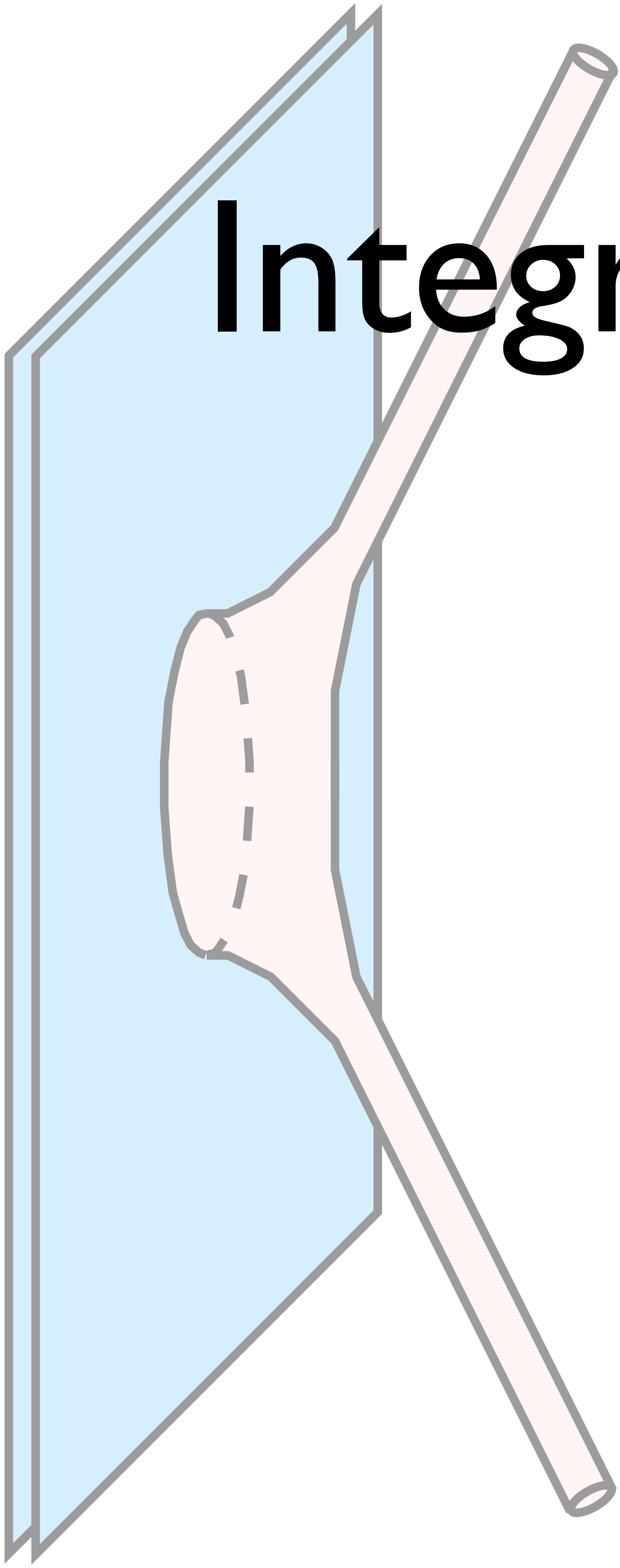
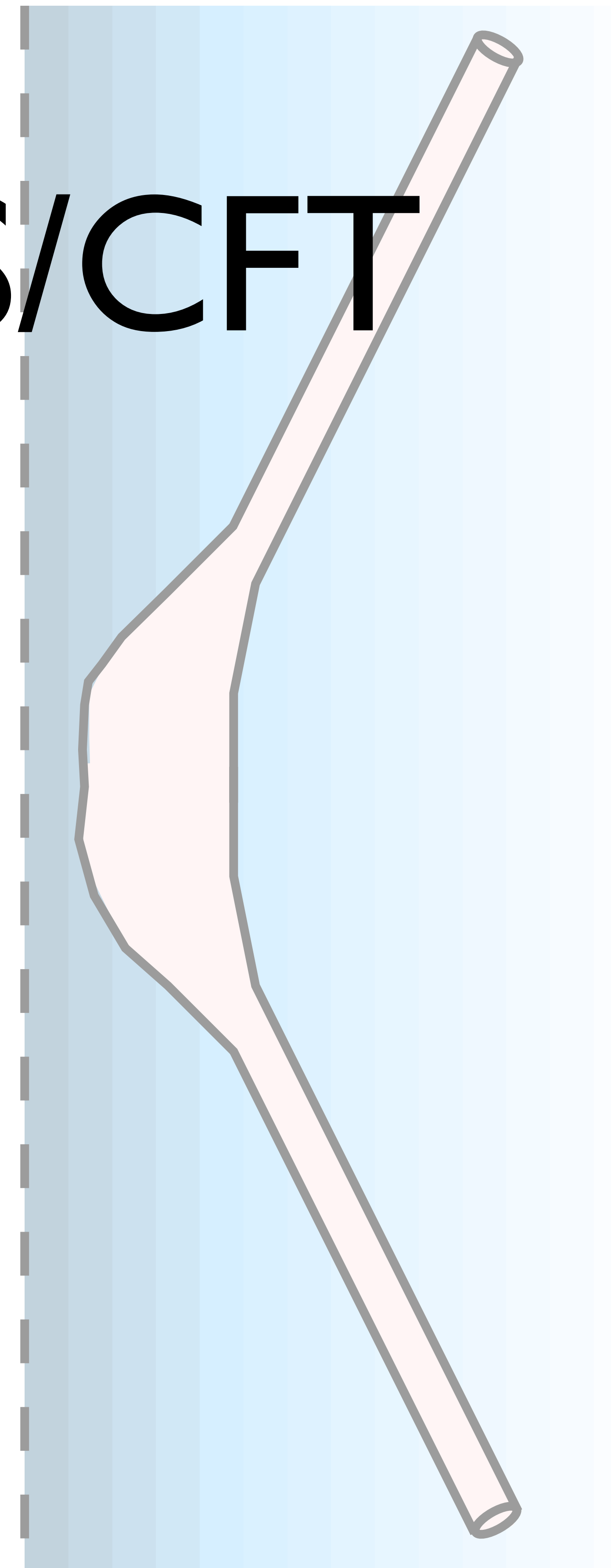
Integrability in AdS/CFT



Pedro Vieira

ENS, Paris and CFP, Porto

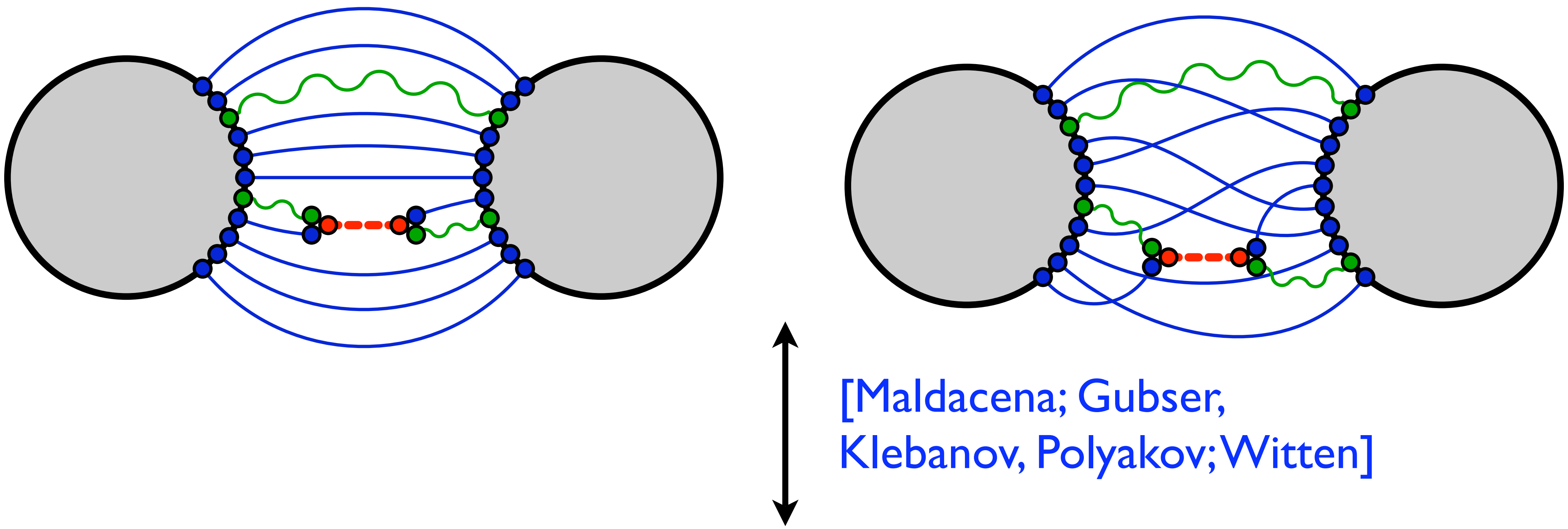


Mars 2008, Edimburg

N=4 Supersymmetric Yang-Mill Theory

(Superconformal 4d gauge theory)

g

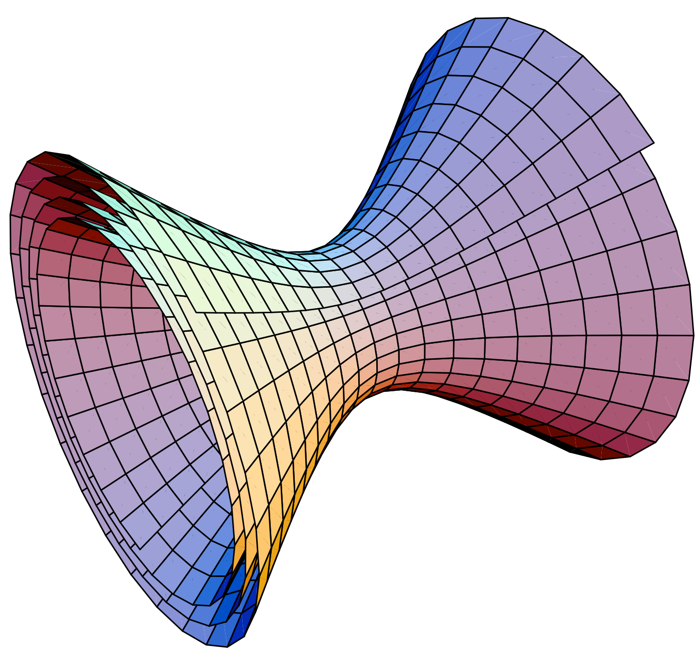


[Maldacena; Gubser, Klebanov, Polyakov; Witten]

