Capacity equilibria in energy production under risk aversion & risk trading

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Outline

Motivation from electricity capacity equilibria under uncertainty & perfect competition

Electricity capacity expansion is kind of stochastic equilibrium

Invest Today: In stage 1, generator (genco) makes investments in different technologies (power plants)

- Later consider 3 technologies: Coal Steam Turbine (CST), Combined Cycle Gas Turbine (CCGT), Gas Turbine (GT) (Can deal with any no. of gencos, consumers, technologies)

Operate in Uncertain Tomorrow: In stage 2, operating cost of portfolio of plants is stochastic, depends on scenario ω

- Fuel & C prices, weather (demand) depend on ω — stochastic data
- Perfect competition sets price P_ω that clears energy market
  - Endogenous to equilibrium
  - Agents do not see their affect on price: perfect competition

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Start with review of risk neutral capacity equilibrium

We will review Risk Neutral (RN) perfectly competitive case

- To assess uncertain outcomes, take an average
- Economic interpretation of **optimization** as **equilibrium**
- Basis for stochastic MARKAL — long term capacity planning

Genco's two stage RN capacity problem

Genco minimises **Investment** + **Average Operating** costs:

$$\min_{x} \sum_j \mathbf{I_j}(\mathbf{x_j}) + \mathbb{E}_{\mathbf{\Pi_0}}\left[\mathcal{Q}_{\mathbf{g}}(\mathbf{x}, \mathbf{P})\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

Stage 1, investment

- There are $j = 1, \ldots, J$ energy technologies (plants)
- $I_j(x_j) :=$ convex investment cost of plant j, capacity x_j
- $\mathcal{X} :=$ closed convex set of feasible technology designs, any $x = \left(x_j\right)_j \in \mathcal{X}$ specifies portfolio of plants

Stage 2, uncertain cost of production

- $\Pi_0 =$ probability density (PD) over scenarios $\omega = 1, .., K$
- $P = \left(P_\omega\right) =$ prices in all future scenarios
- $\mathcal{Q}_g(x, P) := \left(\mathcal{Q}_{g\omega}(x, P_\omega)\right)_\omega$ has expectation $\mathbb{E}_{\Pi_0}\left[\mathcal{Q}_g(x, P)\right]$
  $\mathcal{Q}_{g\omega}(x, P_\omega) :=$ generator's operating costs net of revenue, or negative profit, in scenario ω

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Stage 2: Spot market in scenario ω

Fix plant capacities $x \geq 0$ and spot market scenario ω.

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$$\mathcal{Q}_{\mathbf{g}\omega}(\mathbf{x}, \mathbf{P}_\omega) := \min_{Y_\omega} \sum_j C_{j\omega}(Y_{j\omega}) - P_\omega \sum_j Y_{j\omega} \quad \text{s.t.} \quad 0 \leq Y_{j\omega} \leq x_j, \forall j$$

where $C_{j\omega}(y_j) :=$ convex production cost of technology j

Consumer optimises unserved (shed) load U_ω given price P_ω

$$\mathcal{Q}_{\mathbf{c}\omega}(\mathbf{P}_\omega) := \min_{U_\omega} (\hat{p} - P_\omega) U_\omega \quad \text{s.t.} \quad U_\omega \geq 0$$

where $\hat{p} :=$ positive price cap or Value of Lost Load (exog. data)
$D_\omega :=$ inelastic demand (exog. data)

Price of electricity P_ω clears the market given Y_ω, U_ω

$$0 \leq \sum_j Y_{j\omega} + U_\omega - D_\omega \quad \perp \quad P_\omega \geq 0$$

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Stage 2: Spot market in scenario ω — System optimisation

Assume C_{jw} as convex & increasing; $\hat{p} > 0$; $D_\omega > 0$

Economics 101

Standard welfare theory for perfectly competitive market says

Theorem (Spot equilibrium $\iff$ Spot cost minimization)

Fix plant capacities $x \geq 0$ and spot market scenario ω.

Then $Y_\omega = \left(Y_{j\omega}\right)_j$, U_ω, P_ω is spot equilibrium $\iff$ Y_ω, U_ω solve

$$\begin{array}{rcll} \mathcal{Q}_{\mathbf{s}\omega}(\mathbf{x}) & := & \min\limits_{Y_\omega, U_\omega} & \sum_j C_{j\omega}(Y_{j\omega}) + \hat{p}U_\omega \\ & & \text{s.t.} & 0 \leq Y_{j\omega} \leq x_j \ \forall j, \ \ 0 \leq U_\omega \\ & & & 0 \ \leq \ \sum_j Y_{j\omega} \ + \ U_\omega \ - \ D_\omega \end{array}$$

and P_ω is KKT multiplier for demand constraint.

