Chapter Two. Option Valuation Preliminaries

Outline Solutions to odd-numbered exercises from the book:
An Introduction to Financial Option Valuation:
Mathematics, Stochastics and Computation,
by Desmond J. Higham, Cambridge University Press, 2004
ISBN 0521 83884 3 (hardback)
ISBN 0521 54757 1 (paperback)

2.1 We have

$$e^{rt} D_0 = \left(1 + \frac{r_c t}{m}\right)^m D_0.$$

This rearranges to

$$r_c = m\left(e^{rt/m} - 1\right)/t.$$

Then, using the given approximation,

$$r_c \approx m\left(1 + rt/m - 1\right)/t = r.$$

2.3 Suppose the time-zero value of $\widehat{\pi}_B$ is greater than the time-zero value of π_A, that is, $S > C + Ee^{-rT}$. Then we could sell $\widehat{\pi}_B$ and buy π_A at time zero. This gives us a profit at time zero. But we know that the portfolio $\widehat{\pi}_B$ has a payoff that is never greater than that of π_A. Hence, we can be certain that at expiry we will not to have to pay out more than we gain. So we have locked into a guaranteed profit, at no cost. This violates the no-arbitrage assumption. Hence, by contradiction, we must have $S \le C + Ee^{-rT}$.

2.5 Consider

- holding a European put option with exercise price E_1, and
- holdng a European put option with exercise price E_3, and
- writing two European put options with exercise price $E_2 = \frac{1}{2}(E_1 + E_3)$.

The value at expiry is

$$\max(E_1 - S, 0) + \max(E_3 - S, 0) - 2\max(\tfrac{1}{2}(E_1 + E_3) - S, 0)$$

To show that this matches the payoff in Exercise 1.3, we note that it is piecewise linear with corners at $S = E_1$, $S = E_3$ and $S = \frac{1}{2}(E_1 + E_3)$. At $S = E_1$ the payoff is

$$0 + E_3 - E_1 - 2(\tfrac{1}{2}(E_1 + E_3) - E_1) = 0.$$

At $S = \frac{1}{2}(E_1 + E_3)$ the payoff is

$$0 + E_3 - \tfrac{1}{2}(E_1 + E_3) + 0 = \tfrac{1}{2}(E_3 - E_1).$$

At $S = E_3$ the payoff is

$$0 + 0 + 0 = 0.$$

Hence, we have matched the payoff from Exercise 1.3.

Let $C(E_1, 0)$ denote the time-zero value of the call with exercise price E_1. With this notation, the difference between the time zero values of the two portfolios is

$$[C(E_1,0) - P(E_1,0)] + [C(E_3,0) - P(E_3,0)] - 2\,[C(E_2,0) - P(E_2,0)]\,. \quad (1)$$

Now put-call parity (2.2) says (with $t = 0$):

$$\begin{aligned} C(E_1,0) - P(E_1,0) &= S_0 - E_1 e^{-rT}, \\ C(E_2,0) - P(E_2,0) &= S_0 - E_2 e^{-rT}, \\ C(E_3,0) - P(E_3,0) &= S_0 - E_3 e^{-rT}. \end{aligned}$$

In (1) we get

$$S_0 - E_1 e^{-rT} + S_0 - E_3 e^{-rT} - 2(S_0 - E_2 e^{-rT}) = e^{-rT}\left(2E_2 - (E_1 + E_3)\right) = 0,$$

as required.