Homework 3

October 27, 2014

(due on Friday 10/17, 2.10pm, before class starts):
1) Suppose that M is a closed subspace of a Hilbert space H.

a) Show that $M = (M^\perp)^\perp$.

b) Is there a similar statement for subspaces M which are not necessarily closed?

2) Let H be a Hilbert space and $A : H \to H$ be a linear bounded operator such that Ax is defined for every $x \in H$ (in other words the domain of A is the whole space H). Prove that

$$\|A\| = \sup_{x,y \in H, x \neq 0, y \neq 0} \frac{|\langle Ax, y \rangle|}{\|x\|\|y\|}$$