Note. $\mathcal{Q}_{s\omega}(x)$ is convex in $x \geq 0$.

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RN capacity equilibrium ⇔ RN capacity optimization

Assume $\mathcal{X} \subset \mathbb{R}^J_+$ is nonempty, closed & convex
Assume $C_{j\omega}$ is convex & increasing; $\hat{p} > 0$; $D_\omega > 0$ ($\forall\ j, \omega$)

Economics 102 Two stage risk neutral capacity equilibrium

Genco finds $x = (x_j)$ and $(Y_{j\omega})_j$ for all ω given $P = (P_\omega)$:

$$\min_x \sum_j I_j(x_j) + \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_g(x, P)\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

Consumer sets U_ω for all ω given P

Spot price P_ω clears market for all ω given all $Y_\omega = (Y_{j\omega})_j$, U_ω

Theorem (RN capacity equilibrium ⇔ RN capacity optimization)

Then x is a RN capacity equilibrium (for some (Y_ω), (U_ω), (P_ω)) $\iff x$ solves

$$\min_x \sum_j I_j(x_j) + \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_s(x)\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

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Why is optimization important?

Optimization is important because — in the convex case (here) — it leads to **tractable** problems
Economic consistency of social planning, or system optimization, with agents' investment decisions makes it **credible**

MARKAL, MARket ALlocation, [Fishbone-Abilock-81] is prototype software package implementing the theorem above

- Long term planning model under perfect competition
- Deterministic stagewise linear program (optimization) when functions are piecewise linear
- googlescholar: 4.3k publications mention MARKAL
- Stochastic MARKAL [Kanudia-Loulou-98]
- General MARKAL review [Seebregts-etal-01]

Outline

Risk averse agents

In risk neutral capacity equilibrium, given x and $P = (P_\omega)_{\omega=1}^K$,

- cost to generator of 2nd stage $= \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_g(x,P)\right]$
- cost to consumer of 2nd stage $= \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_c(P)\right]$

where Π_0, $\mathcal{Q}_g(x,P)$, $\mathcal{Q}_c(P)$ have dimension K

What if expectation is replaced by **coherent risk measure** (CRM), $r : \mathbb{R}^K \to \mathbb{R}$?

[Artzner-et-al-99] characterise r as a **worst case expectation**:

- $r(Z) = \max_{\Pi \in \mathcal{D}} \mathbb{E}_\Pi[Z]$ for any cost $Z \in \mathbb{R}^K$ ... risk averse
- $\mathcal{D}$ is nonempty closed convex set of PDs; **risk set** of r
- CVaR/AVaR/E Tail Loss is famous CRM in optimization [Roc-Uryasev-00]
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This is inescapably equilibrium not convex optimization

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There exists a solution of a risk averse capacity equilbrium

Theorem (Ehrenmann-Smeers-11a)

Under the same conditions given for RN case:
There exists an investment solution (x_j), along with spot market equilibria (Y_ω), (U_ω) (P_ω), of the risk averse capacity equilibrium

Proof is via Kakutani's fixed point theorem.

But **equilibrium solutions are tricky**

- How does a solution relate to risk neutral (optimization) case?
- How to find/interpret multiple equilibria?
- Computationally can be hard to find any solution
  - Use PATH: Write equilibrium as large complementarity problem (use KKT conditions for genco & consumer)
  - Diagonalisation/Round Robin/Jacobi iteration: solve each equilibrium condition in turn and update [recent Ferris-Wathen]

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Introducing trading of financial products

Fix $x \geq 0$ and spot prices $P = (P_\omega)$.
Genco has risky cost $Z_g = \mathcal{Q}_g(x, P)$. **How to manage risk?**

- Genco is risk averse: $r_g\left(Z_g\right) = \max_{\Pi \in \mathcal{D}_g} \left[Z_g\right]$.
- What if Genco could buy contracts or securities $W_g \in \mathbb{R}^K$ to change $r_g\left(Z_g\right)$ to $r_g\left(Z_g - W_g\right)$
  - Eg, natural gas futures to hedge cost of CCGT or GT
  - May buy a bundle of hedges: W_g is a vector

