Parallelising the dual revised simplex method

Julian Hall[1] Qi Huangfu[2] Miles Lubin[3]

[1]School of Mathematics, University of Edinburgh

[2]FICO

[3]MIT

Convex Optimization and Beyond

Edinburgh

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Parallelising the dual revised simplex method: Overview

- Background
- Three approaches
  - Multiple iteration parallelism for general LP
  - Single iteration parallelism for general LP
  - Data parallelism for stochastic LP
- Conclusions

Linear programming (LP)

$$\begin{array}{ll} \text{minimize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} = \mathbf{b} \quad \mathbf{x} \geq \mathbf{0} \end{array}$$

Background

- Fundamental model in optimal decision-making
- Solution techniques
  - Simplex method (1947)
  - Interior point methods (1984)
- Large problems have
  - 10^3–$10^{\cancel{7}8}$ variables
  - 10^3–$10^{\cancel{7}8}$ constraints
- Matrix A is (usually) sparse

Example

STAIR: 356 rows, 467 columns and 3856 nonzeros

Solving LP problems

$$\begin{array}{llll} \text{minimize} & f_P = \mathbf{c}^T\mathbf{x} & & \\ \text{subject to} & A\mathbf{x} = \mathbf{b} \quad \mathbf{x} \geq \mathbf{0} & (P) \end{array} \qquad \begin{array}{ll} \text{maximize} & f_D = \mathbf{b}^T\mathbf{y} \\ \text{subject to} & A^T\mathbf{y} + \mathbf{s} = \mathbf{c} \quad \mathbf{s} \geq \mathbf{0} \quad (D) \end{array}$$

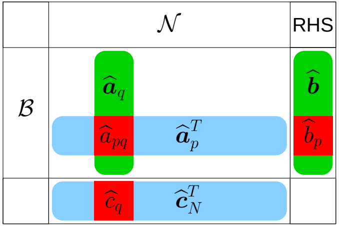
Optimality conditions

- For a partition $\mathcal{B} \cup \mathcal{N}$ of the variable set with nonsingular **basis matrix** B in

$$B\mathbf{x}_B + N\mathbf{x}_N = \mathbf{b} \text{ for (P)} \qquad \text{and} \qquad \begin{bmatrix} B^T \\ N^T \end{bmatrix} \mathbf{y} + \begin{bmatrix} \mathbf{s}_B \\ \mathbf{s}_N \end{bmatrix} = \begin{bmatrix} \mathbf{c}_B \\ \mathbf{c}_N \end{bmatrix} \text{ for (D)}$$

  with $\mathbf{x}_N = \mathbf{0}$ and $\mathbf{s}_B = \mathbf{0}$
  - Primal basic variables $\mathbf{x}_B$ given by $\widehat{\mathbf{b}} = B^{-1}\mathbf{b}$
  - Dual non-basic variables $\mathbf{s}_N$ given by $\widehat{\mathbf{c}}_N^T = \mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} N$
- Partition is optimal if there is
  - Primal feasibility $\widehat{\mathbf{b}} \geq \mathbf{0}$
  - Dual feasibility $\widehat{\mathbf{c}}_N \geq \mathbf{0}$

Simplex algorithm: Each iteration



Dual algorithm: Assume $\widehat{\mathbf{c}}_N \geq \mathbf{0}$ Seek $\widehat{\mathbf{b}} \geq \mathbf{0}$

Scan $\widehat{b}_i$, $i \in \mathcal{B}$, for a good candidate p to leave $\mathcal{B}$ `CHUZR`

Scan $\widehat{c}_j/\widehat{a}_{pj}$, $j \in \mathcal{N}$, for a good candidate q to leave $\mathcal{N}$ `CHUZC`

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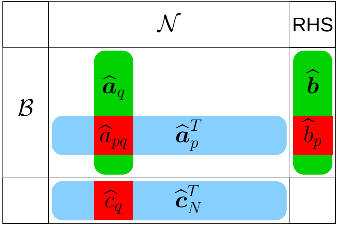

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Update: Exchange p and q between $\mathcal{B}$ and $\mathcal{N}$

Update $\widehat{\mathbf{b}} := \widehat{\mathbf{b}} - \theta_p \widehat{\mathbf{a}}_q$ $\quad \theta_p = \widehat{b}_p/\widehat{a}_{pq}$ `UPDATE-PRIMAL`

