dynamical rules introduce some noises (irrational choices) in the system evolution. Now an additional (superimposing) noisy term is introduced by allowing the appearance of mutants with probability  $P$ . In fact, the effect of two different external constraints (mutation mechanisms) will be studied in models A and B as a function of  $P$ .

## Model A

In the first model, the above evolutionary rule is modified as follows. Each randomly chosen player adopts with probability  $P$  a randomly chosen strategy among the two other strategies. With probability  $1 - P$  it follows the old rule.

This model can describe the behavior of those biological and economical problems where the appearance of mutants cannot be neglected [4,2]. The main feature of this model is that this mutation mechanism does not allow the extinction of any

strategy.

## Model B

In the second model the mutation mechanism is restricted to the adoption of  $C$  strategies [19]. In other words, the randomly chosen player adopts the  $C$  strategy with probability  $P$ , otherwise it adopts one of its neighboring strategy if this neighbor has higher income. Note that in this case the extinction of the  $D$  and/or  $T$  strategies is permitted.

Model B is devoted to describe the effect of an external constraint which enforces the cooperative behavior naively by supporting an unconditional cooperation. Such a phenomenon can be observed in human societies in which any kind of social pressure enforces the  $D$  and  $T$  players to choose the  $C$  strategy. Furthermore, a  $T$  player surrounded by only cooperating strategies ( $C$  or  $T$ ) is motivated to adopt the  $C$  strategy also because of its convenience. In fact, playing  $C$  is simpler than playing  $T$ , which requires the knowledge of the previous decision of your neighbors.

## MEAN-FIELD APPROXIMATION

In the classical mean-field approximation the system is described by the strategy concentrations which satisfy the normalization condition  $c_D(t) + c_C(t) + c_T(t) = 1$ . In this approach the average payoffs are given as:

$$\begin{aligned} m_D &= bc_C , \\ m_C &= c_C + c_T , \\ m_T &= c_C + c_T . \end{aligned} \tag{4}$$

For model A, the time dependent concentrations satisfy the following equations of motion:

$$\begin{aligned} \dot{c}_D &= \frac{P}{2}(c_C + c_T - 2c_D) \mp (1 - P)c_D(c_C + c_T) , \\ \dot{c}_C &= \frac{P}{2}(c_T + c_D - 2c_C) \pm (1 - P)c_Dc_C , \\ \dot{c}_T &= \frac{P}{2}(c_D + c_C - 2c_T) \pm (1 - P)c_Dc_T , \end{aligned} \tag{5}$$

where the upper (lower) signs are valid if  $m_D < m_C = m_T$  ( $m_D > m_C = m_T$ ). In these expressions the first terms describe the effect of external constraint, the second terms come from the Darwinian selection mechanism.

After some algebraic manipulations one can easily get the following stationary solution (for  $P < 1$ ):

$$\begin{aligned}
c_D &= \frac{1 + P/2 - \sqrt{1 - P + 9P^2/4}}{2(1 - P)} , \\
c_C = c_T &= \frac{1 - c_D}{2} .
\end{aligned} \tag{6}$$

Here all the three strategies are present for arbitrary values of  $P$ . Notice that the concentrations of  $C$  and  $T$  strategies are the same due to the symmetries of Eqs. (5). In the limit  $P \rightarrow 0$ , however, the concentration of  $D$  strategy vanishes. Evidently, the concentration of the three strategies becomes equal when the evolution is governed exclusively by the mutation ( $P = 1$ ).

For model B the corresponding equations of motion are similar to those given by Eqs. (5), the differences appear in the first terms proportional to  $P$ . Namely,

$$\begin{aligned}
\dot{c}_D &= -Pc_D \mp (1 - P)c_D(c_C + c_T) , \\
\dot{c}_C &= +P(c_T + c_D) \pm (1 - P)c_Dc_C , \\
\dot{c}_T &= -P2c_T \pm (1 - P)c_Dc_T ,
\end{aligned} \tag{7}$$

where the average payoff values are given by Eqs. (4) and the conditions of validity of the upper and lower signs are defined as above. The analytical solution of these equations predicts strikingly different behavior in the stationary state [19], that is, for  $0 < P < 1/2$

