Chapter Nine. More on hedging

Outline Solutions to odd-numbered exercises from the book:
An Introduction to Financial Option Valuation:
Mathematics, Stochastics and Computation,
by Desmond J. Higham, Cambridge University Press, 2004
ISBN 0521 83884 3 (hardback)
ISBN 0521 54757 1 (paperback)

9.1 Since d_1 is finite and the integrand is positive, we must have

$$0 < N(d_1) := \int_{-\infty}^{d_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2}\, ds < 1$$

and the result follows.

9.3 Repeating the arguments used in Exercise 8.3, we have, as $t \to T^-$:

Case 1: If $S < E$ then $\log(S/E) < 0$, so $d_1 \to -\infty$ and $N(d_1)$ tends to zero.

Case 2: If $S = E$ then $d_1 \to 0$ and $N(d_1)$ tends to $\frac{1}{2}$.

Case 3: If $S > E$ then $\log(S/E) > 0$, so $d_1 \to \infty$ and $N(d_1)$ tends to one.

9.5 For $t \approx T$ there is little time left for the asset value to change—if it is currently in/out-of-the-money then it will probably remain in/out-of-the-money. In particular, if the put option is in-the-money then any (upward or downward) movement in the asset corresponds almost directly to an equal and opposite (downward or upward) movement in the payoff. In other words, the put option and the asset have exactly the opposite risk. Since the portfolio is designed to replicate the risk in the option, it follows that it will hold approximately -1 unit of asset, so $\Delta_i \approx -1$. Conversely, if the put option is out-of-the-money close to expiry then the payoff is very likely to be zero whatever happens to the asset—there is no risk, so we should not be holding any asset.