Update $\widehat{\mathbf{c}}_N^T := \widehat{\mathbf{c}}_N^T - \theta_d \widehat{\mathbf{a}}_p^T$ $\quad \theta_d = \widehat{c}_q/\widehat{a}_{pq}$ `UPDATE-DUAL`

Standard simplex method (SSM): Computation

Major computational component

Update of tableau:

$$\widehat{N} := \widehat{N} - \frac{1}{\widehat{a}_{pq}} \widehat{\mathbf{a}}_q \widehat{\mathbf{a}}_p^T$$

where $\widehat{N} = B^{-1}N$

- Hopelessly inefficient for sparse LP problems
- Prohibitively expensive for large LP problems

Revised simplex method (RSM): Computation

Major computational components

$\boldsymbol{\pi}_p^T = \mathbf{e}_p^T B^{-1}$	BTRAN	$\widehat{\mathbf{a}}_p^T = \boldsymbol{\pi}_p^T N$	PRICE
$\widehat{\mathbf{a}}_q = B^{-1}\mathbf{a}_q$	FTRAN	Invert B	INVERT

Revised simplex method (RSM): Computation

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Hyper-sparsity

- Vectors $\mathbf{e}_p$ and $\mathbf{a}_q$ are always sparse
- B may be highly reducible; B^{-1} may be sparse
- Vectors $\boldsymbol{\pi}_p$, $\widehat{\mathbf{a}}_p^T$ and $\widehat{\mathbf{a}}_q$ may be sparse
- Efficient implementations must exploit these features

H and McKinnon (1998–2005), Bixby (1999)
Clp, Koberstein and Suhl (2005–2008)

Revised simplex method (RSM): Algorithmic enhancements

Row selection: Dual steepest edge (DSE)

- Weight $\widehat{b}_i$ by w_i: measure of $\|B^{-1}\mathbf{e}_i\|_2$
- Requires additional `FTRAN` but can reduce iteration count significantly

Column selection: Bound-flipping ratio test (BFRT)

- Minimizes the dual objective whilst remaining dual feasible
    - Dual variables may change sign if corresponding primal variables can flip bounds
- Requires additional `FTRAN` but can reduce iteration count significantly

Exploiting parallelism: Background

Data parallel standard simplex method

- Good parallel efficiency *was* achieved
- Only relevant for dense LP problems

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- When $n \gg m$ significant speed-up *was* achieved — Bixby and Martin (2000)

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Task parallel revised simplex method

- Overlap computational components for different iterations
  Wunderling (1996), H and McKinnon (1995-2005)
- Modest speed-up *was* achieved on general sparse LP problems

Parallelising the dual revised simplex method: Overview

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Single iteration parallelism for general LP

- Pure dual revised simplex
- **Data parallelism:** Form $\pi_p^T N$
- **Task parallelism:** Identify serial computation which can be overlapped

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Multiple iteration parallelism for general LP

- Dual revised simplex with minor iterations of dual standard simplex
- **Data parallelism:** Form $\pi_p^T N$ and update (slice of) dual standard simplex tableau
- **Task parallelism:** Identify serial computation which can be overlapped

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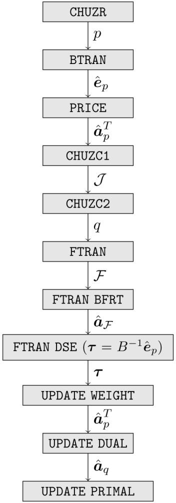
Data parallelism for stochastic LP

- Pure dual revised simplex for column-linked block angular LP problems
- **Data parallelism:** Solve $B^T\boldsymbol{\pi} = \mathbf{e}_p$, $B\widehat{\mathbf{a}}_q = \mathbf{a}_q$ and form $\boldsymbol{\pi}_p^T N$