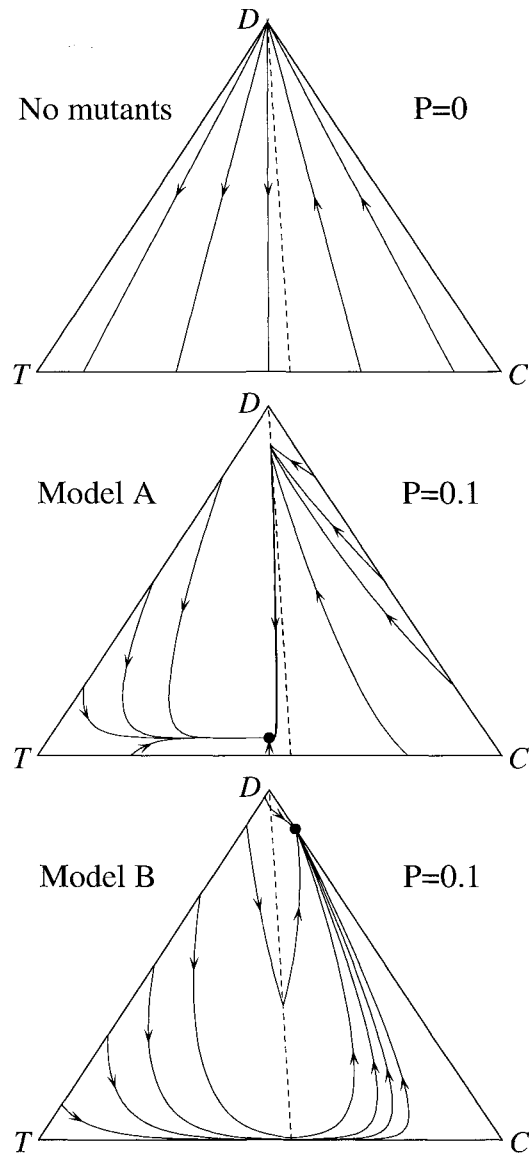
$$\begin{aligned}
c_D &= \frac{1 - 2P}{1 - P} , \\
c_C &= \frac{P}{1 - P} , \\
c_T &= 0 ,
\end{aligned} \tag{8}$$

while the system goes to the absorbing state ( $c_C = 1$  and  $c_D = c_T = 0$ ) for  $P > 1/2$ . The most surprising result is the extinction of  $T$  strategy if  $P > 0$ .

We have to emphasize the non-analytical behavior in the limit  $P \rightarrow 0$ . As illustrated in the upper plot of Figure 3, without the mutation ( $P = 0$ ) the system evolves toward either a homogeneous  $D$  state ( $c_D = 1$ ) or a mixed state composed of  $C$  and  $T$  strategies with a ratio depending on the initial conditions. However, the homogeneous  $D$  state is unstable against  $T$  invasions, therefore in the close vicinity of this state some small perturbations can drive the system toward the state of  $C+T$ . Conversely, this mixed state becomes unstable at a given concentration (where  $m_D = m_C = m_T$ ) against small perturbations increasing  $c_C$  and  $c_D$  simultaneously. In other words, the system evolves toward the  $D$  dominance when the state is positioned on the right hand side of dashed line (see the upper plot in Figure 3 as a results of fluctuations. This feature explains why the system is so sensitive to applied external constraints.

Figure 3 illustrates that for model A the mutation drives the (concentration) trajectories away from the boundaries. On the right hand side of the dashed line





**FIGURE 3.** Trajectories describing the time evolution in ternary diagrams for respectively the model with no mutation, model A ( $P = 0.1$ ) and model B ( $P = 0.1$ ). The dashed lines separate the regions where  $D$  dominates or is dominated. The fixed points are denoted by bullets.

( $m_D > m_C = m_T$ ),  $c_C$  and  $m_D$  decrease while  $c_D$  and  $c_T$  increase until one crosses the dashed line. On the left hand side all the initial states tend toward the only fixed point given by Eq. (6). For model B, however, there is no fixed point on the left hand side. In this region the external insertion of  $C$  strategies increases the value of  $c_C$  until  $m_D$  becomes larger than  $m_C = m_T$  and then the  $D$  invasion drives the system toward the fixed point defined by Eq. (8). During the  $D$  invasion the external constraint can compensate only the loss of  $C$  strategies. Consequently, the  $T$  strategies die out exponentially fast.

Notice that the variation of  $b$  leaves the fixed points unchanged, but modifies only the slope of the dashed line separating the two regions mentioned above.

Within the framework of mean-field theory, the extinction of  $T$  strategies is a consequence of the fact that here  $m_T = m_C$  [see Eq. (4)] in contrary to the spatially extended case, as illustrated in Figure 1.