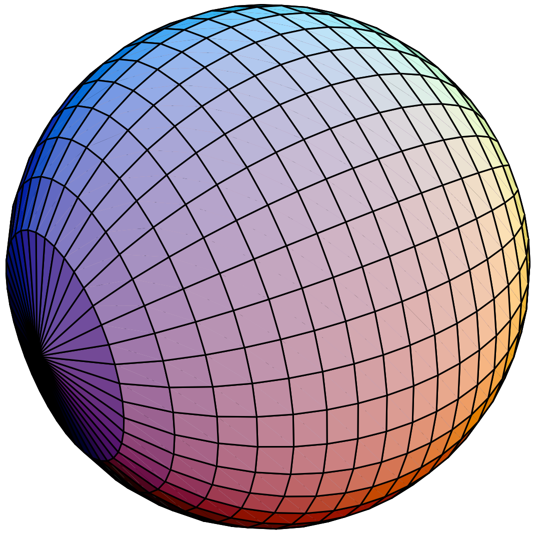
Type IIB superstrings in AdS_5xS^5

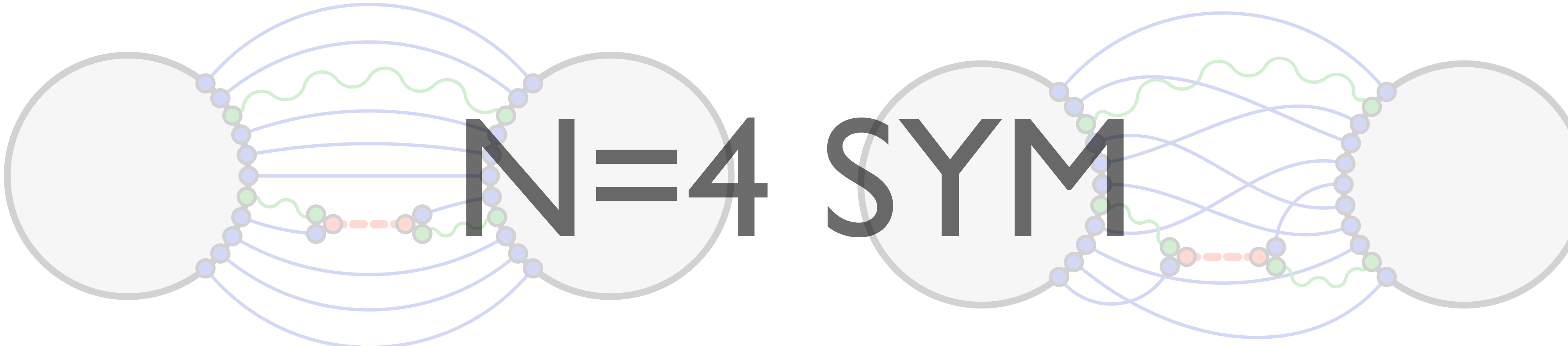
(Two dimensional conformal sigma model)

1/g



x

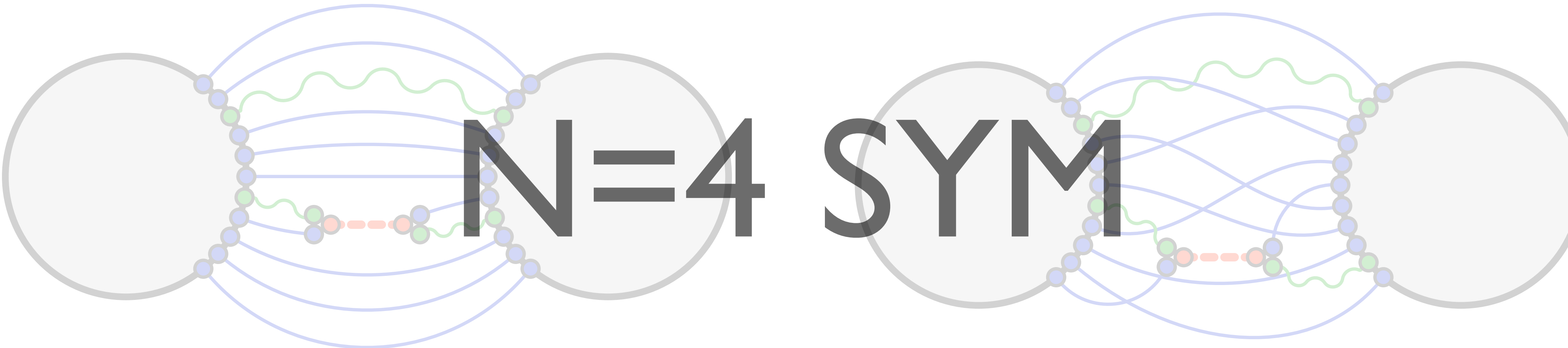




N=4 SYM

$$\mathcal{L} \sim \mathrm{tr}\ \left(\mathrm{F}_{\mu\nu}^2 + (\mathrm{D}_\mu \Phi_\mathrm{i})^2 - [\Phi_\mathrm{i}, \Phi_\mathrm{j}]^2 + \Psi \Gamma_\mu \mathcal{D}_\mu \Psi\right)$$

N=4 SYM



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The conformal symmetry $SO(2,4) \simeq SU(2,2)$

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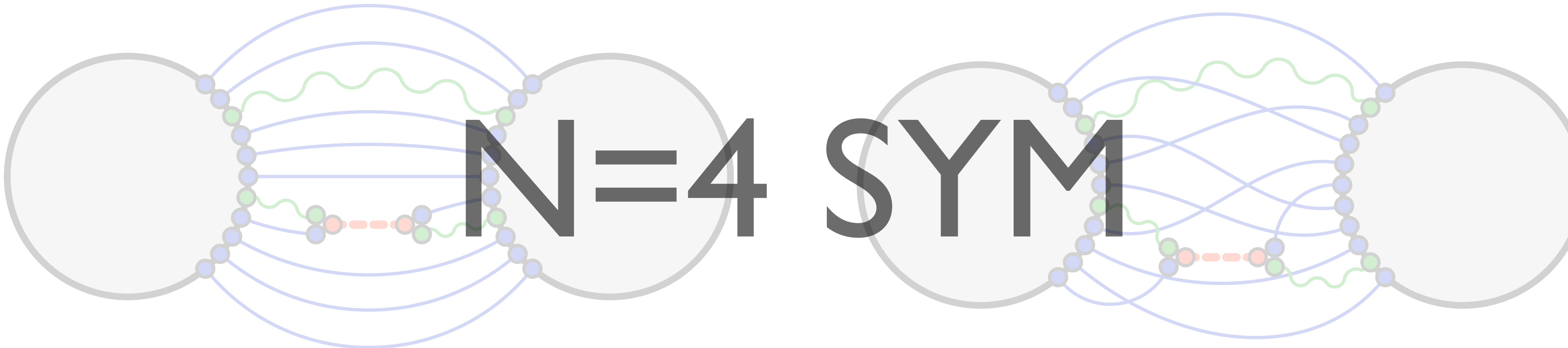

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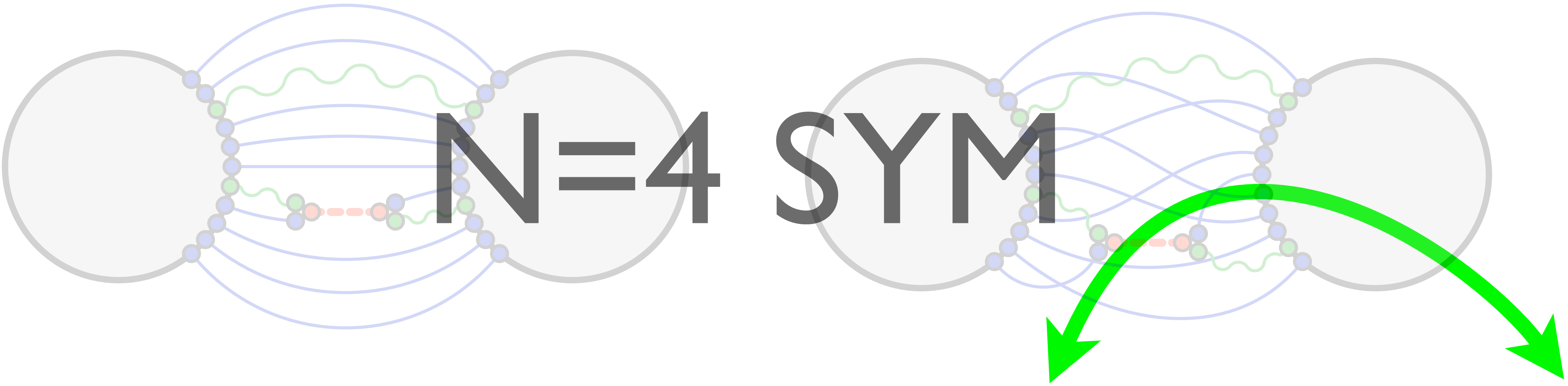
N=4 SYM



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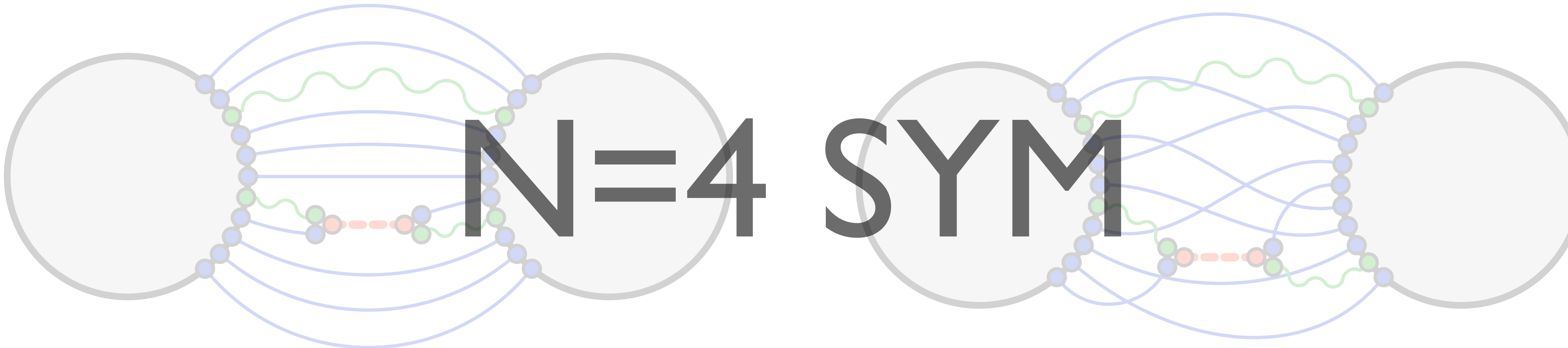
Combine with the N=4 supersymmetry