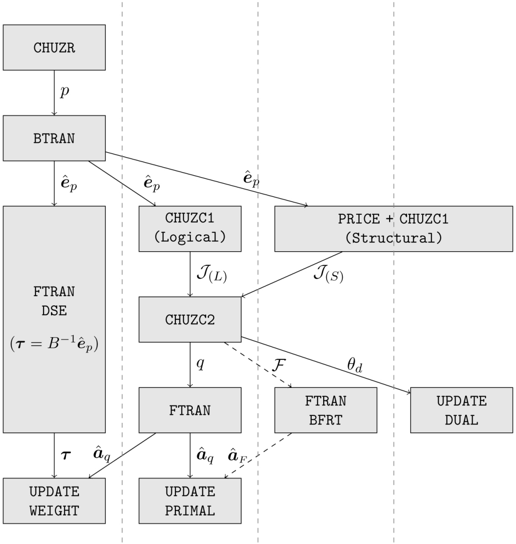
Single iteration parallelism

Single iteration parallelism: Dual revised simplex method



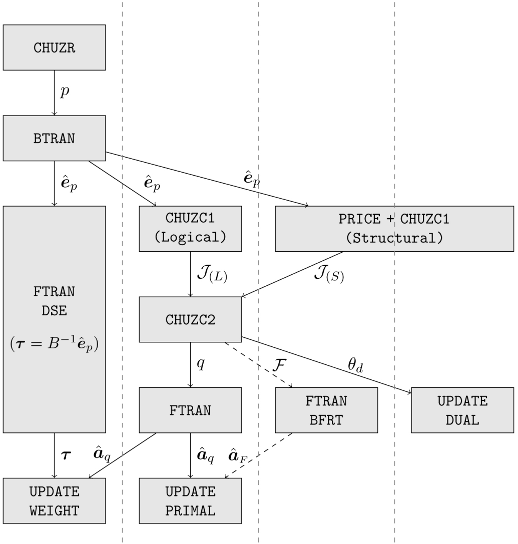
- Computational components appear sequential
- Each has highly-tuned sparsity-exploiting serial implementation
- Exploit “slack” in data dependencies

Single iteration parallelism: Computational scheme



- Parallel PRICE to form $\hat{\mathbf{a}}_p^T = \boldsymbol{\pi}_p^T N$
- Other computational components serial
- Overlap any independent calculations

Single iteration parallelism: Computational scheme



- Parallel PRICE to form $\hat{\mathbf{a}}_p^T = \pi_p^T N$
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- Only four worthwhile threads unless $n \gg m$ so PRICE dominates

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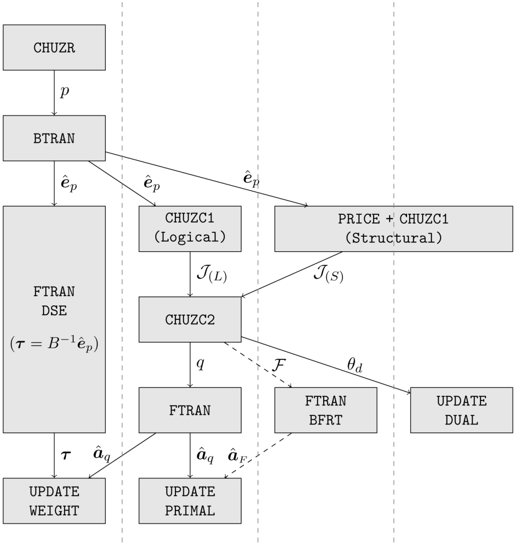


- Parallel PRICE to form $\hat{\mathbf{a}}_p^T = \boldsymbol{\pi}_p^T N$
- Other computational components serial
- Overlap any independent calculations
- Only four worthwhile threads unless $n \gg m$ so PRICE dominates
- More than Bixby and Martin (2000)
- Better than Forrest (2012)

Single iteration parallelism: 4-core `sip` vs 1-core `hsol`

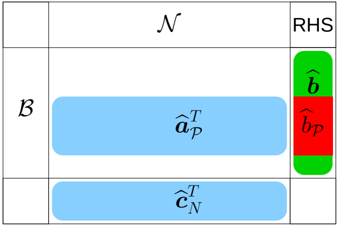
Model	Speedup	Model	Speedup	Model	Speedup
sgpf5y6	0.67	maros-r7	1.12	world	1.27
stormG2-125	0.76	stp3d	1.15	dfl001	1.28
watson_2	0.78	nug12	1.16	l30	1.28
ken-18	0.79	pds-40	1.16	Linf_520c	1.31
watson_1	0.80	dbic1	1.21	pilot87	1.31
qap12	0.83	fome12	1.22	self	1.36
stormG2-1000	0.84	dcp2	1.23	lp22	1.45
pds-80	1.05	ns1688926	1.23	dano3mip_lp	1.49
pds-100	1.06	fome13	1.24	truss	1.58
cre-b	1.08	mod2	1.25	stat96v4	2.05