## MONTE CARLO SIMULATIONS

Systematic Monte Carlo simulations have been performed on a square lattice consisting of  $L \times L$  sites with periodic boundary conditions,  $L$  varying from 200 to 1500. The larger sizes were used in the vicinity of the critical points. Each run started from a random initial state. During the simulations we have monitored the number of players playing a given strategy ( $N_\alpha$ ;  $\alpha = D, C$  or  $T$ ) and the payoffs related to a given strategy. After some relaxation time we have determined the average concentrations

$$c_\alpha = \langle N_\alpha \rangle / L^2 \quad (9)$$

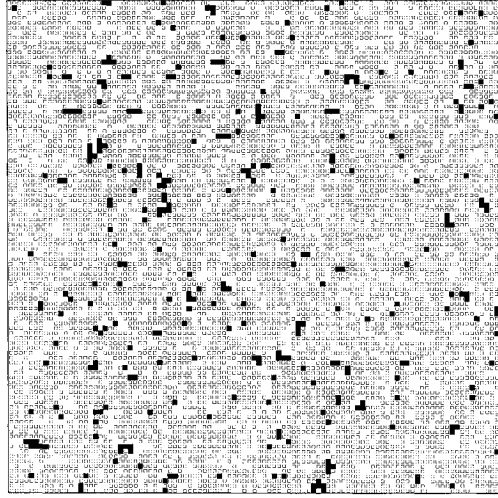
and the fluctuations

$$\chi_\alpha = L^2 \langle (N_\alpha / L^2 - c_\alpha)^2 \rangle \quad (10)$$

by averaging over a sampling time varying between  $10^4$  and  $10^6$  Monte Carlo steps (MCS) per sites. The results obtained respectively for model A and B are the following.

### Results for Model A

Figure 4 shows a typical strategy distribution for the stationary state at a small value of  $P$ . In contrary to the mean-field prediction [see Eqs. (6)] the system is dominated by the  $T$  strategies. The randomly inserted  $D$  and  $C$  strategies form small islands. Occasionally the larger  $C$  islands are occupied by  $D$ s, however, a consecutive  $T$  invasion will eliminate the larger  $D$  territories and maintains the  $T$  dominance. At the same time this process prevents the formation of large  $C$  islands inside a  $T$  domain.



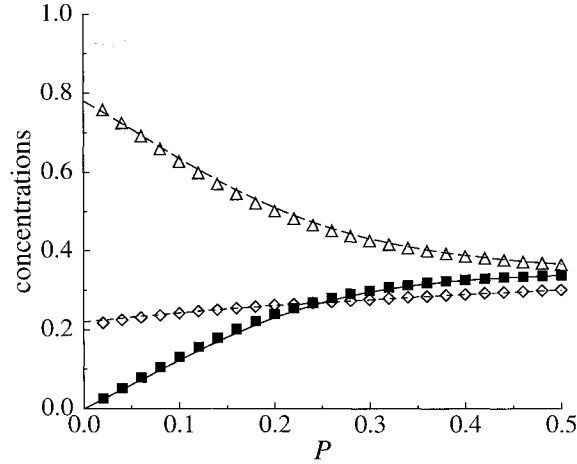
**FIGURE 4.** Typical stationary state distribution of defector (closed black square), cooperator (white area), and Tit for Tat (open square) strategies in model A, on a  $100 \times 100$  portion of a large system for  $P = 0.02$  and  $b > 3/2$ .

One can observe in Figure 5 that when increasing the value of  $P$ , the concentration of  $D$  and  $C$  strategies increases monotonously. In the limit  $P \rightarrow 1$ , the strategy distribution on the lattice tends toward a random (uncorrelated) one  $c_D = c_C = c_T = 1/3$  in agreement with the classical mean-field theory [see Eq. (6)]. In this case, instead of the neighbor invasions, the system evolution is ruled by the stochastic mutation mechanism.

As shown in Figure 5, the Monte Carlo data agree remarkably well with the results of the pair approximation. This pair approximation is considered as a generalized mean-field theory taking the nearest-neighbor correlations explicitly into account. The details of this calculation are available in many previous works [18–20]. The good agreement refers to the absence of long-range correlations which is observable in the “homogeneous” strategy distribution (see Figure 4). It is worth mentioning that the pair approximation is capable to describe the dominance of  $T$  strategies in the limit  $P \rightarrow 0$ .

