Chapter Four. Computer Simulation

Outline Solutions to odd-numbered exercises from the book:
An Introduction to Financial Option Valuation:
Mathematics, Stochastics and Computation,
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4.1 Using the subs. $t = s/\sqrt{2}$, we have

$$\begin{aligned} \operatorname{erf}(x/\sqrt{2}) = \frac{2}{\sqrt{\pi}} \int_0^{x/\sqrt{2}} e^{-t^2}\, dt &= \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-s^2/2}\, ds/\sqrt{2} \\ &= 2\left(\frac{1}{\sqrt{2\pi}} \int_0^{x} e^{-s^2/2}\, ds\right) \\ &= 2\left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-s^2/2}\, ds - \frac{1}{2}\right) \\ &= 2\left(N(x) - \frac{1}{2}\right). \end{aligned}$$

This rearranges to

$$N(x) = \frac{1}{2}\left(1 + \operatorname{erf}(x/\sqrt{2})\right).$$

4.3 We have $N(z(p)) = p$. So, using Exercise 4.1,

$$\frac{1 + \operatorname{erf}(z(p)/\sqrt{2})}{2} = p.$$

Hence,

$$\operatorname{erf}(z(p)/\sqrt{2}) = 2p - 1.$$

So,

$$z(p)/\sqrt{2} = \operatorname{erfinv}(2p - 1)$$

and the result follows immediately.