- Geometric mean speedup is 1.13
- Performance is generally poor for problems with high hyper-sparsity
- Performance is generally good for problems with low hyper-sparsity

Multiple iteration parallelism

Multiple iteration parallelism

- `sip` has too little work to be performed in parallel to get good speedup
- Perform standard dual simplex minor iterations for rows in set $\mathcal{P}$ ($|\mathcal{P}| \ll m$)
- Suggested by Rosander (1975) but never implemented efficiently *in serial*



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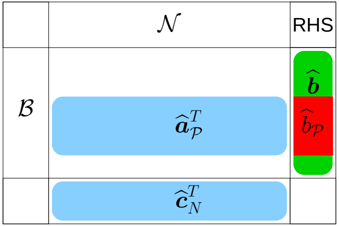


- Task-parallel multiple `BTRAN` to form $\boldsymbol{\pi}_{\mathcal{P}} = B^{-1}\mathbf{e}_{\mathcal{P}}$
- Data-parallel `PRICE` to form $\widehat{\mathbf{a}}_p^T$ (as required)
- Data-parallel tableau update
- Task-parallel multiple `FTRAN` for primal, dual and weight updates

Multiple iteration parallelism: 8-core pami vs 1-core pami

Model	Speedup	Model	Speedup	Model	Speedup
KEN-18	1.54	LINF_520C	2.00	WORLD	2.54
MAROS-R7	1.56	PDS-40	2.00	FOME12	2.58
CRE-B	1.62	NS1688926	2.10	TRUSS	2.67
STORMG2-125	1.70	FOME13	2.20	L30	2.74
WATSON_2	1.72	STORMG2-1000	2.25	DFL001	2.74
SELF	1.81	STP3D	2.33	LP22	2.75
WATSON_1	1.83	DBIC1	2.36	QAP12	2.75
PDS-100	1.88	SGPF5Y6	2.40	NUG12	2.81
DCP2	1.89	PILOT87	2.48	DANO3MIP_LP	3.10
PDS-80	1.92	MOD2	2.53	STAT96V4	3.50

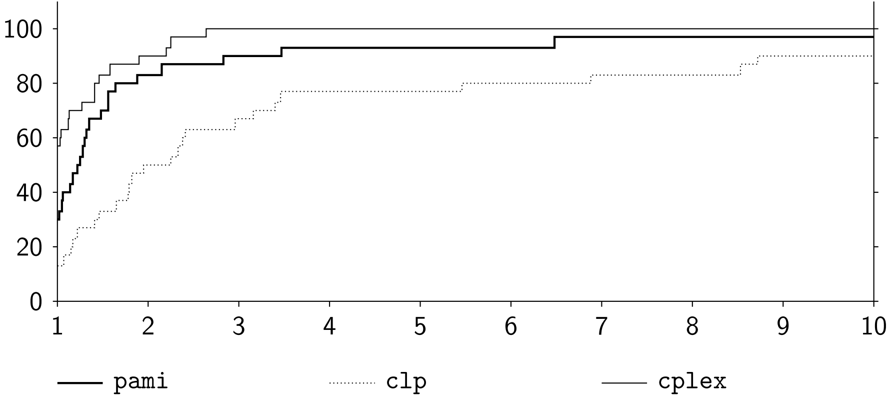
- Speed-up for all problems
- Geometric mean speedup is 2.23

Multiple iteration parallelism: 8-core pami vs 1-core hsol

Model	Speedup	Model	Speedup	Model	Speedup
maros-r7	0.47	pds-40	1.35	lp22	1.67
Linf_520c	0.75	world	1.37	nug12	1.78
self	1.07	stormG2-125	1.44	dfl001	1.81
pds-80	1.16	pilot87	1.50	sgpf5y6	1.90
ns1688926	1.26	dcp2	1.52	truss	1.94
pds-100	1.29	fome13	1.52	cre-b	1.95
mod2	1.29	watson_1	1.55	dano3mip_lp	2.12
l30	1.29	watson_2	1.61	stat96v4	2.33
ken-18	1.30	fome12	1.61	stp3d	2.41
dbic1	1.31	stormG2-1000	1.66	qap12	2.53

- Geometric mean speedup is 1.49
- Lower than speedup relative to 1-core pami
  - Geometric mean speed of 1-core pami relative to 1-core hsol is 0.67

Multiple iteration parallelism: Performance profile benchmarking



- `pami` is plainly better than `clp`
- `pami` is comparable with `cplex`
- `pami` ideas have been incorporated in FICO Xpress (Huangfu 2014)