## Results for Model B

In order to visualize the relevant differences between the two models at small  $P$  values the strategy distribution for model B is displayed in Figure 6. When comparing the corresponding snapshots (Figures 4 and 6) the reader can easily



**FIGURE 5.** Stationary strategy concentrations in model A as a function of  $P$  for  $b > 3/2$ . The Monte Carlo results are represented by closed squares ( $D$ ), open diamonds ( $C$ ), and open triangles ( $T$ ). The prediction of pair approximation is indicated by solid, dashed, and long-dashed lines respectively.

recognize the most striking differences. Namely, the appearance of a strongly correlated spatial structure for model B. In this case the formation of large  $C$  domains inside the sea of  $T$  strategies is not prevented by the random appearance of  $D$  mutants as happened in the previous case. The large  $C$  domains (white areas in Figure 6), however, are unprotected against the  $D$  invasion. Figure 6 shows some  $D$  domains (black areas) invading the  $C$ 's territories. These  $D$  domains are “strip-like” because their territories are invaded simultaneously by the  $T$  strategies. This invasion process is similar also for larger values of  $P$  (but  $0 < P < P_{c1}$ , see later), and only the average invasion velocity changes. On the other hand, the randomly inserted  $C$  strategies survive and accumulate in the  $T$  domains. Consequently, far behind the  $T$ - $D$  invasion front the  $T$ 's territory will be occupied by the externally inserted  $C$ s and then this area becomes unprotected against the  $D$  invasion. Sooner or later this area will be invaded by  $D$ s and the above process repeats itself. This means that the cyclic invasion maintains a self-organizing domain structure. Here we have to emphasize that this cyclic (rock-scissors-paper game like) dominance is provided by this external constraint.

Similar processes are observed in the forest-fire models [21,22] introduced by Bak et al. [23] to model the phenomenon of self-organized criticality. In these models each cell can be in one of the following three states: non-burning tree, burning tree and ash. The dynamics are governed by cyclic dominance, similarly to our model B. Note that the consequence of cyclic invasion with three (or more) states are studied in Lotka-Volterra models [24–26,20] and in cyclically dominated voter



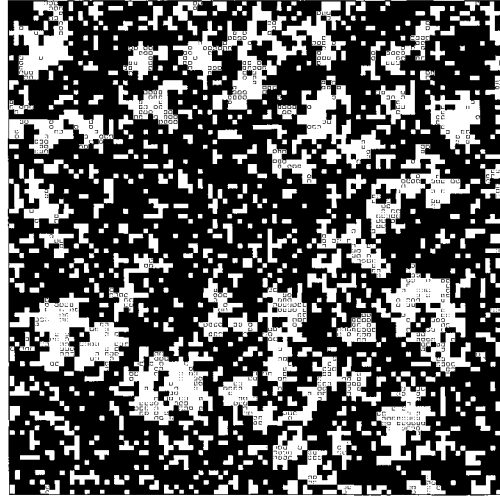
**FIGURE 6.** Typical snapshot for the stationary state of model B at  $P = 0.02$  and for  $b > 3/2$ . The symbols are the same as in Figure 4.

models [27–31].

In model B the transition from the  $T$  to  $C$  state introduces a characteristic length and time unit, both proportional to  $1/P$ . In other words, this length unit is characteristic to the typical (linear) size of the  $T+C$  domain, and the time unit corresponds to periodic time of cyclic invasion processes at a given site.

When increasing the value of  $P$ , the typical size of  $T+C$  domains decreases and the concentration of  $D$ 's increases. It is found that the  $T$  strategies die out if  $P > P_{c1} = 0.1329(1)$ . Figure 7 shows a typical snapshot in the vicinity of this critical value. In this case the external support is sufficiently strong to maintain small  $C$  clusters inside the  $D$  domains. The most remarkable feature of this snapshot is that the  $T$ 's form non-uniformly distributed small (isolated) colonies. The observation of time evolution of configuration shows that these  $T$  colonies walk randomly, they can extinct spontaneously, a single colony can split into two, or two colonies can merge. This phenomenon is analogous to the branching annihilating random walks (BARW) exhibiting a critical transition when varying the control parameters [32,33]. The corresponding critical transitions, both for our model B and for BARW, belongs to the so-called directed percolation universality class [34,35].