Eg, there are $K = 4$ equally likely scenarios, and $Z_g = (-2, 0, 2, 4)$

- Taking $W_g = (0, 0, 0, 2)$ gives $Z_g - W_g = (-2, 0, 2, 2)$
  Ie, your worst outcome is not so bad
- Value gained is $r_g\left((-2, 0, 2, 4)\right) - r_g\left((-2, 0, 2, 2)\right)$
- What this is worth depends on risk set $\mathcal{D}_g$

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A market for risk

Consumer has same question,

- $Z_c = \mathcal{Q}_c(P)$, quantified as $r_c(Z_c)$
- What to pay for W_c to change $r_c(Z_c)$ to $r_c(Z_c - W_c)$?

Economics answer:
A market gives **price of risk** $P^{\mathrm{r}} \in \mathbb{R}^K$,

- Genco pays $P^{\mathrm{r}}[W_g] := \sum_\omega P^{\mathrm{r}}_\omega W_{g\omega}$ & consumer pays $P^{\mathrm{r}}[W_c]$
- Trades balance: $W_g + W_c = 0$

A risk market puts a price on risk (securities)

Where does price of risk P^{r} come from? Risk market:

- Genco finds W_g: $\min_{W_g} P^{\mathrm{r}}[W_g] + r_g(Z_g - W_g)$
- Consumer finds W_c: $\min_{W_c} P^{\mathrm{r}}[W_c] + r_c(Z_c - W_c)$
- Price of risk P^{r} clears market: $W_g + W_c = 0$

[Arrow-60] characterised this as system risk minimization

$$r_s(Z_g, Z_c) := \min_{W_g, W_c} r_g\big(Z_g - W_c\big) + r_c\big(Z_c - W_c\big) \quad \text{s.t.} \quad W_g + W_c = 0$$

Remarkable recent work on CRMs gives something more concrete [Heath-Ku-04, Barrieu -El Karoui-05, Cherny-06, Burgert-Ruschendorf-08, Filipovíc-et-al-08, Dana-le-Van-10]

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Risk market:

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Theorem (Finance: Risk market under CRMs $\Longleftrightarrow$ System CRM)

If r_g and r_c are CRMS:
Finding risk equilibrium $\Rightarrow$ evaluating system risk using

system CRM	$r_s(Z_g + Z_c)$
where	$r_s(\cdot) := \max_{\Pi \in \mathcal{D}_s} \mathbb{E}_\Pi[\cdot]$
and **system risk set**	$\mathcal{D}_s := \mathcal{D}_g \cap \mathcal{D}_c$ *is nonempty.*

Converse holds under mild technical conditions,
eg, risk sets polyhedral or relative interiors intersect

Complete risk market

The last result assumes any uncertainty is priced in risk market

- Terminology: Risk market is **complete**
- Mathematical meaning: $W_g, W_c \in \mathbb{R}^K$
- Practical meaning: all significant risks can be contracted or covered by mixing contracts

We'll return to this assumption later . . .

Outline

Risky capacity equilibrium

Introduce risk trading into capacity equilibrium

Risky capacity equilibrium:

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Consumer finds $W_c \in \mathbb{R}^K$ and $U\omega$ for all ω given P:

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Spot price P_ω clears market for all ω given all $Y_\omega = (Y_{j\omega})_j$, U_ω

Price of risk P^{r} clears risk market: $W_g + W_c = 0$

Risky capacity equilibrium

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Price of risk P^r clears risk market: $W_g + W_c = 0$

Risky capacity equilibria $\Longleftrightarrow$ Risky capacity optimization

Some work that combines

- economic theory of RN capacity equilibria
- finance theory of complete risk markets with CRMs

Theorem (Ehren-Smeers-11b, R-Smeers-12 . . .)

Under assumptions for RN case + completeness of risk market:

x solves risky capacity equilibrium (for some (Y_ω), (U_ω), (P_ω), (W_g, W_c)) $\Longrightarrow$ x solves

$$\min_x \sum_j I_j(x_j) + r_s\Big[Q_s(x)\Big] \quad \text{s.t.} \quad x \in \mathcal{X}$$

Converse holds under mild technical conditions.