Data parallelism for stochastic LPs

Stochastic MIP problems: General

Two-stage stochastic LPs have column-linked block angular structure

$$
\begin{array}{rcccccccccccc}
\text{minimize} & \mathbf{c}_0^T\mathbf{x}_0 & + & \mathbf{c}_1^T\mathbf{x}_1 & + & \mathbf{c}_2^T\mathbf{x}_2 & + & \dots & + & \mathbf{c}_N^T\mathbf{x}_N & & \\
\text{subject to} & A\mathbf{x}_0 & & & & & & & & & = & \mathbf{b}_0 \\
& T_1\mathbf{x}_0 & + & W_1\mathbf{x}_1 & & & & & & & = & \mathbf{b}_1 \\
& T_2\mathbf{x}_0 & & & + & W_2\mathbf{x}_2 & & & & & = & \mathbf{b}_2 \\
& \vdots & & & & & & \ddots & & & & \vdots \\
& T_N\mathbf{x}_0 & & & & & & & + & W_N\mathbf{x}_N & = & \mathbf{b}_N \\
& \mathbf{x}_0 \geq \mathbf{0} & & \mathbf{x}_1 \geq \mathbf{0} & & \mathbf{x}_2 \geq \mathbf{0} & & \dots & & \mathbf{x}_N \geq \mathbf{0} & &
\end{array}
$$

- Variables $\mathbf{x}_0 \in \mathbb{R}^{n_0}$ are **first stage** decisions
- Variables $\mathbf{x}_i \in \mathbb{R}^{n_i}$ for $i = 1, \dots, N$ are **second stage** decisions
  Each corresponds to a **scenario** which occurs with modelled probability
- The objective is the expected cost of the decisions
- In stochastic MIP problems, some/all decisions are discrete

Stochastic MIP problems: For Argonne

- Power systems optimization project at Argonne
- Integer second-stage decisions
- Stochasticity comes from availability of wind-generated electricity
- Initial experiments carried out using model problem
- Number of scenarios increases with refinement of probability distribution sampling
- Solution via branch-and-bound
  - Solve root node using parallel IPM solver `PIPS` Lubin, Petra *et al.* (2011)
  - Solve subsequent nodes using parallel dual simplex solver `PIPS-S` Lubin, H *et al.* (2013)

Stochastic MIP problems: General

Convenient to permute the LP thus:

$$
\begin{array}{rccccccccccc}
\text{minimize} & \mathbf{c}_1^T\mathbf{x}_1 & + & \mathbf{c}_2^T\mathbf{x}_2 & + & \dots & + & \mathbf{c}_N^T\mathbf{x}_N & + & \mathbf{c}_0^T\mathbf{x}_0 & & \\
\text{subject to} & W_1\mathbf{x}_1 & & & & & & & + & T_1\mathbf{x}_0 & = & \mathbf{b}_1 \\
 & & & W_2\mathbf{x}_2 & & & & & + & T_2\mathbf{x}_0 & = & \mathbf{b}_2 \\
 & & & & & \ddots & & & \vdots & & & \vdots \\
 & & & & & & & W_N\mathbf{x}_N & + & T_N\mathbf{x}_0 & = & \mathbf{b}_N \\
 & & & & & & & & & A\mathbf{x}_0 & = & \mathbf{b}_0 \\
 & \mathbf{x}_1 \ge \mathbf{0} & & \mathbf{x}_2 \ge \mathbf{0} & & \dots & & \mathbf{x}_N \ge \mathbf{0} & & \mathbf{x}_0 \ge \mathbf{0} & &
\end{array}
$$

Exploiting problem structure: Basis matrix inversion

- Inversion of the basis matrix B is key to revised simplex efficiency
- For column-linked BALP problems

$$B = \begin{bmatrix} W_1^B & & & T_1^B \\ & \ddots & & \vdots \\ & & W_N^B & T_N^B \\ & & & A^B \end{bmatrix}$$

- W_i^B are columns corresponding to n_i^B basic variables in scenario i
- $\begin{bmatrix} T_1^B \\ \vdots \\ T_N^B \\ A^B \end{bmatrix}$ are columns corresponding to n_0^B basic first stage decisions

