ADMM-based Unit and Time Decomposition for Price Arbitrage by Cooperative Price-Maker Electricity Storage Units

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Joint work with Miguel F. Anjos[1] and James R. Cruise[2]

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Introduction - The Electric Power System

- Electric power systems must be in close balance at any time:

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- Traditional approach: Keep flexible power plants on call:
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- Challenges:
  1. Hydro resources are limited.
  2. Other resources produce carbon emissions.

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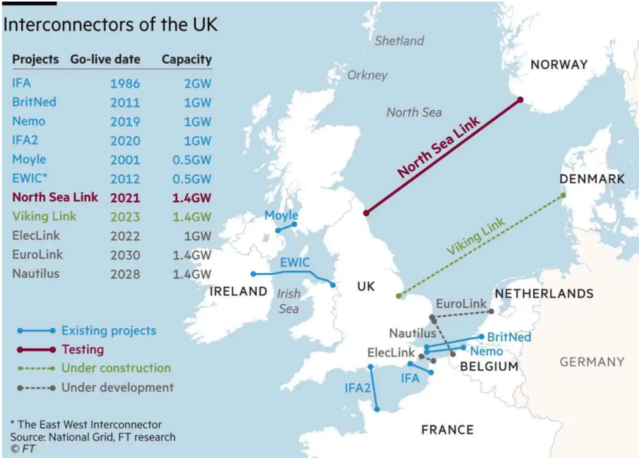
This increases flexibility needs for two reasons:

1. Higher shares of wind, solar and/or nuclear.
2. Reduction of the sources that currently provide a substantial amount of flexibility: gas, diesel and coal.

Introduction - *Green* Sources of Flexibility

1. A well-connected grid



Projects	Go-live date	Capacity
IFA	1986	2GW
BritNed	2011	1GW
Nemo	2019	1GW
IFA2	2020	1GW
Moyle	2001	0.5GW
EWIC*	2012	0.5GW
North Sea Link	2021	1.4GW
Viking Link	2023	1.4GW
ElecLink	2022	1GW
EuroLink	2030	1.4GW
Nautilus	2028	1.4GW

Figure: Interconnectors between the UK and other European countries: operational (blue and purple) and under construction or planned (other).

Introduction - *Green* Sources of Flexibility

2. Electric energy storage

Figure: A pumped-storage hydro station.

Figure: A lithium-ion battery.

Introduction - *Green* Sources of Flexibility

3. Demand-side response

Figure: The canal network and an electric vehicle.

Figure: A fridge and an electric water heater.

Introduction - *Green* Sources of Flexibility

4. Fossil fuels + carbon capture and storage

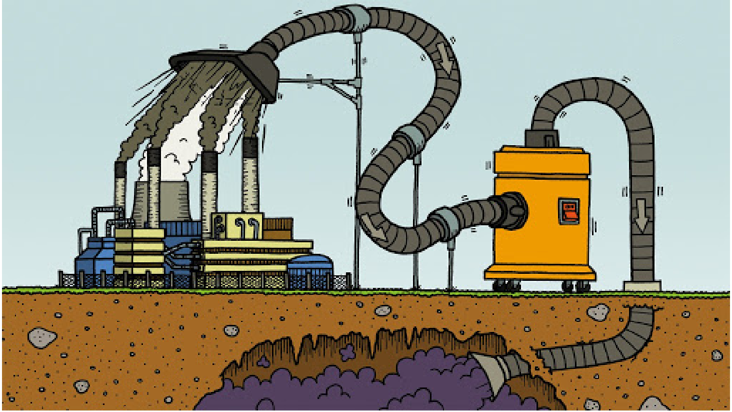


Figure: Liberal representation of carbon capture and storage.

