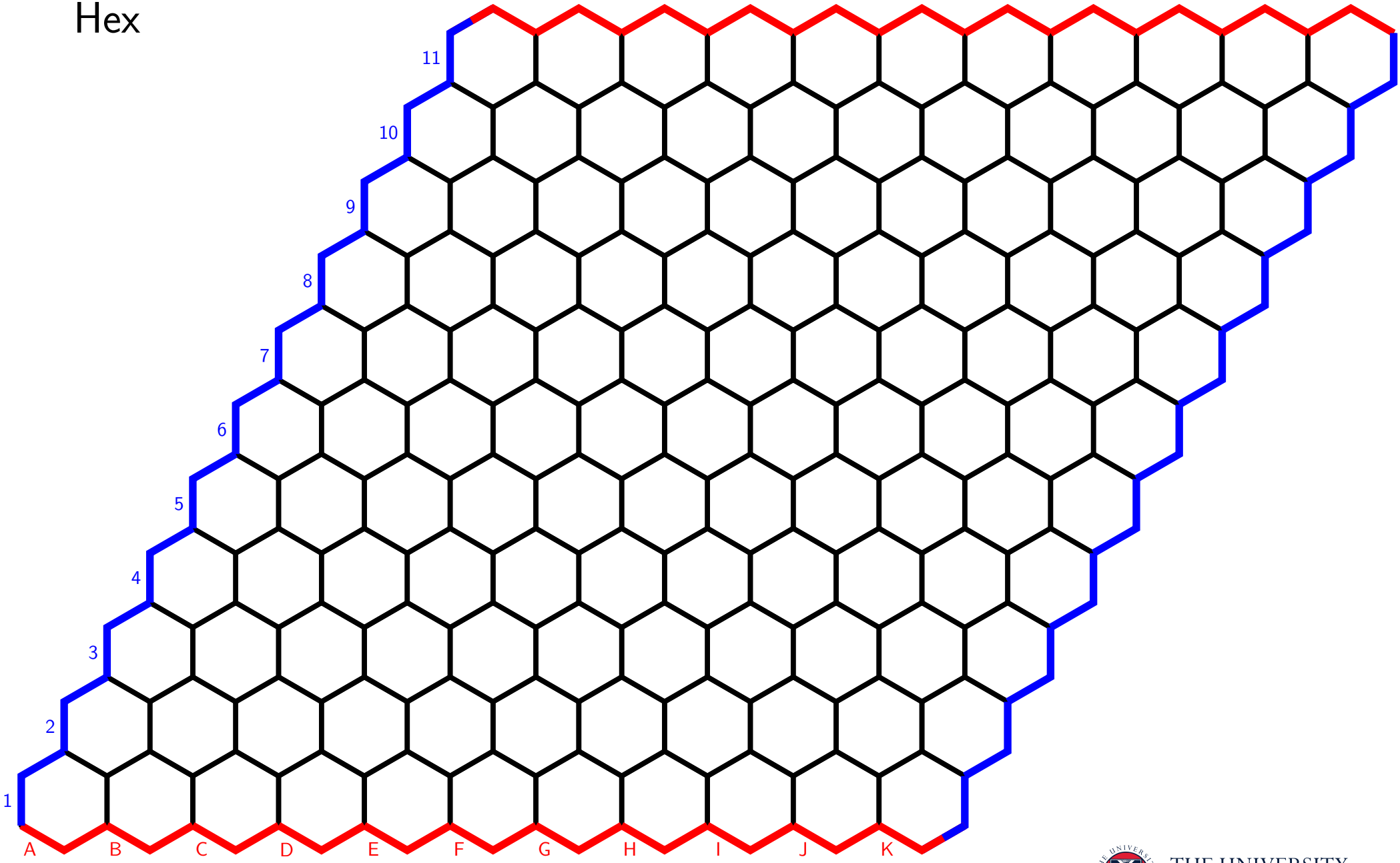
Hex



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Hex rules

Hex is a connection strategy game for two players.