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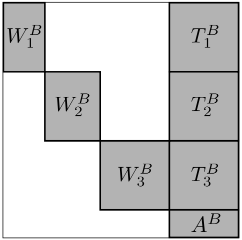
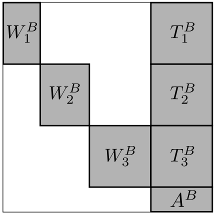


- B is nonsingular so
  - W_i^B are "tall": full column rank
  - $\begin{bmatrix} W_i^B & T_i^B \end{bmatrix}$ are "wide": full row rank
  - A^B is "wide": full row rank
- Scope for parallel inversion is immediate and well known

Duff and Scott (2004)

Exploiting problem structure: Basis matrix inversion

- Eliminate sub-diagonal entries in each W_i^B (independently)



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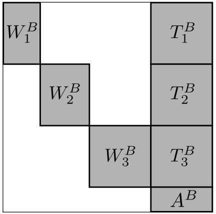

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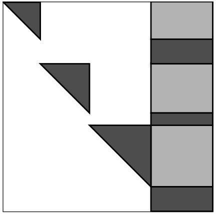

- Apply elimination operations to each T_i^B (independently)

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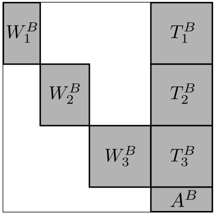

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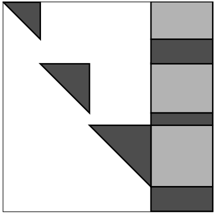

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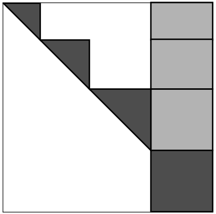

- Accumulate non-pivoted rows from the W_i^B with A^B and complete elimination

Exploiting problem structure: Basis matrix inversion

- After Gaussian elimination, have invertible representation of

$$B = \left[\begin{array}{ccc|c} S_1 & & & C_1 \\ & \ddots & & \vdots \\ & & S_N & C_N \\ \hline R_1 & \dots & R_N & V \end{array}\right] = \left[\begin{array}{c|c} S & C \\ \hline R & V \end{array}\right]$$

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- Specifically
  - $L_i U_i = S_i$ of dimension n_i^B

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  - LU factors of the Schur complement $M = V - RS^{-1}C$ of dimension n_0^B

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  - $\widehat{C}_i = L_i^{-1} C_i$
  - $\widehat{R}_i = R_i U_i^{-1}$
  - LU factors of the Schur complement $M = V - RS^{-1}C$ of dimension n_0^B
- Scope for parallelism since each GE applied to $\left[W_i^B \mid T_i^B \right]$ is independent

Exploiting problem structure: Solving $B\mathbf{x} = \mathbf{b}$

FTRAN for $B\mathbf{x} = \mathbf{b}$

Solve $\begin{bmatrix} S & C \\ R & V \end{bmatrix} \begin{bmatrix} \mathbf{x}_\bullet \\ \mathbf{x}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_\bullet \\ \mathbf{b}_0 \end{bmatrix}$ as

1. $L_i \mathbf{y}_i = \mathbf{b}_i, \; i = 1, \ldots, N$
2. $\mathbf{z}_i = \widehat{R}_i \mathbf{y}_i, \; i = 1, \ldots, N$
3. $\mathbf{z} = \mathbf{b}_0 - \sum_{i=1}^{N} \mathbf{z}_i$
4. $M\mathbf{x}_0 = \mathbf{z}$
5. $U_i \mathbf{x}_i = \mathbf{y}_i - \widehat{C}_i \mathbf{x}_0, \; i = 1, \ldots, N$

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- Appears to be dominated by parallelizable
  - Solves $L_i \mathbf{y}_i = \mathbf{b}_i$ and $U_i \mathbf{x}_i = \mathbf{y}_i - \widehat{C}_i \mathbf{x}_0$
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  - In simplex, $\mathbf{b}_\bullet$ is from constraint column

    Either $\begin{bmatrix} \mathbf{t}_{1q} \\ \vdots \\ \mathbf{t}_{Nq} \end{bmatrix}$

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    Either $\begin{bmatrix} \mathbf{t}_{1q} \\ \vdots \\ \mathbf{t}_{Nq} \end{bmatrix}$ or, more likely, $\begin{bmatrix} \mathbf{0} \\ \mathbf{w}_{iq} \\ \mathbf{0} \end{bmatrix}$