Research Question

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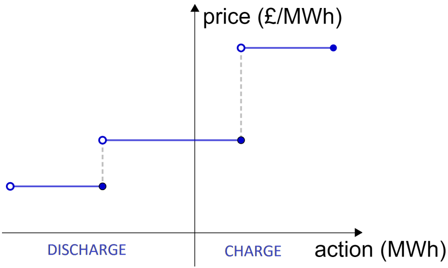
We extend previous work on a single unit by Cruise et al. (2019), and our previous work on two units (Anjos, Cruise, SV (2020)).

Modelling Price-Makers

Residual Demand Curve



Given an action, provides the market clearing price.

Modelling Price-Makers

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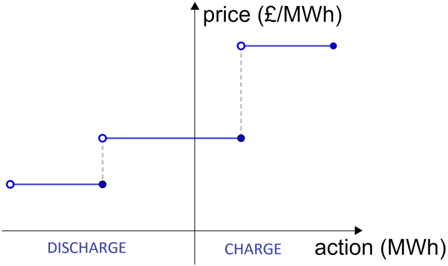

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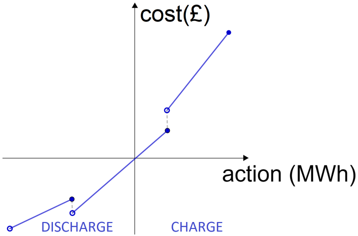
Cost Function C_t



Given an action, provides the total cost or reward of that action.

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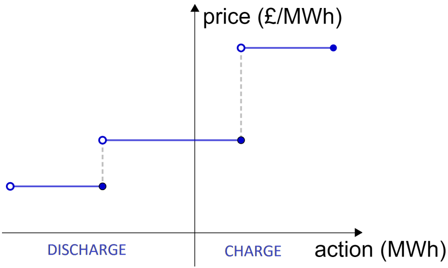

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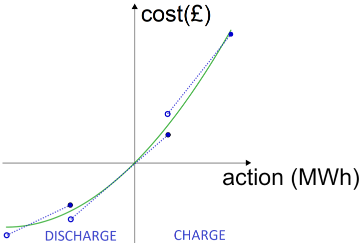

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Key assumption: C_t is convex.

Mathematical Formulation

Let $x_{j,t} \in \mathbb{R}$ be the action taken by unit $j \in \mathcal{S}$ at time $t \in \mathcal{T}$.

$$\underset{x_{j,t}}{\text{Minimize}} \qquad \sum_{t \in \mathcal{T}} C_t \Big(\sum_{j \in \mathcal{S}} x_{j,t} \Big)$$

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*The model can account for round-trip efficiencies, leakage and negative electricity prices.

Decomposition by Unit

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$$\underset{x_{j,t},z_t}{\text{Minimize}} \quad \sum_{t\in\mathcal{T}} C_t(z_t)$$

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Decomposition by Unit

$$\underset{x_{j,t},z_t}{\text{Minimize}} \quad \sum_{t\in\mathcal{T}}\left[C_t(z_t)+\nu_t\left(\sum_{j\in\mathcal{S}}x_{j,t}-z_t\right)+\frac{\gamma}{2}\left(\sum_{j\in\mathcal{S}}x_{j,t}-z_t\right)^2\right]$$

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This results in $N + 1$ 1-Unit Subproblems:

- A subproblem for every storage unit.
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They are solved iteratively until the linking constraints are satisfied.

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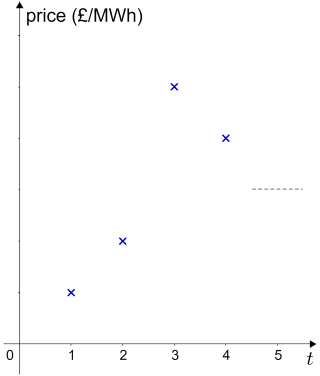

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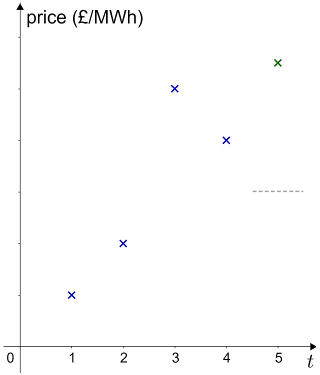