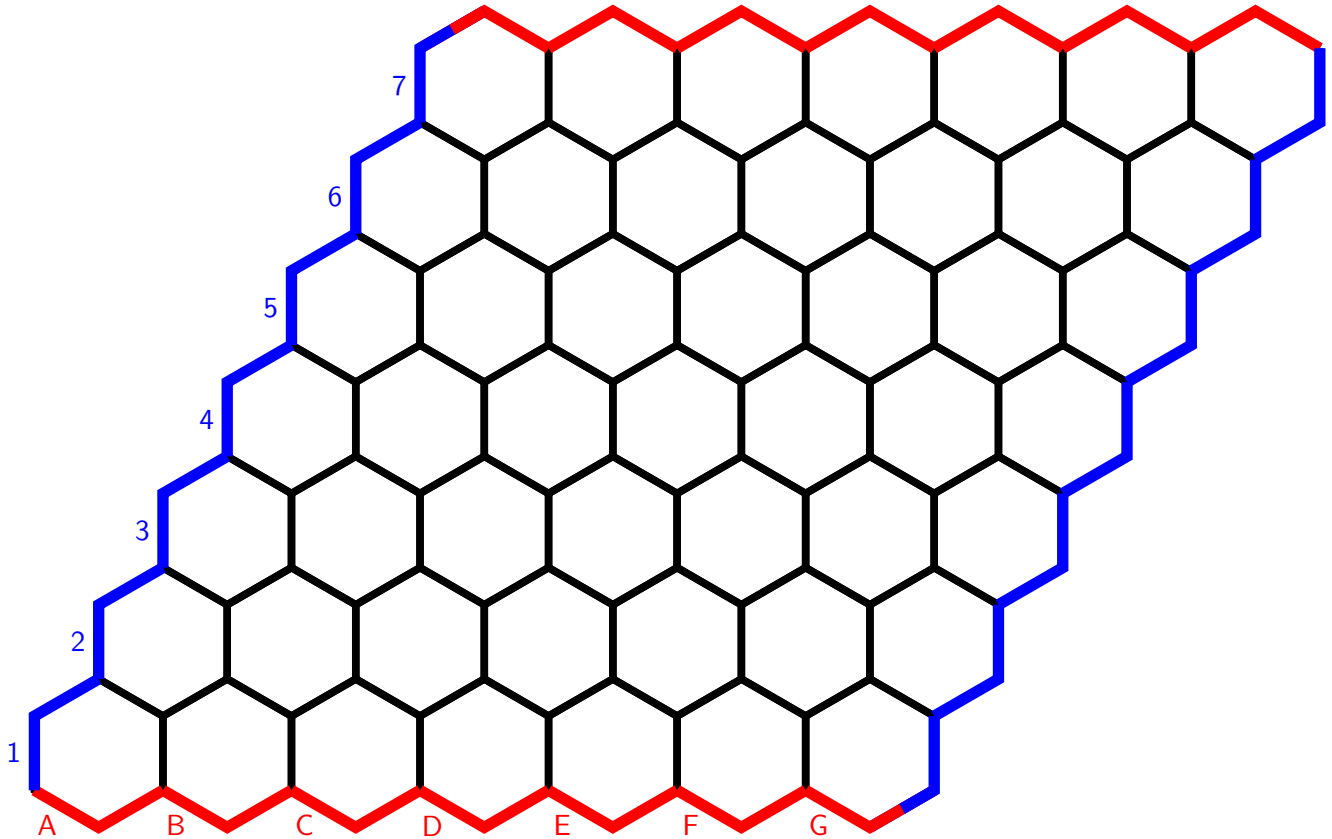
Players choose a colour and take turns. On each turn one counter is placed in an empty hexagonal cell.

Counters may not be moved except with the *swap rule*.

The first player to form a connected path of their counters linking the opposing sides of the board marked by their colour wins.

The four corner hexagons belong to both adjacent sides.

Swap rule: on their first move the second player may move normally, or choose to swap their piece with that placed by the first player. [This encourages the first player to only choose a moderately strong first move and so reduces any advantage of going first. Ignore the swap rule for the first few games.]



Bridges

The configuration below is known as a *bridge*. These two counters are strongly connected. If blue takes one empty hex then red immediately takes the other. But, beware of empty cells which are part of multiple bridges.

Basic strategy

Play defensively: defence is also attack.

Use bridges to make connections between your pieces *and* simultaneously to block your opponent.

If you can think of a strong response to your own move then look for a better one!

Never give up the game until it is clearly over but abandon areas of the board which are hopeless.

Notes

In theory any size board can be used. Start with the 7×7 board above.

For more information see C. Browne. *Hex Strategy: Making the Right Connections*. A K Peters/CRC Press, 2000.