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    Either $\begin{bmatrix} \mathbf{t}_{1q} \\ \vdots \\ \mathbf{t}_{Nq} \end{bmatrix}$ or, more likely, $\begin{bmatrix} \mathbf{0} \\ \mathbf{w}_{iq} \\ \mathbf{0} \end{bmatrix}$
  - In latter case, the $\mathbf{y}_i$ inherit structure
    - Only one $L_i \mathbf{y}_i = \mathbf{w}_{iq}$
    - Only one $\widehat{R}_i \mathbf{y}_i$
- Less scope for parallelism than anticipated

Exploiting problem structure: Solving $B^T\mathbf{x} = \mathbf{b}$

BTRAN for $B^T\mathbf{x} = \mathbf{b}$

Solve $\begin{bmatrix} S^T & R^T \\ C^T & V^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_\bullet \\ \mathbf{x}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_\bullet \\ \mathbf{b}_0 \end{bmatrix}$ as

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3. $\mathbf{z} = \mathbf{b}_0 - \sum_{i=1}^{N} \mathbf{z}_i$
4. $M^T \mathbf{x}_0 = \mathbf{z}$
5. $L_i^T \mathbf{x}_i = \mathbf{y}_i - \widehat{R}_i^T \mathbf{x}_0,\ i = 1, \dots, N$

- Appears to be dominated by parallelizable
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  - Products $\widehat{C}_i^T\mathbf{y}_i$ and $\widehat{R}_i^T\mathbf{x}_0$
- Curse of exploiting hyper-sparsity
  - In simplex, $\mathbf{b} = \mathbf{e}_p$
  - At most one solve $U_i^T\mathbf{y}_i = \mathbf{b}_i$
  - At most one $\widehat{C}_i^T\mathbf{y}_i$
- Less scope for parallelism than anticipated

Exploiting problem structure: Forming $\pi_p^T N$

- PRICE forms

$$\begin{bmatrix} \pi_1^T & \pi_2^T & \dots & \pi_N^T & \pi_0^T \end{bmatrix} \begin{bmatrix} W_1^N & & & & T_1^N \\ & W_2^N & & & T_2^N \\ & & \ddots & & \vdots \\ & & & W_N^N & T_N^N \\ & & & & A^N \end{bmatrix}$$

$$= \begin{bmatrix} \pi_1^T W_1^N & \pi_2^T W_2^N & \dots & \pi_N^T W_N^N & \pi_0^T A^N + \sum_{i=1}^{N} \pi_i^T T_i^N \end{bmatrix}$$

- Dominated by parallelizable products $\pi_i^T W_i^N$ and $\pi_i^T T_i^N$

Exploiting problem structure: Update

- Update of the invertible representation of B is second major factor in revised simplex efficiency
- Each iteration column $\mathbf{a}_q$ of the constraint matrix replaces column $B\mathbf{e}_p$ of B

$$B' = B[I + (\widehat{\mathbf{a}}_q - \mathbf{e}_p)\mathbf{e}_p^T]$$

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- Unfortunately, the structure of B is not generally maintained
- `PIPS-S` uses standard **product form** update

$$B'^{-1} = [I + (\widehat{\mathbf{a}}_q - \mathbf{e}_p)\mathbf{e}_p^T]^{-1}B^{-1} = E^{-1}B^{-1} \quad \text{where} \quad E^{-1} = I - \frac{1}{\widehat{a}_{pq}}(\widehat{\mathbf{a}}_q - \mathbf{e}_p)\mathbf{e}_p^T$$

Exploiting problem structure: Applying update

- Form $\mathbf{x} = E^{-1}\mathbf{b}$ as $x_p = -\dfrac{b_p}{\widehat{a}_{pq}}$ then $\mathbf{x}_{p'} = \mathbf{b}_{p'} + \widehat{\mathbf{a}}_q x_p$

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- Updates $\{E_k\}_{k=1}^{K}$ of B_0 to B_K require $\{\widehat{\mathbf{a}}_{q_k}\}_{k=1}^{K}$ and $\mathcal{P} = \{p_k\}_{k=1}^{K}$, $|\mathcal{P}| \ll m$