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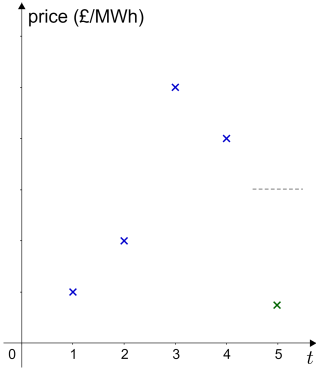

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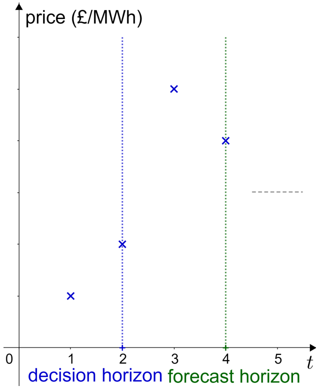

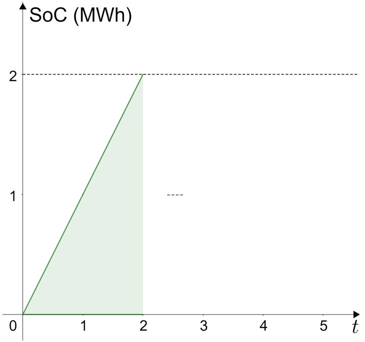

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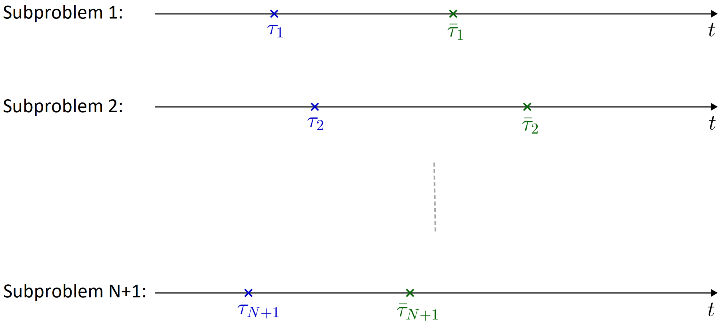

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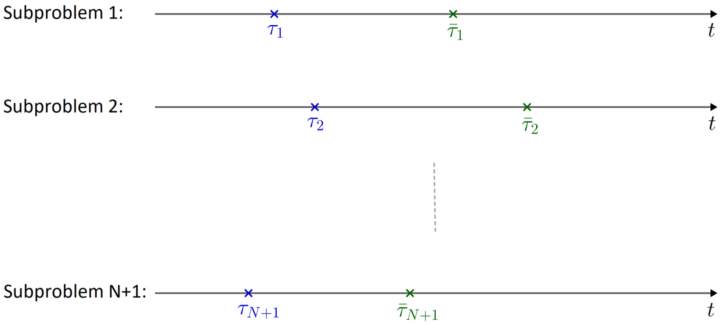

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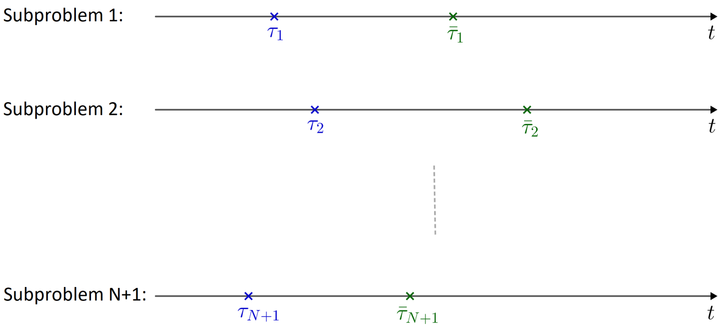

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↓

It is the forecast horizon of the unit with the largest energy-to-power ratio.