A computer version of Hex, called *Hexy*, is available from `http://vanshel.com/Hexy` and is a worthy opponent.

Game of Y

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Y is a connection game for two players.

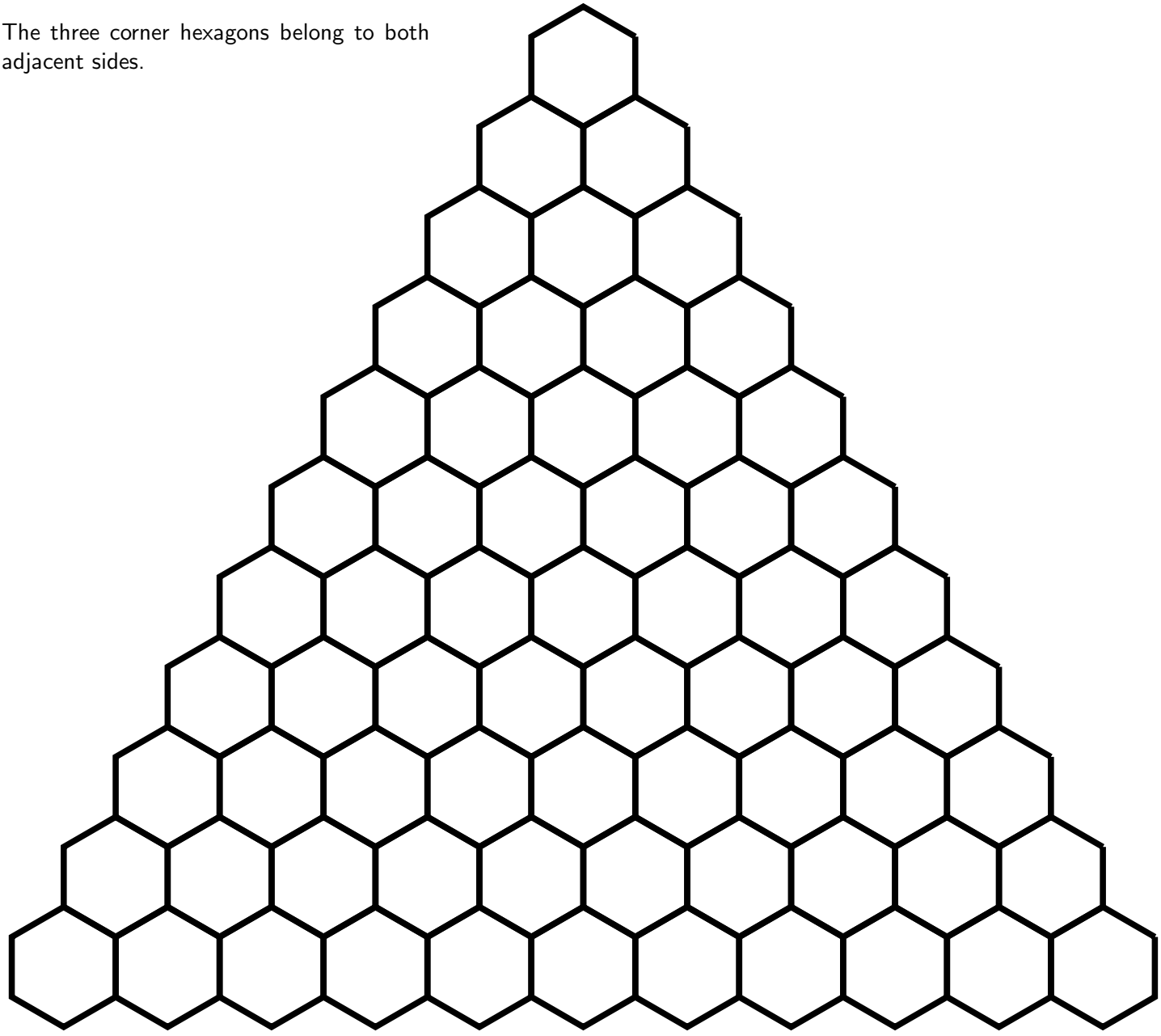
Players choose a colour and take turns. On each turn one counter is placed in an empty hexagonal cell.

Counters may not be moved except with the *swap rule*.

The first player to form a connected path of their counters linking all three opposing sides of the board wins.