N=4 SYM



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The conformal symmetry $SO(2,4) \simeq SU(2,2)$

And the R symmetry $SO(6) \simeq SU(4)$

Combine with the N=4 supersymmetry

To make a superconformal symmetry with symmetry group $PSU(2,2|4)$

Spin chains in N=4

$$\mathrm{tr}\ \left(\Phi_1\Phi_2(D_3)^2(\Phi_1)^5\ldots\right)$$

Spin chains in N=4

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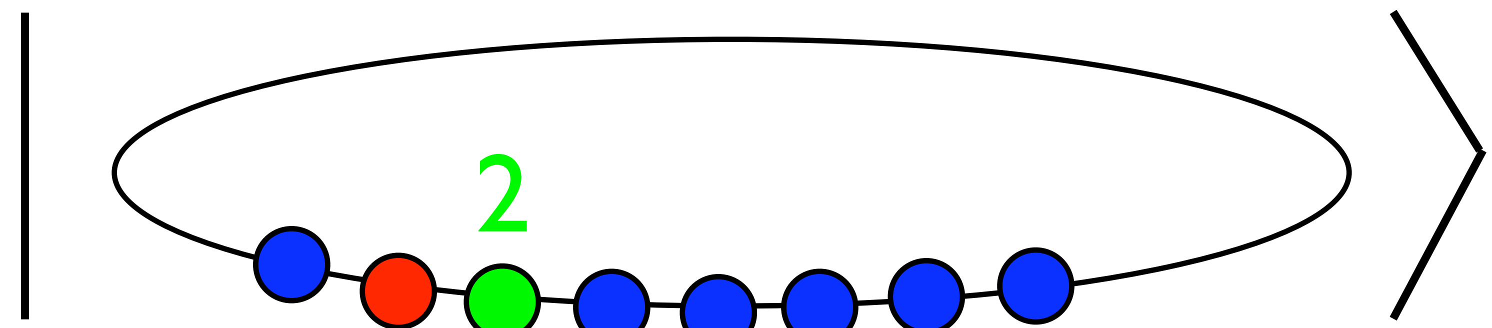
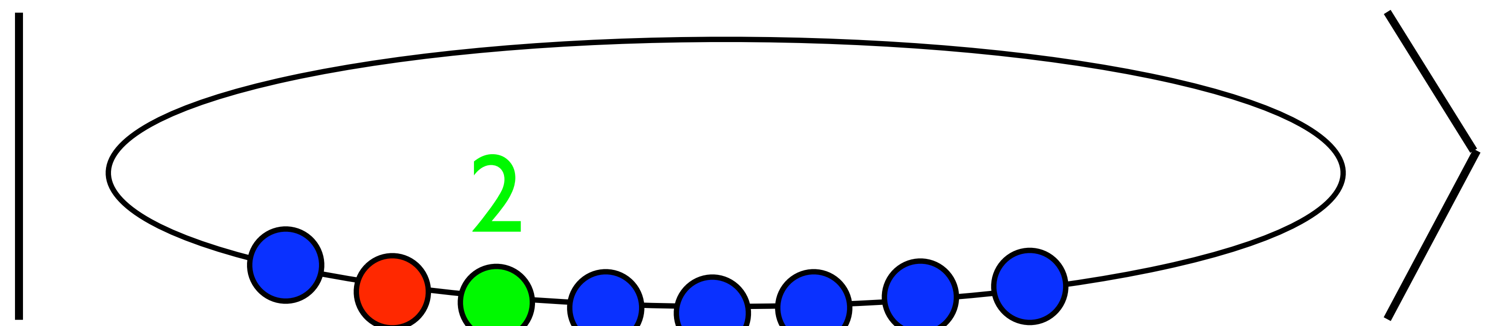


Spin chains in N=4

$$\mathrm{tr}\,\big(\Phi_1\Phi_2(D_3)^2(\Phi_1)^5\ldots\big)$$



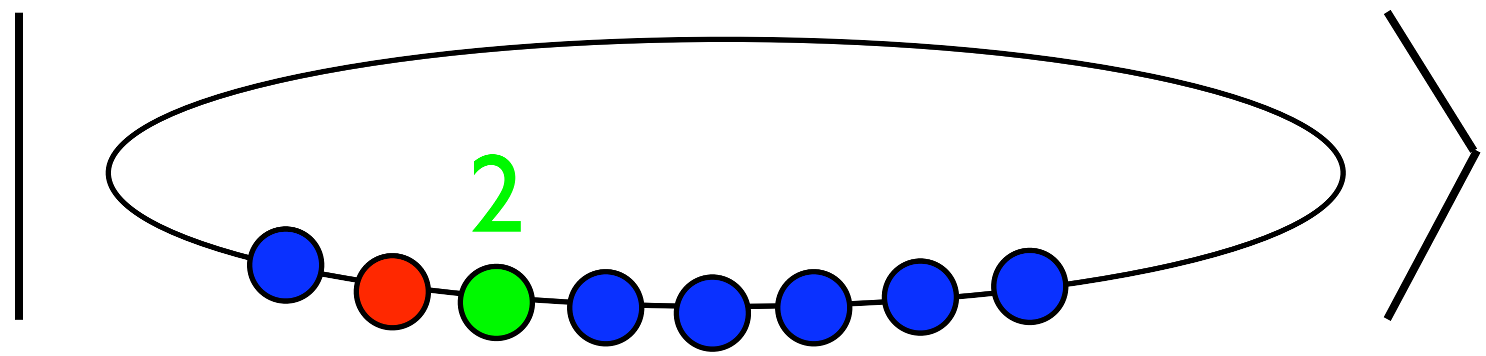
Dilatation operator

$$\mathcal{O}^{ren}_A(x) = (e^{\,\hat{H}\log\Lambda})_{AB}\,\mathcal{O}_B(x)$$

$$\langle \mathcal{O}^{ren}_A(x)\mathcal{O}^{ren}_B(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta}}$$

Spin chains in N=4

$$\mathrm{tr}\,\left(\Phi_1\Phi_2(D_3)^2(\Phi_1)^5\ldots\right)$$



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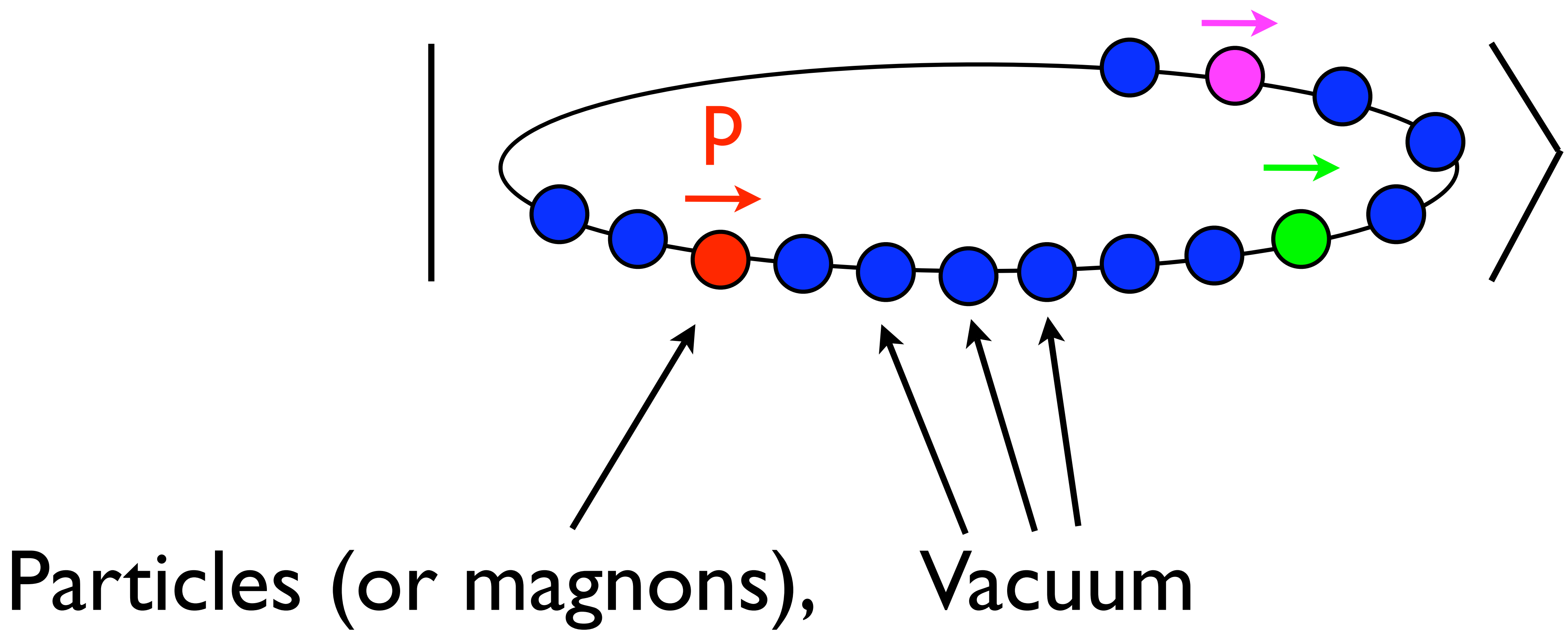
$$\langle \mathcal{O}^{ren}_A(x)\mathcal{O}^{ren}_B(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta}}$$

