

Convergence of Probability Measures and Its Applications

Our goal in this course is to cover the techniques of weak convergence of probability measures and get familiar with applications of these techniques in various fields. The course will be delivered by Dr. Burak Buke from University of Edinburgh in 2-hour blocks every week. The classes will be delivered in traditional lecture style, but in the last two weeks, the students will be asked to present their projects. Depending on the number of students enrolled in the course, the assessment will be carried as either individual or group projects that focus on how weak convergence techniques are used for different applications. The provisional syllabus for the course is as follows:

1. Basics of Weak Convergence
 - a. Measures on Metric Spaces
 - b. Properties of Weak Convergence
 - c. Prokhorov's Theorem
2. The Space of Continuous Functions C
 - a. Uniform topology on C
 - b. Criteria for Compactness and Tightness
 - c. Donsker's Theorem and Brownian Motion
3. The Space of Right-Continuous Functions (Space D)
 - a. Four Skorokhod Topologies
 - b. Tightness in D
 - c. Sequences with Continuous Limits, C -Tightness
4. Convergence of Markov Processes
 - a. Characterization of Markov Processes
 - b. Stone's Lemma for Birth-Death processes
 - c. Convergence of Generators
5. Martingale Methods for Weak Convergence
 - a. Basic properties of Martingales, Quadratic Variations and Basic Inequalities
 - b. Martingale Central Limit Theorem
 - c. Martingale Problems for Markov Processes
6. Applications
 - a. Manufacturing and Service Industry
 - b. Telecommunication Networks
 - c. Mathematical Finance
 - d. Chemical Reactions
 - e. Mathematical Biology
 - f. Statistical Physics and Mean Field Limits