The three corner hexagons belong to both adjacent sides.

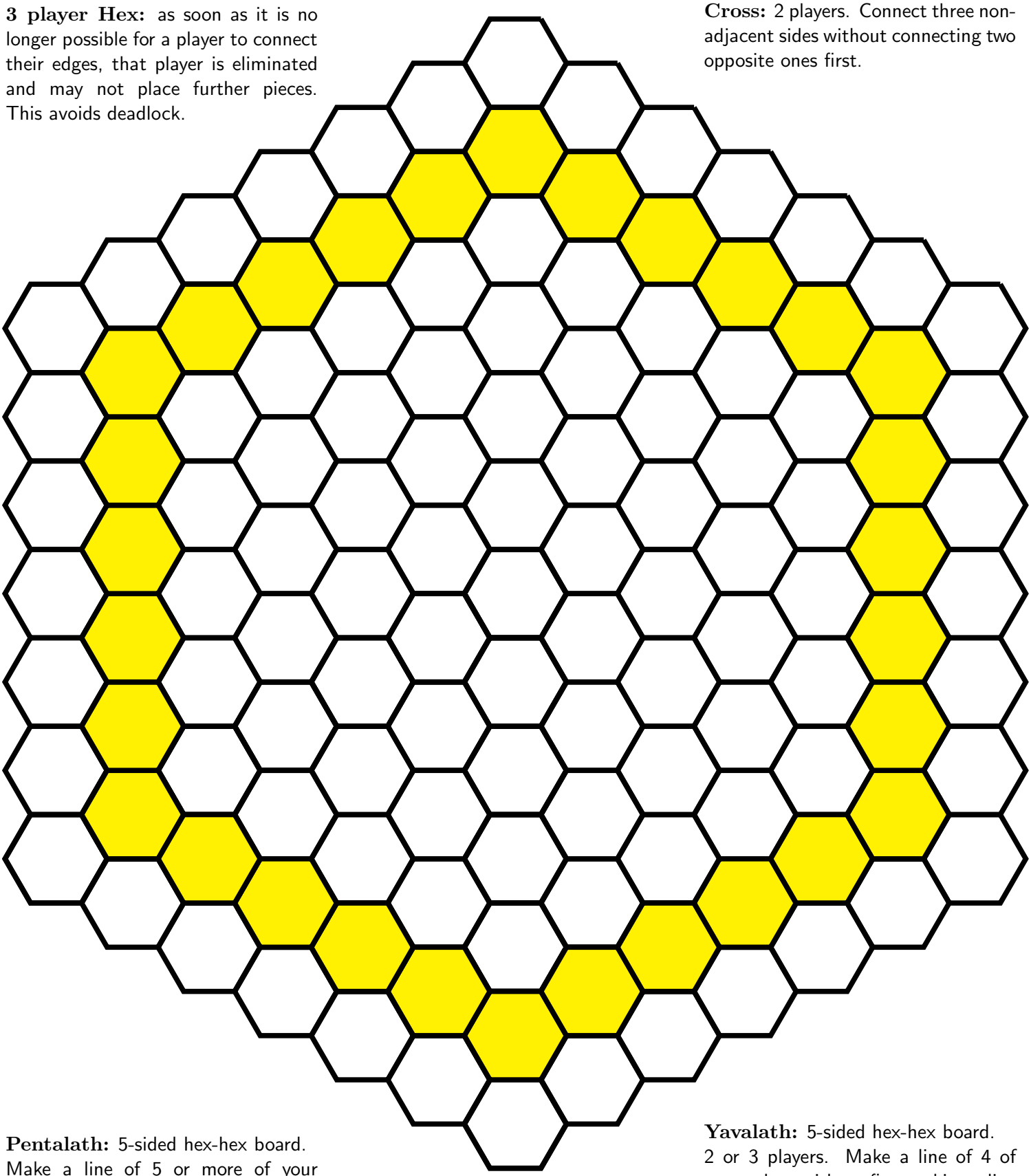
Swap rule: on their first move the second player may move normally, or choose to swap their piece with that placed by the first player. [This encourages the first player to only choose a moderately strong first move and so reduces any advantage of going first. Ignore the swap rule for the first few games.]



Cross/Pentalath/Yavalath/Three player Hex

3 player Hex: as soon as it is no longer possible for a player to connect their edges, that player is eliminated and may not place further pieces. This avoids deadlock.