Integrable Hamiltonian of a PSU(2,2|4) spin chain

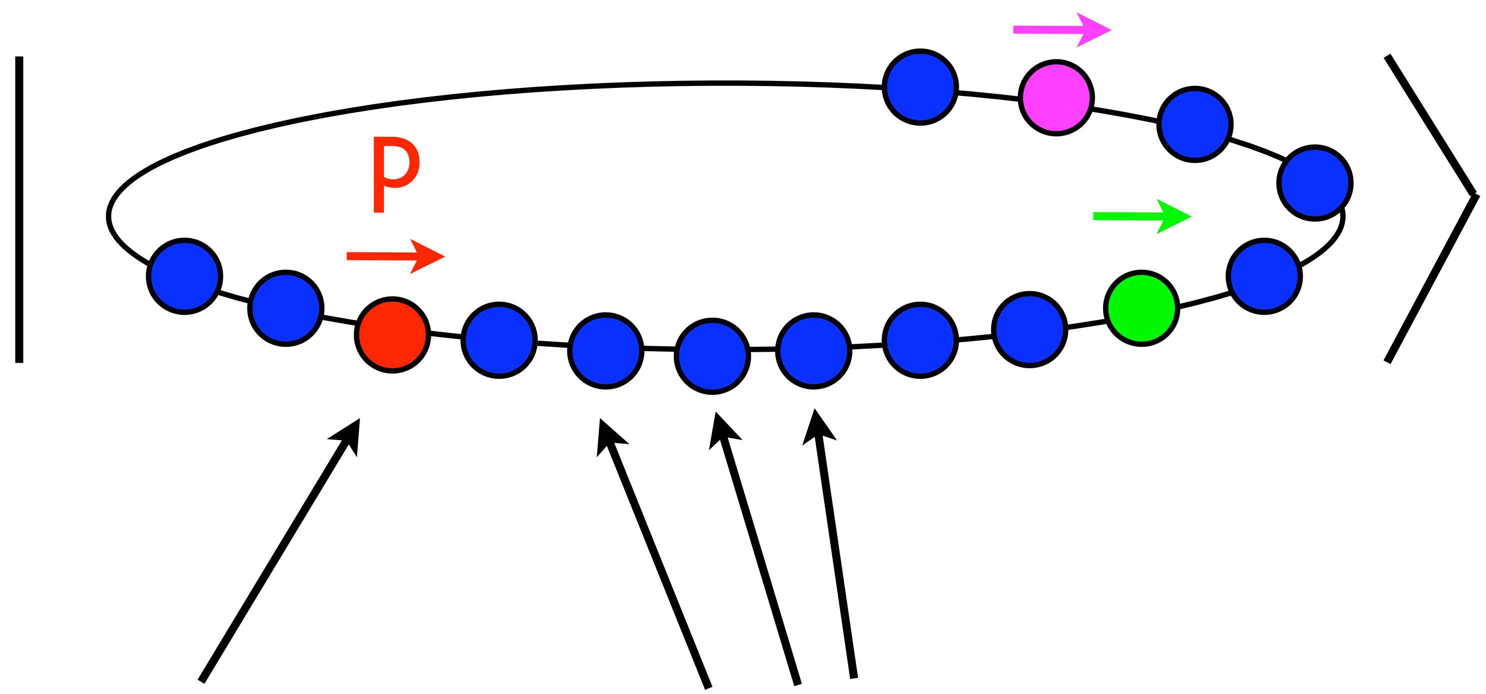
[Minahan, Zarembo; Beisert]

H is nearest neighbors to leading order in perturbation theory, next to nearest neighbors at next to leading order etc...

2d S-matrix in N=4

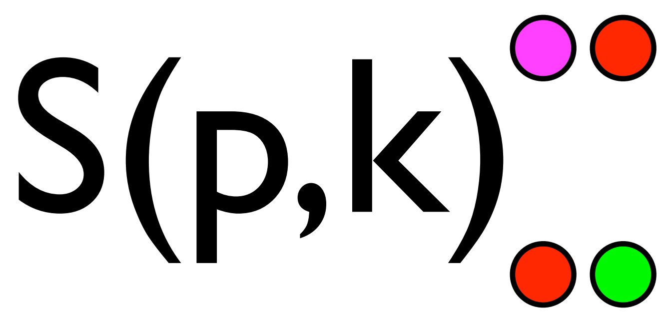


2d S-matrix in N=4



Particles (or magnons), Vacuum

Particles can scatter, e.g.

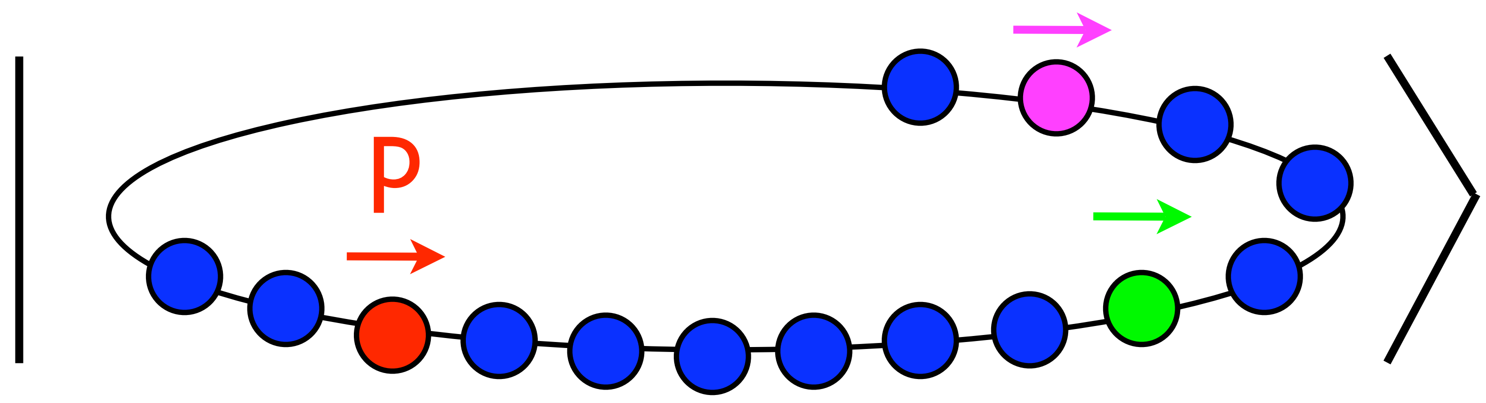


[Staudacher]

Particles transform in PSU(2|2)^2 extended

[Beisert]

2d S-matrix in N=4



H ⟶ S(p,k)

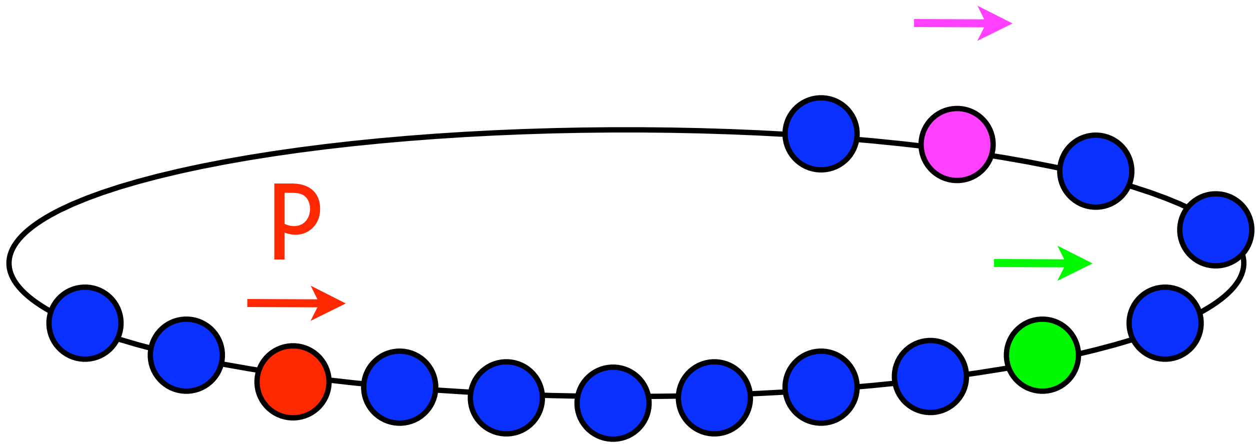
PSU(2,2|4) ⟶ PSU(2|2)^2 extended

S-matrix (up to a scalar factor) and magnon dispersion relation almost fixed by symmetry

[Beisert; Aryutunov, Frolov, Zamaklar]

Bethe Equations

$$0 = \left(e^{iLp_j} \prod_{k \neq j}^{M} \hat{S}(p_j, p_k) - 1 \right) \Big|$$



$$\Big\rangle$$

$$\Delta = J + \sum_{j=1}^{M} \sqrt{1 + \lambda \sin^2 \frac{p_j}{2}} + \ldots$$

[Staudacher; Beisert, Staudacher; Beisert, Eden, Staudacher;
Beisert, Hernandez, Lopez; Arutyunov, Frolov, Zamaklar]

