Chapter Fifteen. Monte Carlo method

Outline Solutions to odd-numbered exercises from the book:
An Introduction to Financial Option Valuation: Mathematics, Stochastics and Computation,
by Desmond J. Higham, Cambridge University Press, 2004
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15.1 Using "sum of means = mean of sums", that is, (3.6), we have

$$\mathbb{E}(a_M) = \mathbb{E}\left(\frac{1}{M}\sum_{i=1}^{M} X_i\right) = \frac{1}{M}\sum_{i=1}^{M}\mathbb{E}(X_i) = a.$$

15.3 Using (3.8),

$$\mathbb{E}(e^Z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^x\, e^{-x^2/2} dx = e^{1/2}\,\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(x-1)^2}{2}} dx = e^{1/2}.$$

(Note that $(1/\sqrt{2\pi})\int_{-\infty}^{\infty} e^{-\frac{(x-1)^2}{2}} dx = 1$ because this is the integral over $(-\infty,\infty)$ of the density function for a N(1,1) random variable—see (3.16).)