This has exactly same form as Risk Neutral case:

- equilibrium $\Leftrightarrow$ convex optimization

Cf related two stage structure [Philpott-Ferris-Wets] in progress

Completeness?

But **energy generation markets are far from complete**!

- Can contract fuel (coal, natural gas) prices out many months, even several years
- Can contract electricity prices somewhat into future
- Cannot contract price or penalty of C or other emissions

What if a major uncertainty is not priced in risk market?

- We'll look at range between "worst case" of no risk trading and "best case" of complete risk trading
- Range indicates uncertainty in long term capacity planning

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Summary of capacity equilibria under uncertainty

3 different cases of capacity equilibria, from **worst to best**

1. **No risk trading**: Risk averse capacity equilibrium
2. **Complete risk trading**: Risky capacity equilibrium
3. **Risk neutral:** RN capacity equilibrium using PD Π_0

Corollary (Easy)

Provided RN probability density Π_0 lies in $\mathcal{D}_s$:

Social cost at equilibrium:

No Risk Trading $\geq$ Complete RT $\geq$ Risk Neutral

Or, welfare at equilibrium:

No Risk Trading $\leq$ Complete RT $\leq$ Risk Neutral

Outline

2 stage capacity equilibrium

Stage 1 Generator sets capacity x of future electricity plants

- There are $J = 3$ technologies: $j = 1$ Coal Steam Turbine, $j = 2$ Combined Cycle Gas Turbine, $j = 3$ Gas Turbine
- $x = (x_j)_j$ specifies plant capacities, so $x \in \mathcal{X} := \mathbb{R}^3_+$
- Cost of capacity is $I(x) := I_1(x_1) + I_2(x_2) + I_3(x_3)$
  $I_3(1) \le I_2(1) \le I_1(1)$ in ratio $1 : 1.5 : 3$

Stage 2, scenario ω

- There are $K = 15$ scenarios, $\omega \in \{1, .., 15\}$
  Fuel prices of Coal & Natural Gas are random (exog. data)
  Price of C emissions is also highly uncertain (exog. data)
  Demand split into 8 random load segments (exog. data)
- CST runs cheaper than CCGT except when high coal & high C prices
- GTs are "peakers", expensive to run

2 stage capacity equilibrium

Stage 1 Generator sets capacity x of future electricity plants

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Good deal risk measure

Adapt *Good Deal* risk measure from [Cochrane-Saa-Requejo-00]

- Given "base" PD Π_0 and scalar $\nu > 0$, define risk set
  $$\mathcal{D}_\nu^{\mathrm{GD}} \quad := \quad \left\{ \zeta\Pi_0 \in \mathcal{P} \;:\; \mathbb{E}_{\Pi_0}\left[\zeta^2\right] \le \nu^2 \right\}$$
  where $\zeta\Pi_0 = \left(\zeta_\omega \Pi_{0\omega}\right)$ and $\zeta^2 = \left(\zeta_\omega^2\right)$
- Taking $\nu = 1$ gives risk neutral case with respect to Π_0
- As ν increases above 1, risk aversion also increases

In results to follow,

- Both generator and consumer use same Good Deal risk set
  - Π_0 is uniform $(1/15, .., 1/15)$
  - ν is 1 (**Risk Neutral**) or 1.2 (**Medium** risk aversion) or 2 (**High**)
- There are approx 500 variables/constraints
- Use CONOPT & PATH: Tried EMP but need more smarts . . .

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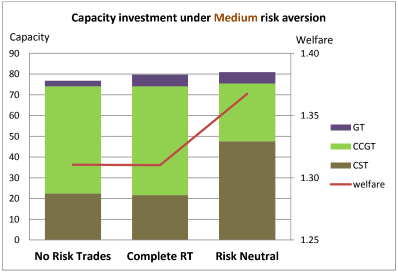
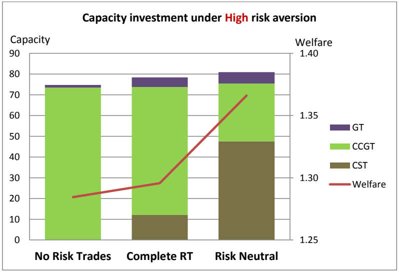
2 stage capacity equilibrium results