Explicitly

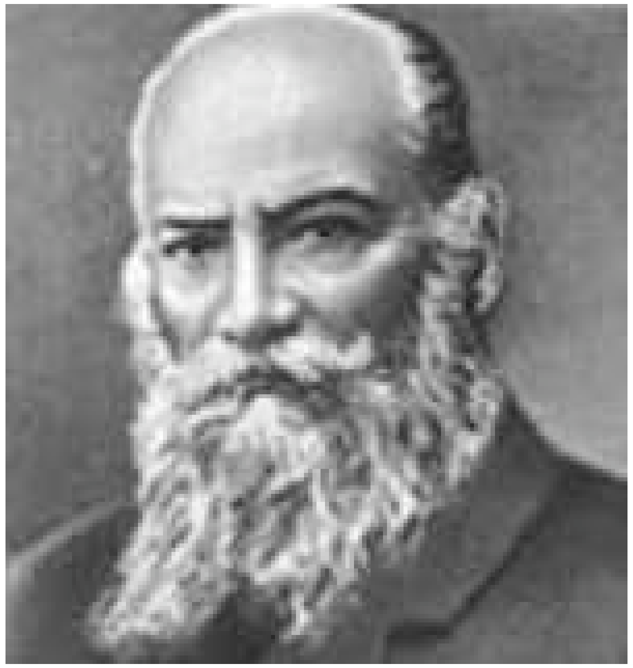
$$\frac{x^+}{x^-} = e^{ip}$$

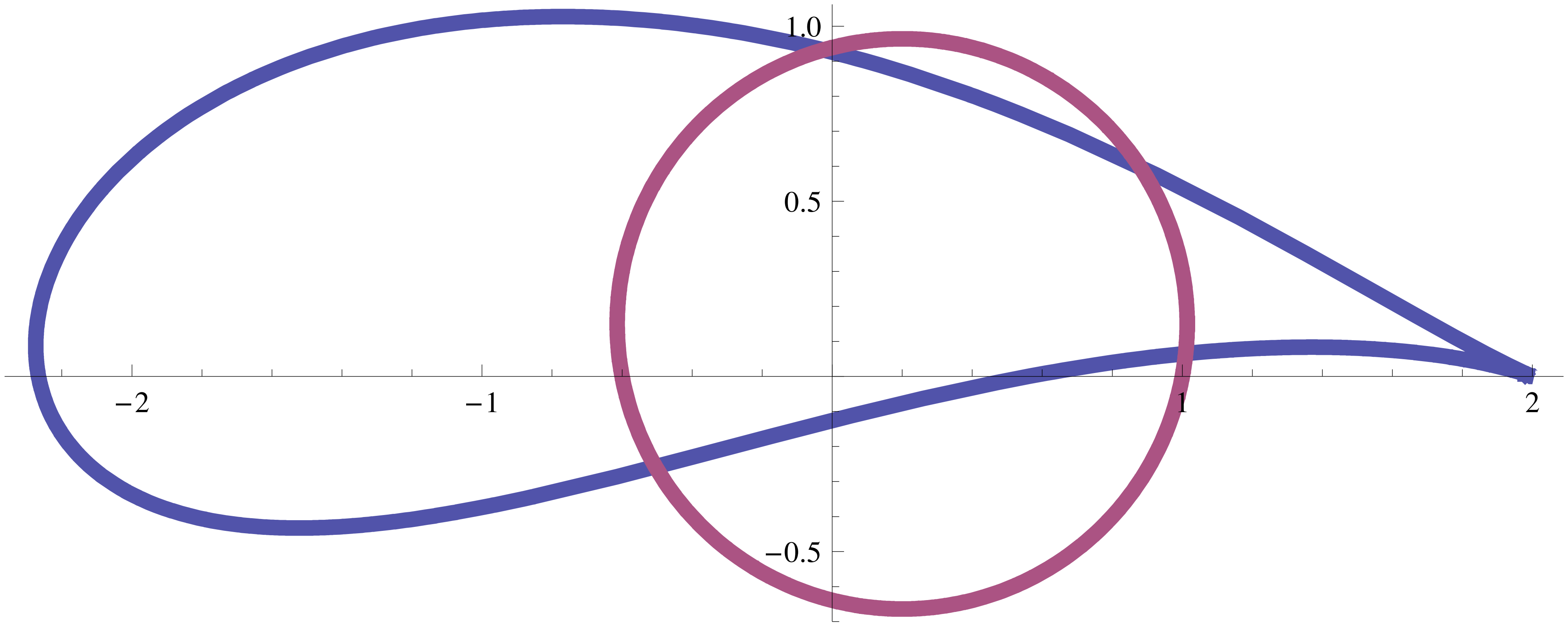
$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{4i}{g}$$

$$x^+ + \frac{1}{x^+} + x^- + \frac{1}{x^-} = \frac{8u}{g} = 2\left(x + \frac{1}{x}\right)$$

The Zhukovsky Map

$$z = x + \frac{1}{x}$$





[Beisert, Staudacher; Beisert, Eden, Staudacher; Beisert, Hernandez, Lopez]

$$e^{i\eta\phi_1 - i\eta\phi_2} = \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}}{u_{1,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - 1/x_{1,k} x_{4,j}^+}{1 - 1/x_{1,k} x_{4,j}^-} ,$$

$$e^{i\eta\phi_2 - i\eta\phi_3} = \prod_{j\neq k}^{K_2} \frac{u_{2,k} - u_{2,j} - i}{u_{2,k} - u_{2,j} + i} \prod_{j=1}^{K_3} \frac{u_{2,k} - u_{3,j} + \frac{i}{2}}{u_{2,k} - u_{3,j} - \frac{i}{2}} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}}{u_{2,k} - u_{1,j} - \frac{i}{2}} ,$$

$$e^{i\eta\phi_3 - i\eta\phi_4} = \prod_{j=1}^{K_2} \frac{u_{3,k} - u_{2,j} + \frac{i}{2}}{u_{3,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3,k} - x_{4,j}^+}{x_{3,k} - x_{4,j}^-} ,$$

$$e^{i\eta\phi_4 - i\eta\phi_5} = \left(\frac{x_{4,k}^-}{x_{4,k}^+}\right)^{\eta L} \prod_{j\neq k}^{K_4} \frac{u_{4,k} - u_{4,j} + i}{u_{4,k} - u_{4,j} - i} \prod_{j}^{K_4} \left(\frac{1 - 1/x_{4,k}^+ x_{4,j}^-}{1 - 1/x_{4,k}^- x_{4,j}^+}\right)^{\eta-1} \left(\sigma^2(x_{4,k}, x_{4,j})\right)^{\eta}$$

$$\times \prod_{j=1}^{K_1} \frac{1 - 1/x_{4,k}^- x_{1,j}}{1 - 1/x_{4,k}^+ x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^- - x_{3,j}}{x_{4,k}^+ - x_{3,j}} \prod_{j=1}^{K_5} \frac{x_{4,k}^- - x_{5,j}}{x_{4,k}^+ - x_{5,j}} \prod_{j=1}^{K_7} \frac{1 - 1/x_{4,k}^- x_{7,j}}{1 - 1/x_{4,k}^+ x_{7,j}} ,$$

$$e^{i\eta\phi_5 - i\eta\phi_6} = \prod_{j=1}^{K_6} \frac{u_{5,k} - u_{6,j} + \frac{i}{2}}{u_{5,k} - u_{6,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{5,k} - x_{4,j}^+}{x_{5,k} - x_{4,j}^-} ,$$

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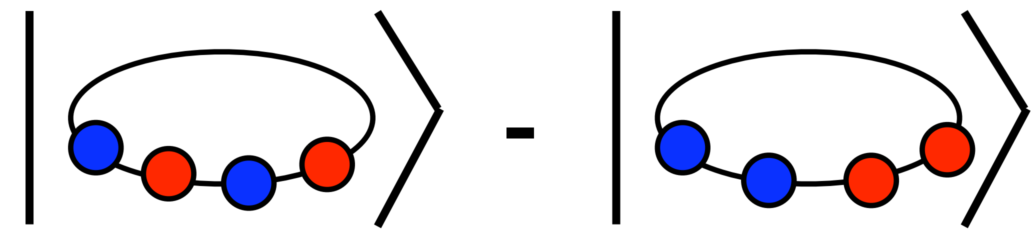
$$e^{i\eta\phi_7 - i\eta\phi_8} = \prod_{j=1}^{K_6} \frac{u_{7,k} - u_{6,j} + \frac{i}{2}}{u_{7,k} - u_{6,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - 1/x_{7,k} x_{4,j}^+}{1 - 1/x_{7,k} x_{4,j}^-} .$$

$$\Delta_{\mathrm{tr}\,(\Phi_{\mathrm{i}}^2)}(g) = ?$$

Konishi as 2 magnon state:



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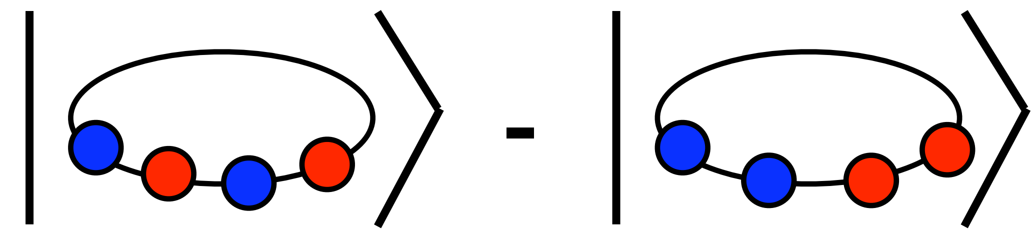

