Homework 2

October 27, 2014

Homework assignment 2 (due on Friday 03/10, 2.10pm, before class starts):

1) Consider the inner product space of continuously differentiable functions $C^1[0,1]$ with the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)\overline{g(x)}\,dx \;+\; \int_0^1 f'(x)\overline{g'(x)}\,dx.$$

Show that $\langle f, \cosh \rangle = f(1)\sinh(1)$ for any $f \in C^1[0,1]$ and use this to show that the subspace

$$\{f \in C^1[0,1] \;:\; f(1) = 0\,\}$$

is a closed subspace of $C^1[0,1]$.

2) Let $(X, \|\cdot\|)$ be a n.l.s and $\{x_n\}$ a sequence in X such that

$$\sum_{i=1}^{\infty} \|x_{n+1} - x_n\| < \infty.$$

Prove that $\{x_n\}$ is Cauchy sequence. Is the converse statement true?

3) Let $(C[0,1], \|\cdot\|_2)$ be the n.l.s. with $\|\cdot\|_2$ norm. For $x \in C[0,1]$ define

$$\|x\| = \left(\int_0^1 v(t)[x(t)]^2 dt\right)^{\frac{1}{2}}$$

where $v(t)$ is continuous on $[0,1]$ and $v(t) \geq \frac{1}{\sqrt{2}}$. Prove that $\|\cdot\|$ is equivalent to $\|\cdot\|_2$.