Cross: 2 players. Connect three non-adjacent sides without connecting two opposite ones first.

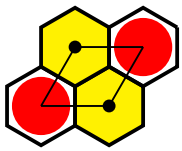


Pentalath: 5-sided hex-hex board. Make a line of 5 or more of your colour.

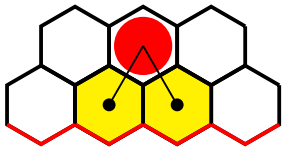
Yavalath: 5-sided hex-hex board. 2 or 3 players. Make a line of 4 of your colour without first making a line of 3.

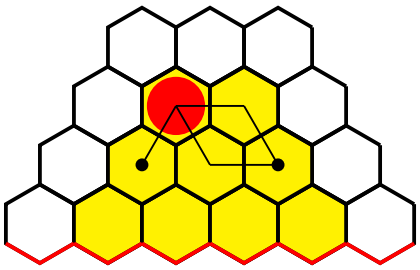
Edge templates

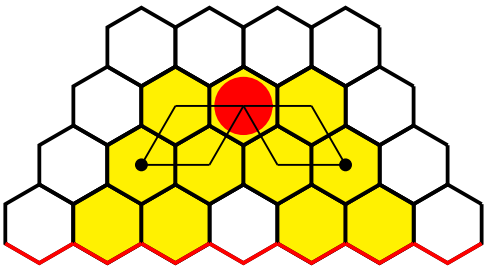
The most important template is the bridge.

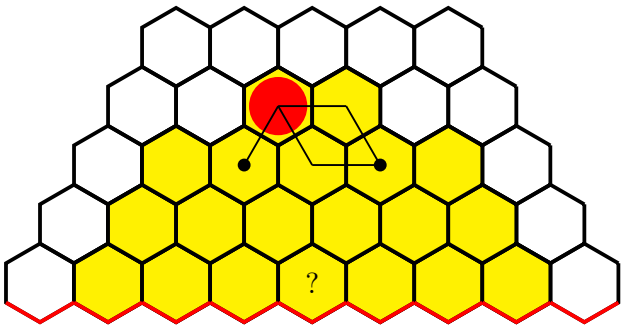


The same reasoning can be applied to *edge templates*.

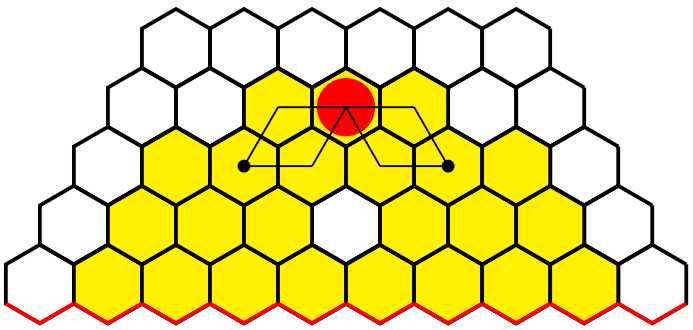


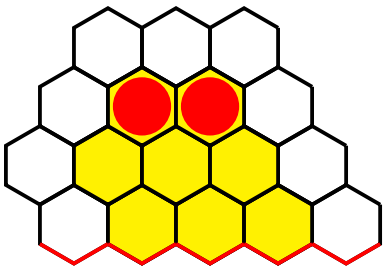


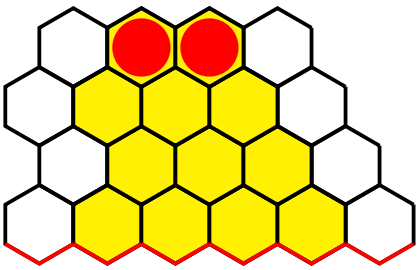




Consider what happens if blue puts a piece in the hex marked "?".