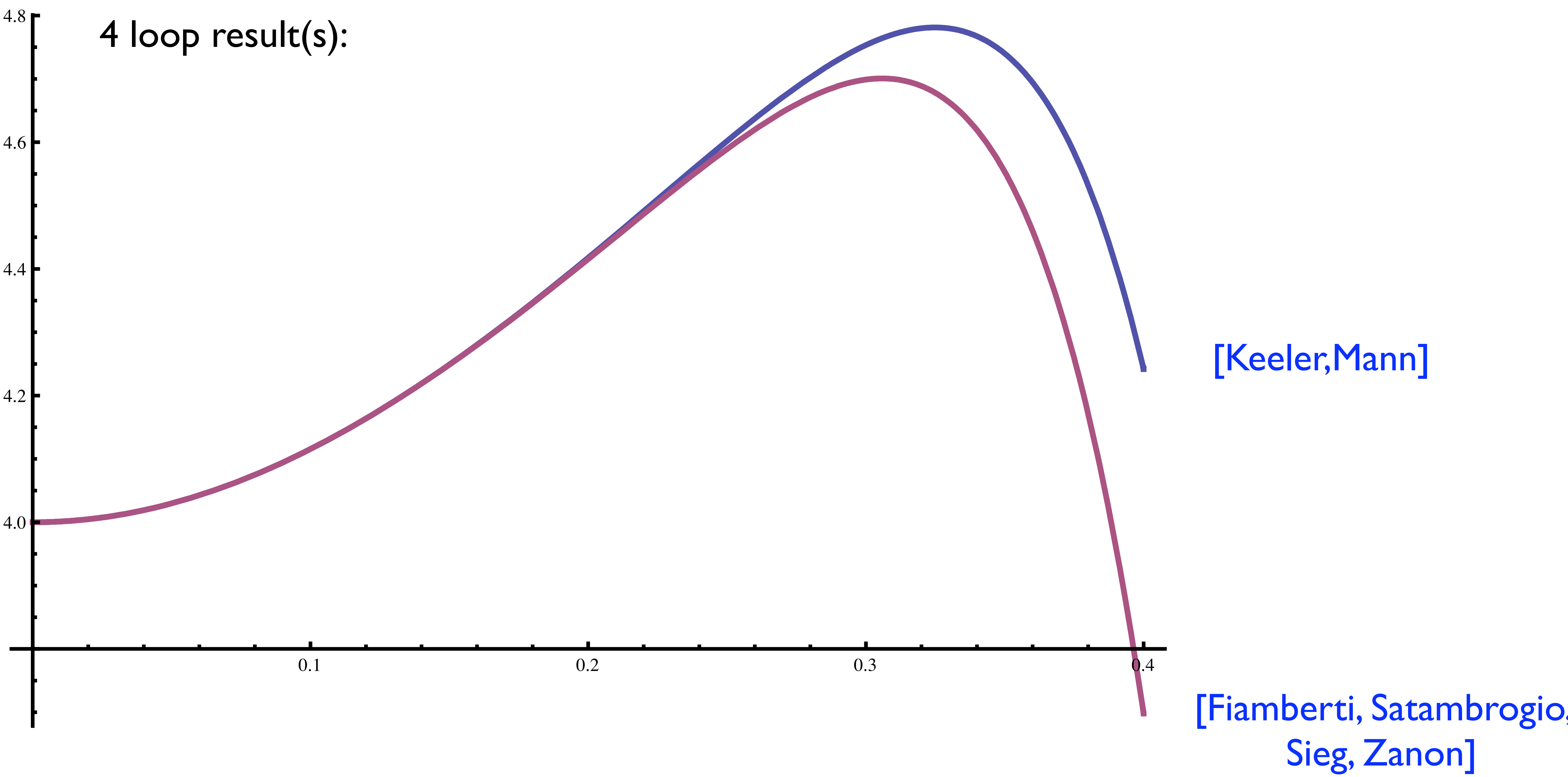

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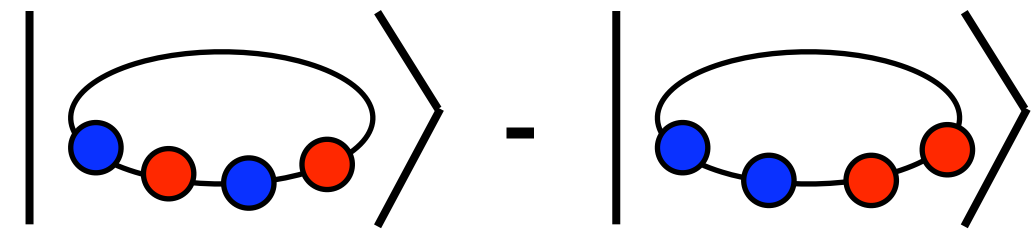
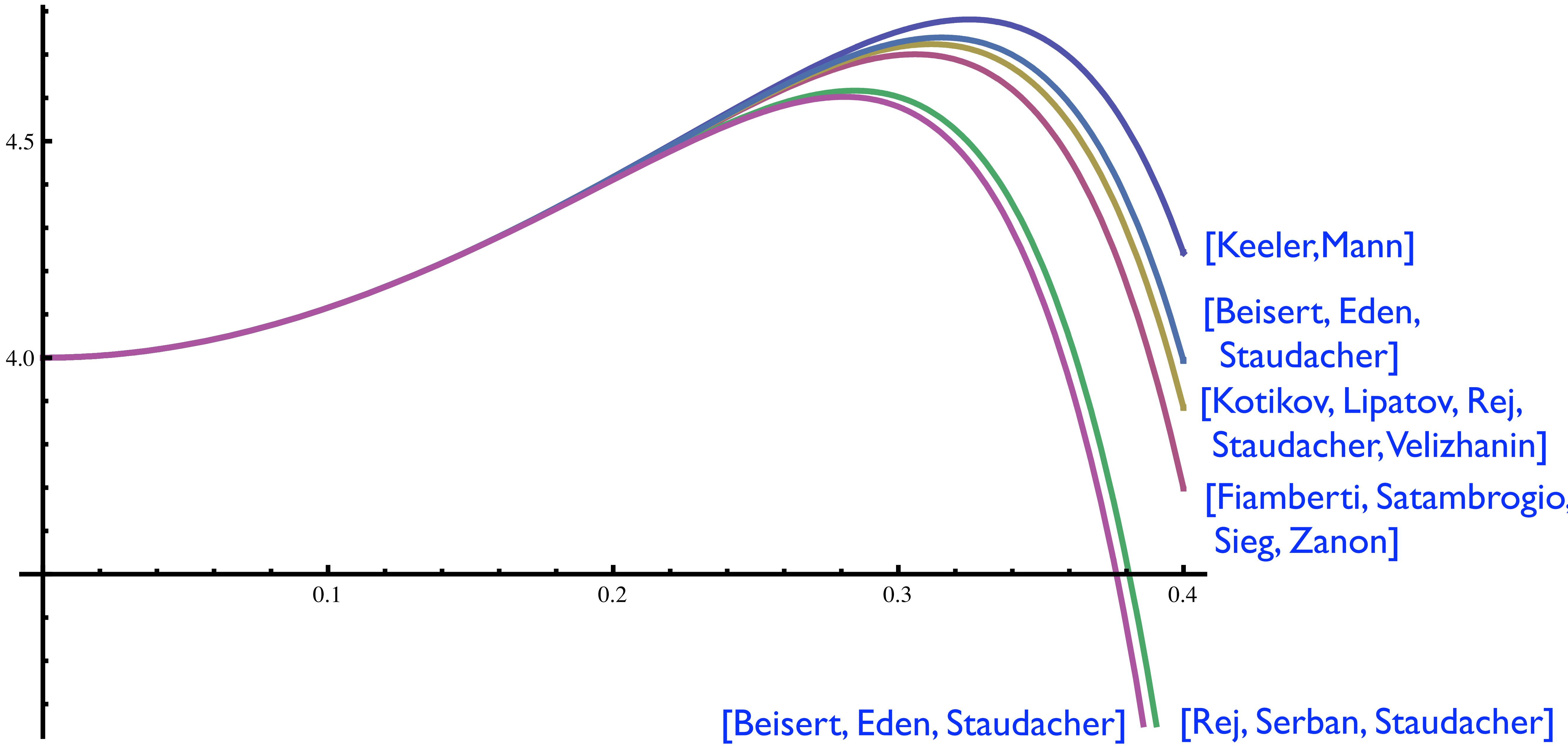
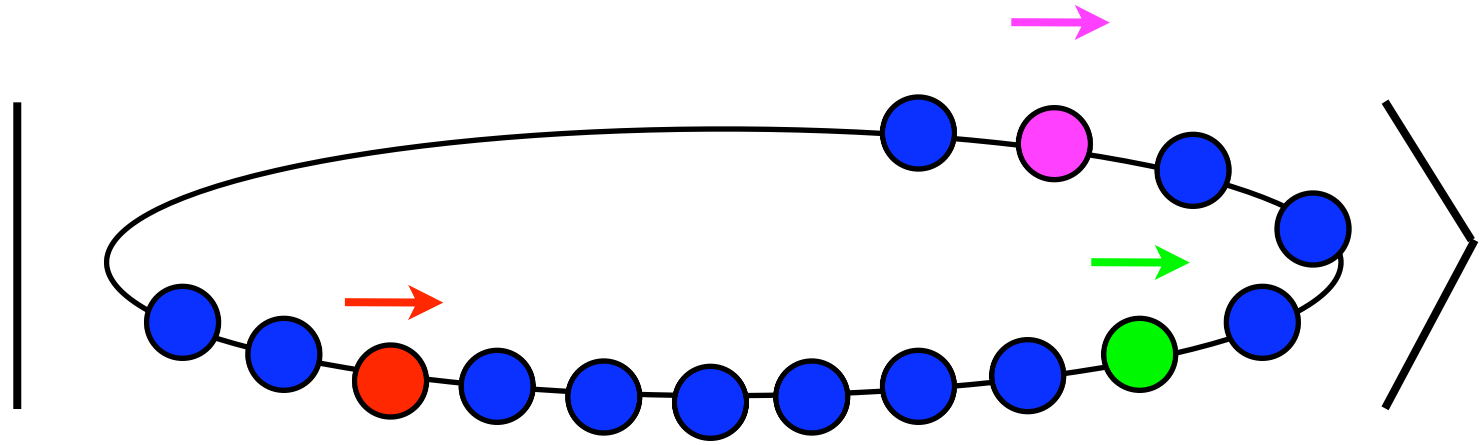