Results

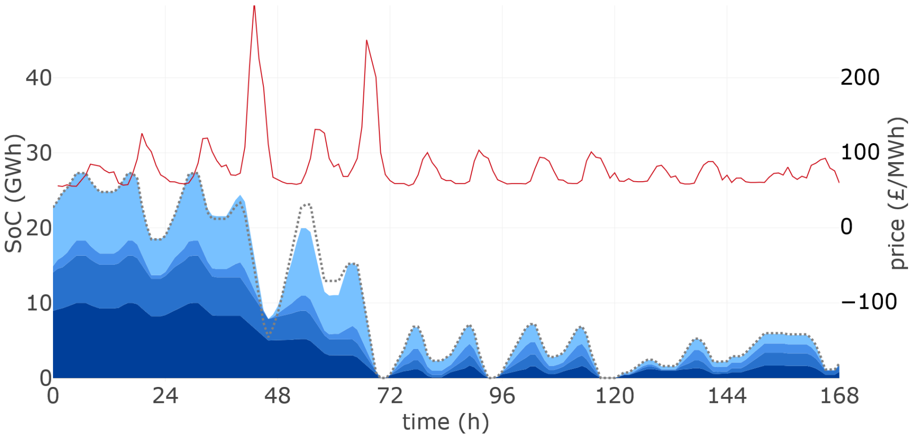

Figure: Left axis: Stacked SoC of unit 1 [very dark blue], unit 2 [dark blue], unit 3 [light blue] and unit 4 [very light blue]. SoC of aggregated unit [dotted grey line]. Right axis: Electricity market-clearing prices [red].

Computational Performance I

Instance	Anjos et al. (2020)	Anjos et al. (2021)	Gap
Oct '19	$\mathbf{-6.231 \times 10^6}$	-6.230×10^6	0.02%
Nov '19	$\mathbf{-5.689 \times 10^6}$	-5.688×10^6	0.02%
Dec '19	———	$\mathbf{-7.091 \times 10^6}$	———
Jan '20	$\mathbf{-5.988 \times 10^6}$	-5.986×10^6	0.03%
Feb '20	-6.558×10^6	$\mathbf{-6.953 \times 10^6}$	−6.03%
Mar '20	$\mathbf{6.384 \times 10^6}$	6.378×10^6	0.09%

Table: Objective function values and gap between the methods in Anjos, Cruise, SV (2020) and Anjos, Cruise, SV (2021). One month instances of 2-Unit Problems.

Computational Performance II

Month	Anjos et al. (2020)	Anjos et al. (2021)
Oct '19	91s	0.83s
Nov '19	89s	0.54s
Dec '19	——	0.45s
Jan '20	112s	0.56s
Feb '20	83s	0.28s
Mar '20	103s	0.19s

Table: Computational time comparison of the solution methods in Anjos, Cruise, SV (2020) and Anjos, Cruise, SV (2021). One month instances of 2-Unit Problems.

Computational Performance III

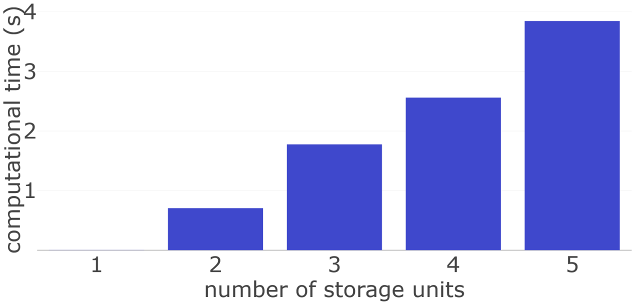

Figure: Average computational time of one month instances for different number of storage units.

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- Linear scaling of computational time w.r.t. the number of units.
- The energy-to-power ratio of storage units plays a crucial role.

References

Anjos MF, Cruise JR, Solà Vilalta A (2020) Control of two energy storage units with market impact: Lagrangian approach and horizons. *Proc. 2020 International Conference on Probabilistic Methods Applied to Power Systems* (PMAPS IEEE, Liege), 1–6.

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