- With respect to equilibrium Welfare (negative system cost),

No risk trading $\leq$ Complete risk trading $\leq$ Risk neutral

- Split between CST, CCGT and GT shows fear of high C price

Outline

1. Review of risk neutral capacity equilibria
2. Risk aversion and risk trading
   - Risk averse capacity equilibria
   - Risk trading and risk markets
   - Risky capacity equilibria in a complete risk market
3. Examples
   - Two stage capacity equilibrium
   - Multi stage capacity equilibrium

Case study of capacity development in Italy

Backdrop of recent mothballing & closures of EU gas plants

- Low coal prices
- High wind penetration
- Low demand (recession)

Model of Italian power system is used to explore this

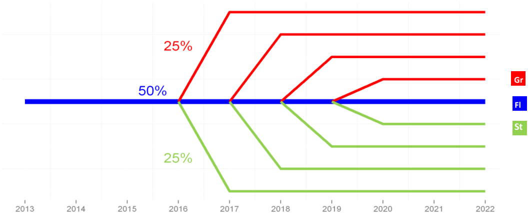
- Runs from 2013 to 2030 with annual decisions
  - Existing plants:
    - Close existing plant?
    - Mothball or demothball?
    - Extend life of plant due for retirement?
    - Convert fuel type?
  - New plants — commission new plant in which technology?

Size of problem in 10's of 000's of variables

Modelling uncertainty between now and 2030

Uncertainty

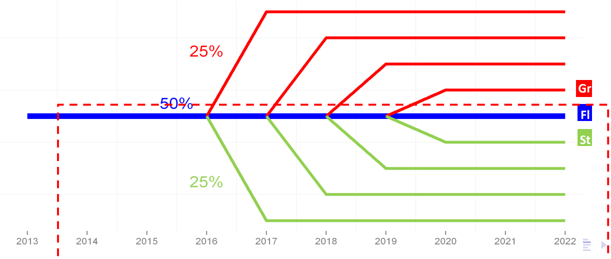
- Uncertainty is mainly seen in demand via 3 economic states
  - Flat, Stagnation or Growth
  - Only 4 branching points, all from Flat
    - 2016, 2017, 2018, 2019
    - You can branch from Flat to either Stagnation or Growth
    - Once you have branched, you stay on that branch deterministically till 2030
- Some uncertainty in fuel prices



Where risk matters — no risk trading

No risk trading

- Risk averse genco delays demothballing in Stagnation
- Risk measure puts a higher probability on Stagnation



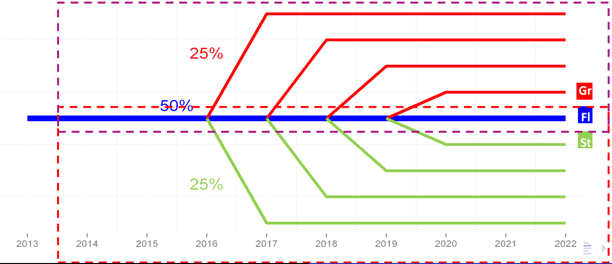
Where risk matters — complete risk market

No risk trading

- Risk averse genco delays the demothballing in Stagnation
- Risk measure puts a higher probability on Stagnation

Complete risk trading

- Risk averse genco wants to avoid under-capacity in Growth
- Financial market incentivizes more capacity in the system
- Risk measure puts a higher probability on Growth



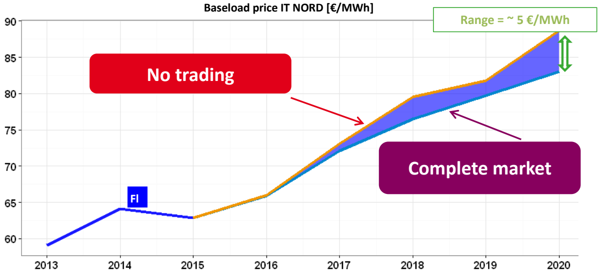
A price corridor spanning the impacts of risk hedging

No risk trading

- The spot is the only signal to incentivise physical assets

Complete risk trading

- A complete market leads to lower spot price
- It is accompanied by risk reduction (not represented here)



CONCLUSION

1. **Managing risk** of **physical** assets with **financial** assets is exciting
   - Combines energy economics with financial markets
   - Risk neutral capacity equilibrium $\Leftrightarrow$ RN optimization
   - ... extends to risk averse case if all risks can be traded:
     Risky capacity equilibrium $\Leftrightarrow$ Risk averse optimization
2. Incomplete risk trading remains a challenge
3. Multi stage likewise challenging

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