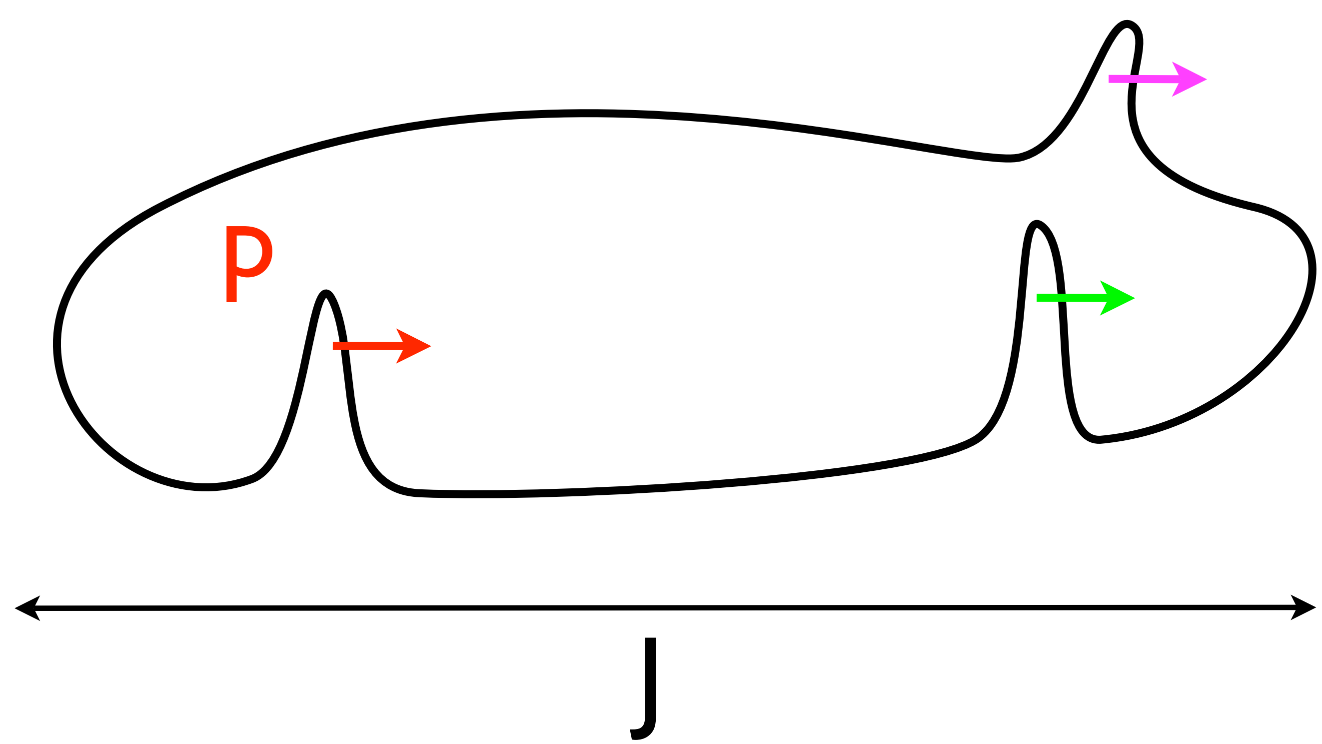


YM, small g



$$\Delta = J + \sum_{j=1}^{M} \sqrt{1 + \lambda \sin^2 \frac{p_j}{2}} + \ldots$$

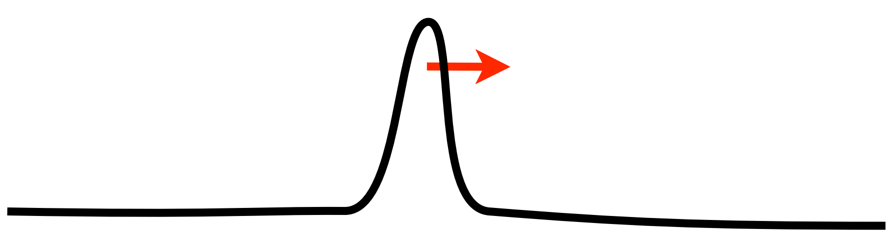
Strings, large g



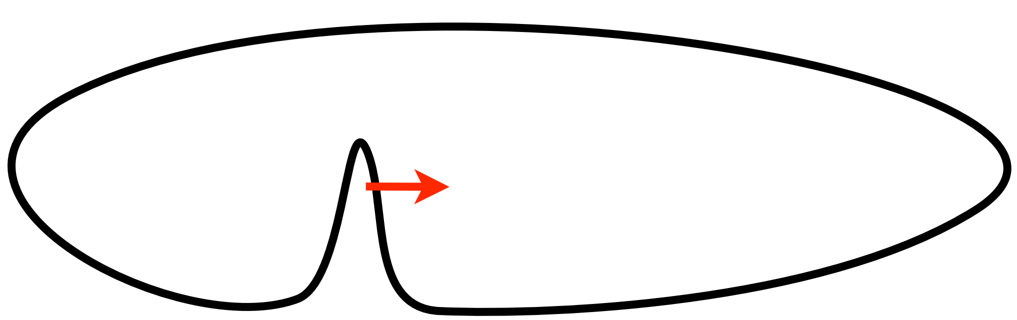
From ∞ to finite volume

[Beisert; Aryutunov, Frolov, Plefka, Zamaklar; Hofman, Maldacena; Vicedo]

$$\epsilon_\infty(p) = \sqrt{1 + \lambda \sin^2 \frac{p}{2}} = \sqrt{\lambda}\sin\frac{p}{2} + 0 + \mathcal{O}(1/\sqrt{\lambda})$$



[Papathanasiou, Spradlin;
Chen, Dorey, Lima Matos]



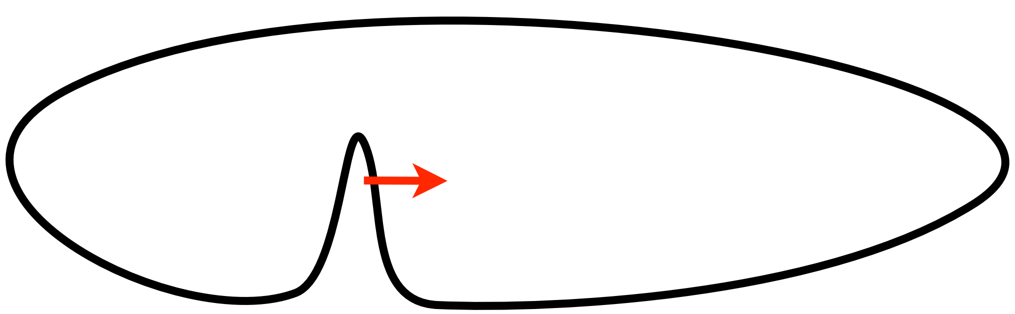
$$\epsilon(p) = \epsilon_\infty(p) + \delta\epsilon(p)$$

[Aryutunov, Frolov, Zamaklar]
[Janik, Lukowski]
[Minahan, Sax; Hatsuda, Suzuki]
[Gromov, Schafer-Nameki, PV]

From ∞ to finite volume

[Beisert; Aryutunov, Frolov, Plefka, Zamaklar; Hofman, Maldacena; Vicedo]

See more in Benoit II



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[Aryutunov, Frolov, Zamaklar]
[Janik, Lukowski]
[Minahan, Sax; Hatsuda, Suzuki]
[Gromov, Schafer-Nameki, PV]

[Beisert, Staudacher; Beisert, Eden, Staudacher; Beisert, Hernandez, Lopez]

$$e^{i\eta\phi_1 - i\eta\phi_2} = \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}}{u_{1,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - 1/x_{1,k} x_{4,j}^+}{1 - 1/x_{1,k} x_{4,j}^-} ,$$

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$$\times \prod_{j=1}^{K_1} \frac{1 - 1/x_{4,k}^- x_{1,j}}{1 - 1/x_{4,k}^+ x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^- - x_{3,j}}{x_{4,k}^+ - x_{3,j}} \prod_{j=1}^{K_5} \frac{x_{4,k}^- - x_{5,j}}{x_{4,k}^+ - x_{5,j}} \prod_{j=1}^{K_7} \frac{1 - 1/x_{4,k}^- x_{7,j}}{1 - 1/x_{4,k}^+ x_{7,j}} ,$$

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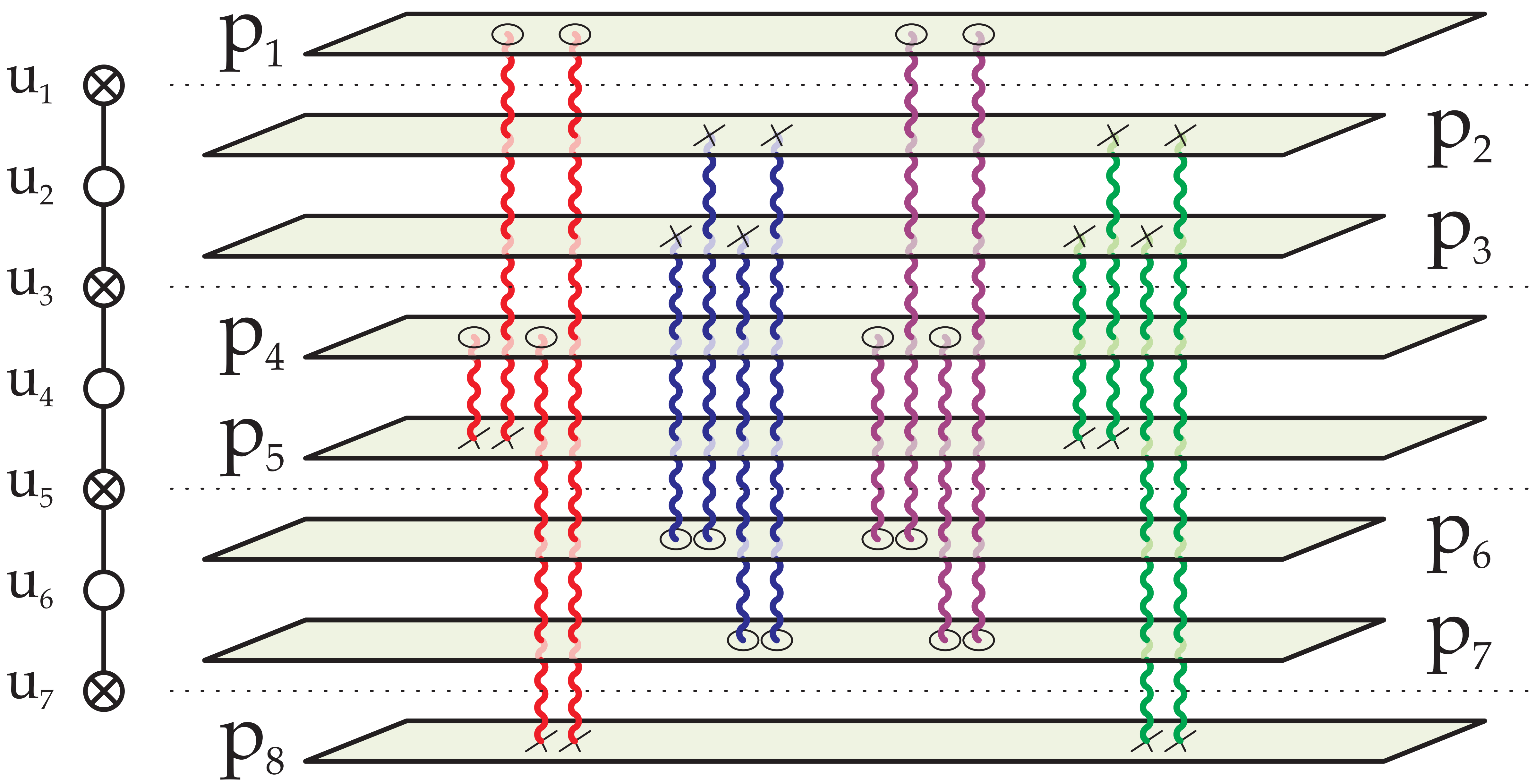
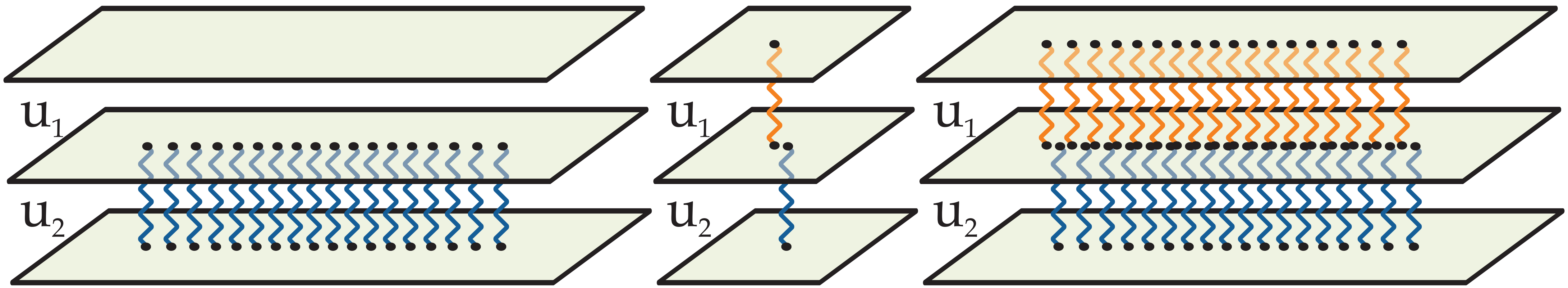
Output