For  $P > P_{c1}$ , the concentration of  $D$  decreases monotonously if  $P$  is increased and vanishes at  $P = P_{c2} = 0.3678(1)$ . This extinction process is similar to the previous one, i.e. it also belongs to the DP universality class. The similarity in the correlations is recognizable in the spatial distribution of the extincting strategies



**FIGURE 7.** Stationary strategy distribution in model B for  $P = 0.13$  and  $b > 3/2$ . The symbols are the same as in Figure 4.

when comparing the snapshots displayed in Figures 7 and 8.

For  $P > P_{c2}$ , any initial state evolves toward the absorbing state where all the players follow the  $C$  strategy.

The results of our systematic investigations are summarized in Figure 9. Systematic numerical investigations in the close vicinity of the critical points show that the vanishing concentrations follow the same power law behavior. Namely,

$$\begin{aligned} c_T &= (P_{c1} - P)^\beta, \\ c_D &= (P_{c2} - P)^\beta, \end{aligned} \quad (11)$$

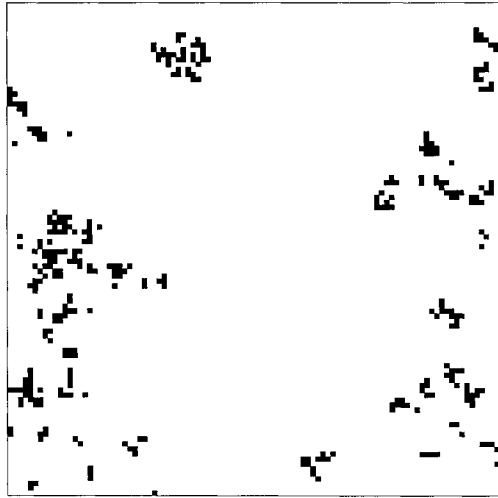
in the limits  $P_{c1} - P \rightarrow 0$  and  $P_{c2} - P \rightarrow 0$  respectively and  $\beta = 0.57(3)$  in both cases [19]. Within the statistical error this value of the exponent  $\beta$  agrees with the one of the 2+1 dimensional directed percolation [36,37].

As expected, these critical transitions are accompanied with the divergence of concentration fluctuations, i.e.

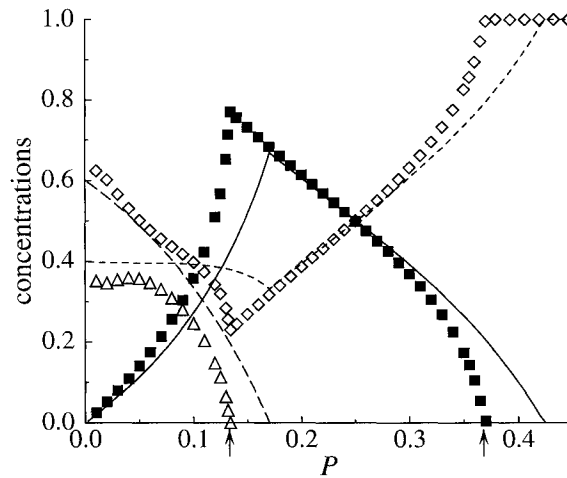
$$\begin{aligned} \chi_T &= (P_{c1} - P)^{-\gamma}, \\ \chi_D &= (P_{c2} - P)^{-\gamma}, \end{aligned} \quad (12)$$

in the vicinity of the corresponding critical points. The numerical fitting yields  $\gamma = 0.37(9)$  in agreement with the DP values [36,37,34,35].

Despite the same universal behavior there is a remarkable difference between the two extinction processes. The second extinction process (at  $P = P_{c2}$ ) results in a



**FIGURE 8.** Stationary distribution of  $C$  (white area) and  $D$  (black boxes) strategies in model B for  $P = 0.366$  and  $b > 3/2$ .



**FIGURE 9.** Simulation and pair-approximation results for the stationary concentration of strategies in model B, versus  $P$ . The notation agrees with those of Fig. 5 and the arrows indicate the critical points where  $T$  and  $D$  strategies extinct.

frozen (time independent) absorbing state. Conversely, the transition at  $P = P_{c1}$  is an example where the extinction of  $T$  strategies happens on a fluctuating background. In other words, the properties of the absorbing state (frozen or fluctuating) do not affect the critical behavior of our model.