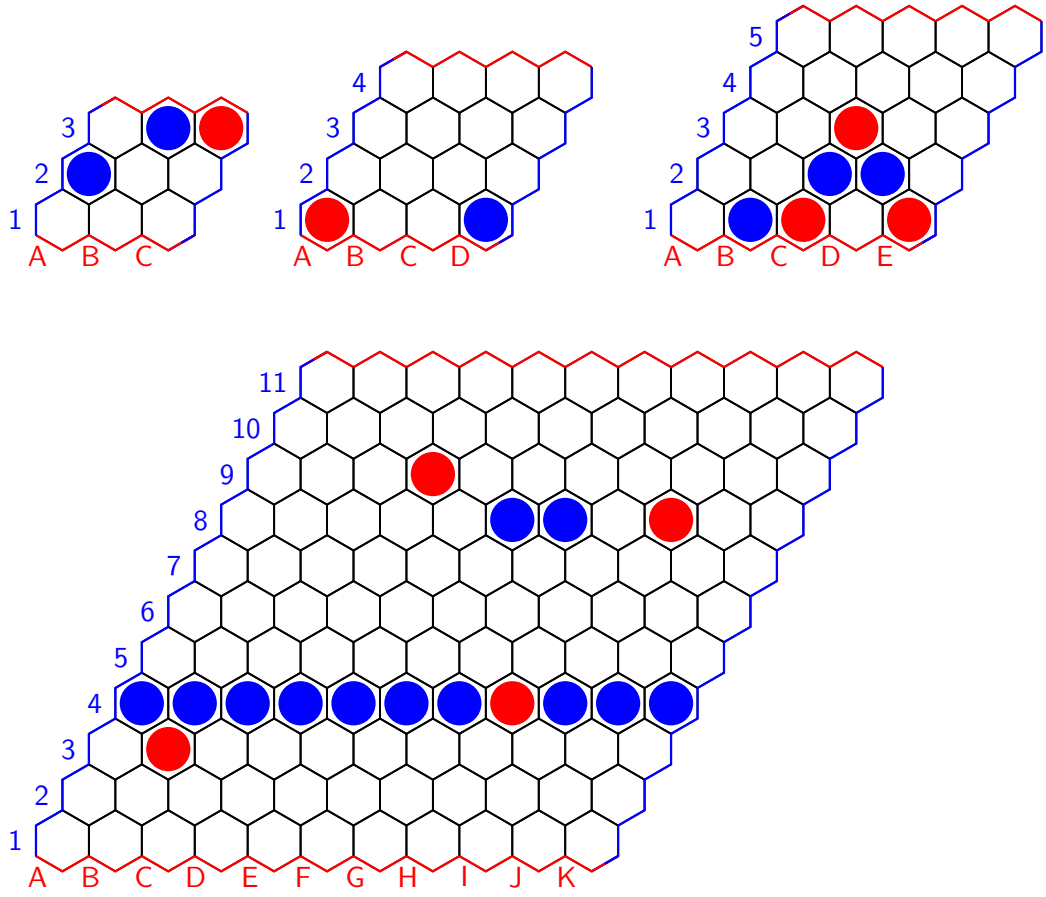






Hex puzzles

In each of these puzzles, red to move and win. The first three are taken from from [**?**], the last is from [**?**].



More puzzles and further references are given in [**?**].

Mathematics of Hex

Hex is deeply mathematical. More information is available from

- C. Browne. *Hex Strategy: Making the Right Connections.* A K Peters/CRC Press, 2000.
- C. Browne. *Connection Games: Variations on a theme.* A K Peters, 2005.

Notes for hex session helpers: things to talk about.

1. Consider the basic logic of examining positions by exhaustive cases. E.g. bridges and basic templates for positions. There are lots of hex puzzles.
2. Compare hex with a similar game on a square board. Can you find a stalemate position for square 3×3? For $n \times n$?
3. Why does hex never end in a stalemate position? What constitutes a proof here (exhaustive cases is theoretically possible, but are you going to enumerate every game?!) What other *arguments about all games* would be valid?
4. Who always wins Hex-3, playing on a 3×3 board?
5. Should the first player always win? There is a difference between knowing the first player wins and knowing a winning strategy.

You don't need expertise in game theory to raise these questions. The difference between being "good at" playing hex (or playing algebra) in practice is different from understanding theoretical concepts about hex (or algebra). This is an opportunity to try to illustrate the difference between school mathematics and the notions of proof which arise at university.

The link between mathematics and games has a long and distinguished history. More information can be found in

1. D. Wells. *Games and Mathematics: Subtle Connections.* Cambridge University Press, 2012.
2. E. R. Berlekamp, J. H. Conway, and R. K. Guy. *Winning Ways for your Mathematical Plays* (2 vols). Academic Press, 1982.

Remember:

> *the best mutual opponents are the ones having the most fun.*