

● means learn everything

● means learn everything but no need to know proofs

● means we didn't cover this material this year

Stochastic Control and Dynamic Asset Allocation*

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Contents

1	Introduction to stochastic control through examples	4
1.1	Discrete space and time	4
1.2	Merton's problem	5
1.3	Optimal liquidation problem	8
1.4	Systemic risk - toy model	10
1.5	Optimal stopping	11
1.6	Basic elements of a stochastic control problem	11
1.7	Exercises	13
1.8	Solutions to Exercises	15
2	Controlled Markov chains	20
2.1	Problem setting	20
2.2	Dynamic programming for controlled Markov chains	20
2.3	Bellman equation for controlled Markov chain	22
2.4	Q-learning for unknown environments	23
2.5	Robbins-Monro algorithm	24
2.6	The Q-learning algorithm	25
2.7	Exercises	26
2.8	Solutions to Exercises	26
3	Stochastic control of diffusion processes	27
3.1	Equations with random drift and diffusion	27
3.2	Controlled diffusions	33
3.3	Stochastic control problem with finite time horizon	34
3.4	Exercises	36

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3.5	Solutions to Exercises	37
4	Dynamic programming and the HJB Equation	40
4.1	Dynamic programming principle	40
4.2	Hamilton-Jacobi-Bellman (HJB) and verification	44
4.3	Solving control problems using the HJB equation and verification theorem	48
4.4	Policy Improvement Algorithm	53
4.5	Exercises	54
4.6	Solutions to Exercises	55
5	Applications in algorithmic trading and market making	60
5.1	Poisson process with controlled jump intensity	60
5.2	Controlled diffusions with jumps	64
5.3	Bellman principle and Bellman PDE for diffusions with jumps	64
5.4	Optimal execution with limit orders	66
5.5	Optimal limit order spread in market making	68
5.6	Solution to Exercises	71
6	Pontryagin maximum principle and BSDEs	73
6.1	Non-rigorous Derivation of Pontryagin's Maximum Principle	73
6.2	Deriving a Numerical Method from Pontryagin's maximum principle	76
6.3	Backward Stochastic Differential Equations (BSDEs)	76
6.4	Pontryagin's Maximum Principle as Sufficient Condition	81
6.5	Variational connection to HJB equation	89
6.6	Exercises	93
6.7	Solutions to the exercises	94
A	Appendix	95
A.1	Basic notation and useful review of analysis concepts	95
A.2	Some useful results from stochastic analysis	99
A.3	Other useful results	109
A.4	Solutions to the exercises	117
	References	117

Reading these notes

The reader is expected to know basic stochastic analysis and ideally a little bit of financial mathematics. The notation and basic results used throughout the notes are in Appendix A.