As demonstrated in Figure 9 the results of Monte Carlo simulations are reproduced qualitatively well by the pair approximation [19]. The striking differences are related to the long-range correlations accompanying the critical transitions at  $P = P_{c1}$  and  $P_{c2}$ . Due to the strongly correlated domain structure, illustrated in Figure 6, the largest deviation can be observed for small  $P$  values. We note that the concentration fluctuations, defined by Eq. (10), also diverge in the limit  $P \rightarrow 0$ . Unfortunately, in this particular case, we could not deduce a reliable value for the exponent  $\gamma$  because of the significant size effects. Further systematic analyzes are required to clarify what happens in this limit.

## CONCLUSIONS

We have studied quantitatively the effect of external constraints on the emergence of cooperation in an evolutionary prisoner's dilemma game with three possible strategies (cooperation, defection and tit for tat). In the present spatial model the players are distributed on a square lattice and their interactions are restricted to nearest neighbors. The Darwinian selection rule is modeled by the adoption of the neighboring successful strategies. This evolutionary process is superimposed by two types of mutation mechanisms (external constraints) whose strength is characterized by a control parameter  $P$ .

The choice of these three possible strategies yields non-analytical behavior in the limit  $P \rightarrow 0$  for both the mean-field approximation and Monte Carlo simulation. The time-dependent predictions of mean-field theory are sensitive to the small perturbations.

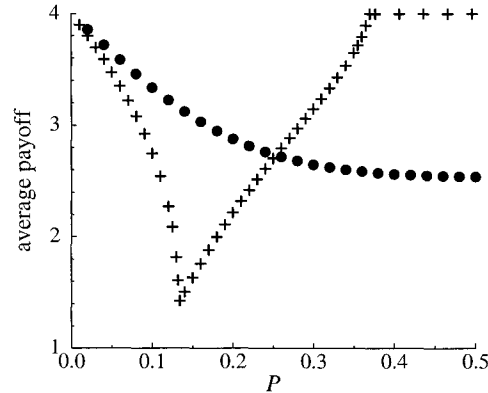
According to the Monte Carlo simulations, in the absence of external constraint the system tends toward a frozen state composed from  $C$  and  $T$  strategies whose ratio depends on the initial concentrations. For both types of external constraints (models A and B) the system evolves toward a stationary state independently of the initial condition, and the defector concentration vanishes linearly as  $P \rightarrow 0$ . In the limit  $P \rightarrow 0$ , however, model A and B will exhibit different ratio of  $C$  and  $T$  strategies. This difference is related to the appearance of self-organizing patterns for model B. The present investigation indicates that such a society of strategies (or species) are very sensitive to the type of external supports (or the ration of mutation rates).

The measure of mutual cooperation can be well characterized by the average payoff whose maximum (4) can be reached only in the absence of defectors. Figure 10 compares the Monte Carlo results for the models A and B. Surprisingly, for weak external support (small  $P$ ) the average payoff is larger for model A than for model B. In contrary to the naive expectation, the weak support of defenseless coopera-



tors results in opposite consequence. Namely, this mechanism feeds the defectors and simultaneously prevents their elimination by the retaliatory ( $T$ ) strategies.

Examples from the political and economical world justifies the above conclusions. In general, the exploiters are preferred by the governmental support for the defenseless layer of a society. The most dangerous effect is the reduction in the  $T$  type population which can maintain the mutual cooperation against the exploiters. From the view point of cooperation, it is better to help those individuals who are able to prevent themselves against the exploitation.



**FIGURE 10.** Comparison of average payoffs as a function of  $P$  for the models A (closed circles) and B (plus symbols). The sharp minimum coincides with the extinction of  $T$  strategies for model B.

Evidently, for sufficiently large  $P$  values the random insertion of  $C$  strategies can provide their dominance. In this case the A type external support is preferred to the B one if we wish to improve cooperation. Above a threshold value this type of external constraints yields a homogeneous  $C$  state which is defenseless against any defector appearing occasionally in a real system. Further systematic research is required to clarify what happens in those models where the mutation mechanism is characterized by three independent control parameters.

The present study confirms that the  $T$  strategy is able to prevent the spreading of defection in the spatial models. We have to emphasize, however, that according to the simplest mean-field theory,  $T$  dies out if the external support is of B type. Consequently, the defectors will dominate those systems where the mean-field theory is exact (e.g. infinite range of interaction, or randomly chosen partnership). In these mean-field like systems, the games between the “parent” and “its offspring” is not emphasized (they are not neighbors), which is an advantage for the defectors comparing to spatially extended models. In the light of this feature our investigations imply many interesting questions related to the transition from the “short range” spatially extended systems to the “long range” of mean-field like ones.

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