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- Exploit parallelism when forming $\mathbf{x} = E_K^{-1} \dots E_1^{-1}\mathbf{b}$ thus
  - Compute $\mathbf{x}_{\mathcal{P}}$ serially
  - Compute $\mathbf{x}_{\mathcal{P}'}$ as a parallel matrix-vector product

$$\mathbf{x}_{\mathcal{P}'} = \mathbf{b}_{\mathcal{P}'} + \begin{bmatrix} \widehat{\mathbf{a}}_{q_1} & \dots & \widehat{\mathbf{a}}_{q_K} \end{bmatrix} \mathbf{x}_{\mathcal{P}}$$

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- Similar trick for parallelising $\mathbf{x}^T = \mathbf{b}^T E_K^{-1} \dots E_1^{-1}$

Lubin, H *et al.* (2013)

Results

Results: Stochastic LP test problems

Test Problem	1st Stage n_0	m_0	2nd-Stage Scenario n_i	m_i	Nonzero Elements A	W_i	T_i
`Storm`	121	185	1,259	528	696	3,220	121
`SSN`	89	1	706	175	89	2,284	89
`UC12`	3,132	0	56,532	59,436	0	163,839	3,132
`UC24`	6,264	0	113,064	118,872	0	327,939	6,264

- `Storm` and `SSN` are publicly available

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- `Storm` and `SSN` are publicly available
- `UC12` and `UC24` are stochastic unit commitment problems developed at Argonne
  - Aim to choose optimal on/off schedules for generators on the power grid of the state of Illinois over a 12-hour and 24-hour horizon
  - In practice each scenario corresponds to a weather simulation
    Model problem generates scenarios by normal perturbations

Zavala (2011)

Results: Baseline serial performance for large instances

Serial performance of `PIPS-S` and `clp`

Problem	Dimensions	Solver	Iterations	Time (s)	Iter/sec
`Storm`	$n =$10,313,849	`PIPS-S`	6,353,593	385,825	16.5
8,192 scen.	$m = 4,325,561$	`clp`	6,706,401	133,047	50.4
`SSN`	$n = 5,783,651$	`PIPS-S`	1,025,279	58,425	17.5
8,192 scen.	$m = 1,433,601$	`clp`	1,175,282	12,619	93.1
`UC12`	$n = 1,812,156$	`PIPS-S`	1,968,400	236,219	8.3
32 scen.	$m = 1,901,952$	`clp`	2,474,175	39,722	62.3
`UC24`	$n = 1,815,288$	`PIPS-S`	2,142,962	543,272	3.9
16 scen.	$m = 1,901,952$	`clp`	2,441,374	41,708	58.5

Results: On Fusion cluster

Speed-up of PIPS-S relative to 1-core PIPS-S and 1-core clp

Cores	Storm	SSN	UC12	UC24
1	1.0	1.0	1.0	1.0
4	3.6	3.5	2.7	3.0
8	7.3	7.5	6.1	5.3
16	13.6	15.1	8.5	8.9
32	24.6	30.3	14.5	

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32	24.6	30.3	14.5	
clp	8.5	6.5	2.4	0.7

Results: On Fusion cluster - larger instances

	Storm	SSN	UC12	UC24
Scenarios	32,768	32,768	512	256
Variables	41,255,033	23,134,297	28,947,516	28,950,648
Constraints	17,301,689	5,734,401	30,431,232	30,431,232

Results: On Fusion cluster - larger instances, from an advanced basis

Speed-up of `PIPS-S` relative to 1-core `PIPS-S` and 1-core `clp`

Cores	Storm	SSN	UC12	UC24
1	1	1	1	1
8	15	19	7	6
16	52	45	14	12
32	117	103	26	22
64	152	181	44	41
128	202	289	60	64
256	285	383	70	80

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256	285	383	70	80
clp	299	45	67	68

Results: On Blue Gene supercomputer - very large instance

- Instance of `UC12`
  - 8,192 scenarios
  - 463,113,276 variables
  - 486,899,712 constraints
- Requires 1 TB of RAM ($\geq$ 1024 Blue Gene cores)
- Runs from an advanced basis

Cores	Iterations	Time (h)	Iter/sec
1024	Exceeded execution time limit		
2048	82,638	6.14	3.74
4096	75,732	5.03	4.18
8192	86,439	4.67	5.14

Parallelising the dual revised simplex method: Conclusions

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Slides: http://www.maths.ed.ac.uk/hall/COB14/

Paper: M. Lubin, J. A. J. Hall, C. G. Petra, and M. Anitescu
Parallel distributed-memory simplex for large-scale stochastic LP problems
Computational Optimization and Applications, 55(3):571–596, 2013

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