

## 2021/22 Semester 2

## Stochastic Control and Dynamic Asset Allocation

## Problem Sheet 4 - Last updated 18th January 2022

Recall the optimal liquidation problem: trader's inventory, an  $\mathbb{R}$ -valued process:

$$dQ_u = -\alpha_u du \text{ with } Q_t = q > 0 \text{ initial inventory.}$$

Here  $\alpha$  will typically be mostly positive as the trader should sell all the assets. We will denote this process  $Q_u = Q_u^{t,q,\alpha}$  because clearly it depends on the starting point  $q$  at time  $t$  and on the trading strategy  $\alpha$ . Asset price, an  $\mathbb{R}$ -valued process:

$$dS_u = \lambda \alpha_u du + \sigma dW_u, \quad S_t = S.$$

We will denote this process  $S_u = S_u^{t,S,\alpha}$  because clearly it depends on the starting point  $S$  at time  $t$  and on the trading strategy. Here  $\lambda \geq 0$  controls how much permanent impact the trader's own trades have on its price. Trader's execution price (for  $\kappa > 0$ ):

$$\hat{S}_t = S_t - \kappa \alpha_t.$$

This means that there is a temporary price impact of the trader's trading: she doesn't receive the full price  $S_t$  but less, in proportion to her selling intensity.

Quite reasonably we wish to maximize (over trading strategies  $\alpha$ ), up to to some finite time  $T > 0$ , the expected amount gained in sales, whilst penalising the terminal inventory (with  $\theta > 0$ ):

$$J(t, q, S, \alpha) := \mathbb{E} \left[ \underbrace{\int_t^T \hat{S}_u^{t,S,\alpha} \alpha_u du}_{\text{gains from sale}} + \underbrace{Q_T^{t,q,\alpha} S_T^{t,S,\alpha}}_{\text{val. of inventory}} - \underbrace{\theta |Q_T^{t,q,\alpha}|^2}_{\text{penalty for unsold}} \right].$$

The goal is to find

$$V(t, q, S) := \sup_{\alpha} J(t, q, S, \alpha).$$

**Exercise 4.1** (Optimal liquidation with no permanent market impact). Solve the optimal liquidation problem above in the case  $\lambda = 0$  (i.e. there is no permanent price impact of our trading on the market price).

*Hint:* Follow the linear-quadratic example in the lecture notes but in this case the Ricatti ODE arising here has an explicit solution.