Chapter Thirteen. Solving a nonlinear equation

Outline Solutions to odd-numbered exercises from the book:
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13.1 The length of the interval decreases by a factor $\frac{1}{2}$ on each interval. So the number of iterations, n, is the smallest integer n such that

$$\frac{L}{2^n} < \epsilon, \quad \text{i.e.} \quad n > \frac{\log(L/\epsilon)}{\log(2)}\,.$$

In Figure 13.2, $\dfrac{\log(L/\epsilon)}{\log(2)} = \dfrac{\log(10^6)}{\log(2)} = 19.9$, giving $n = 20$.

13.3 We know that $N(x^\star) = 2/3$, so, from Exercise 4.1, we have

$$\frac{1 + \mathrm{erf}(x^\star/\sqrt{2})}{2} = \frac{2}{3}.$$

Hence, $\mathrm{erf}(x^\star/\sqrt{2}) = 1/3$, giving $x^\star = \sqrt{2}\,\mathrm{erfinv}(1/3)$.