Scaling limit

[Beisert, Kazakov, Sakai, Zarembo]



Cuts Stacks Cut of Stacks

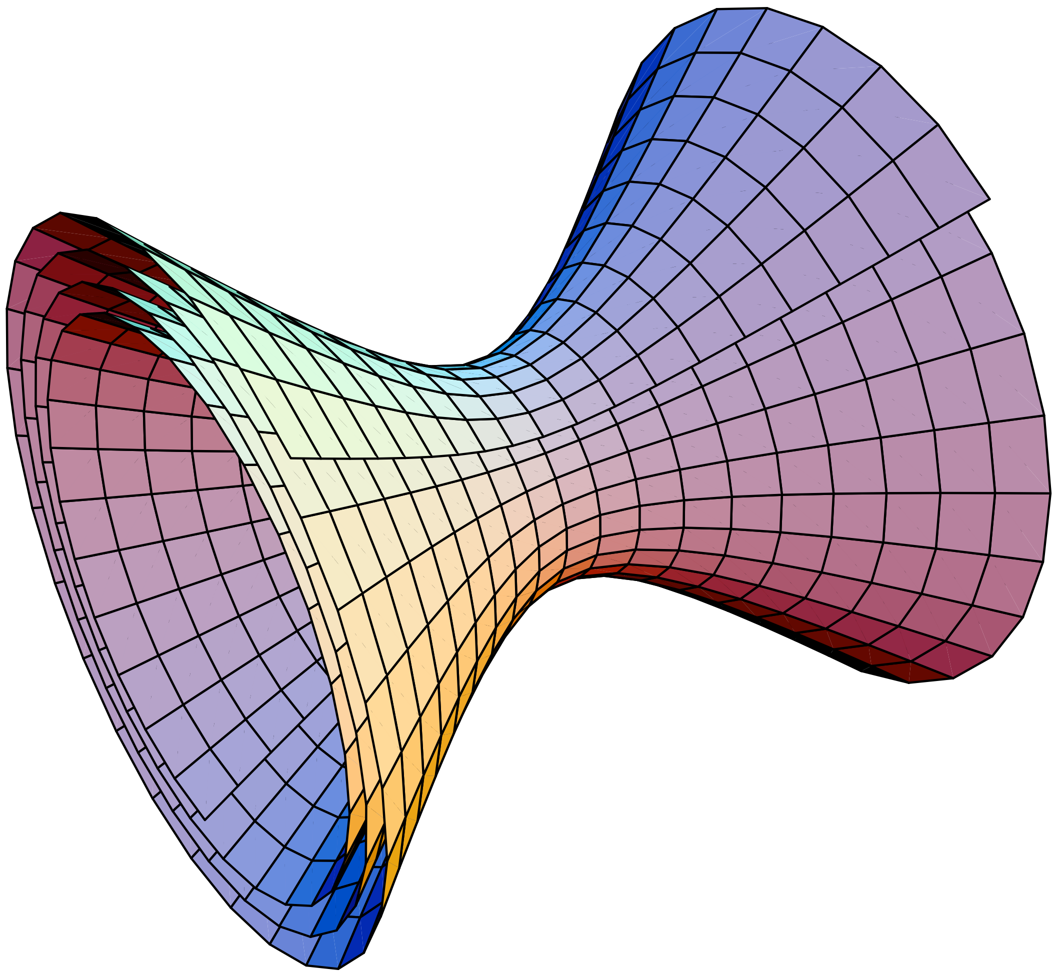
Bethe Equations ⟶ (8-sheet) Riemann Surface

Time to go to the string side!

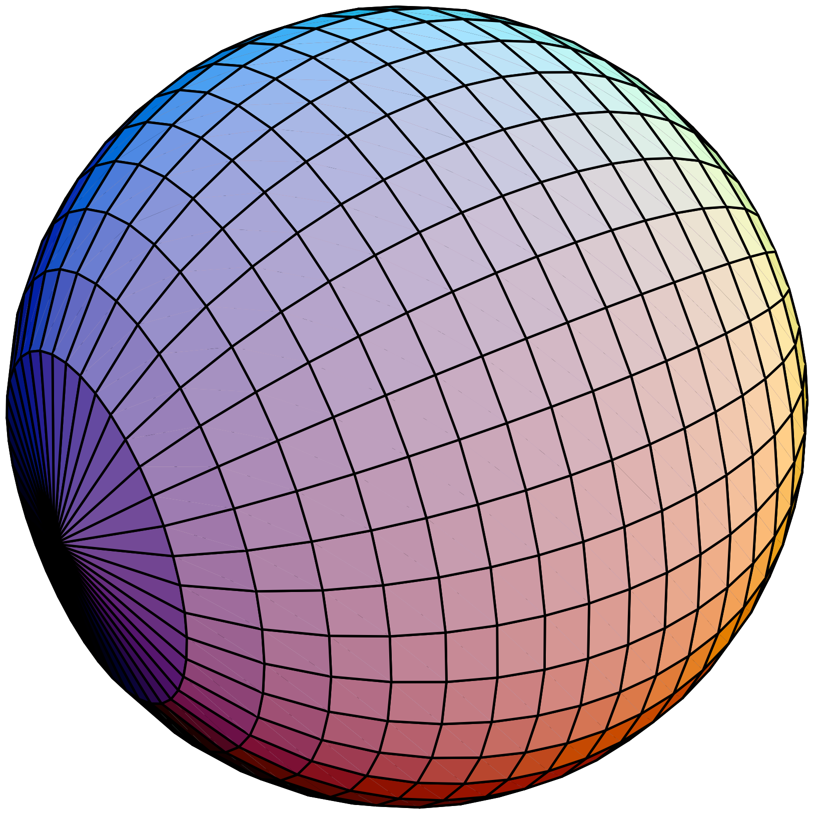
AdS

IIB Superstrings
in AdS_5xS^5

$$S \sim \int d\tau d\sigma \left[(\partial\vec{n})^2 + fermions\right]$$



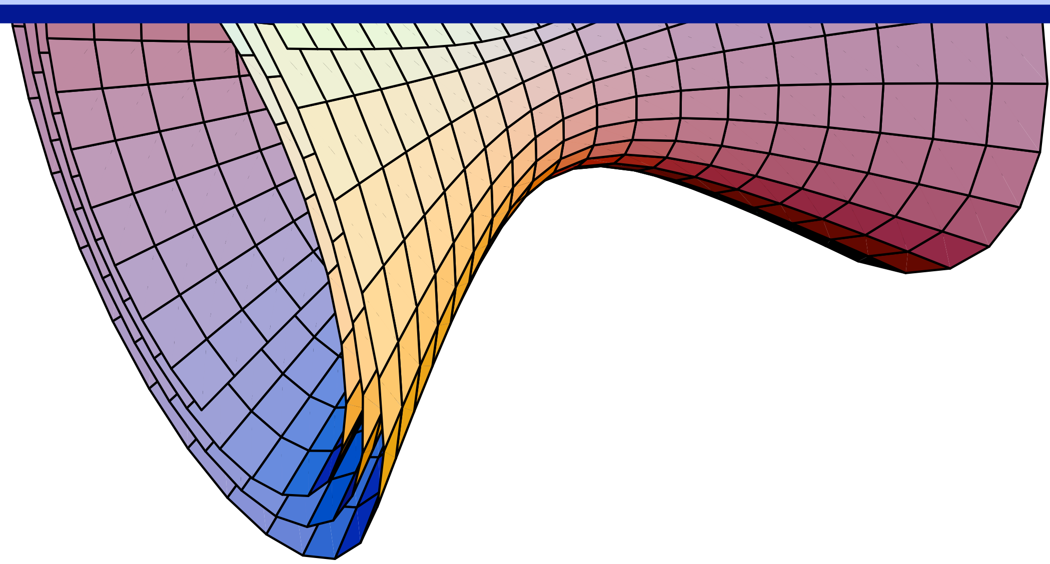
x



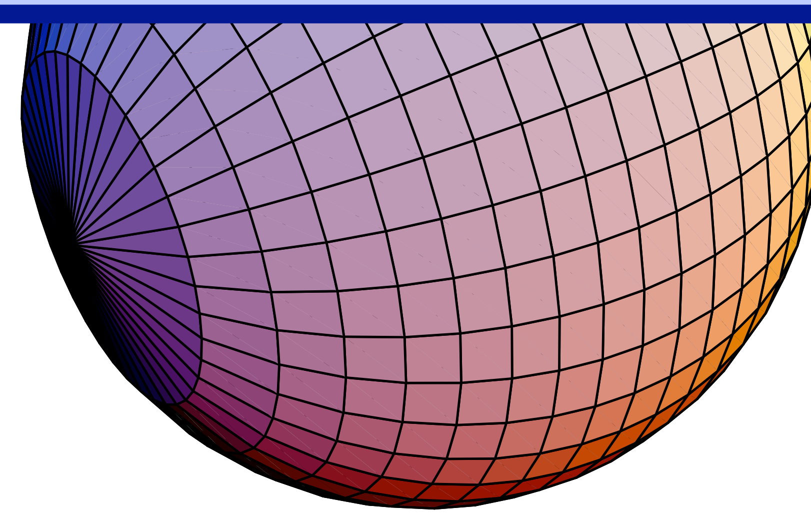
AdS

IIB Superstrings in $AdS_5 x S^5$

Description of the full theory in Pedro III this afternoon

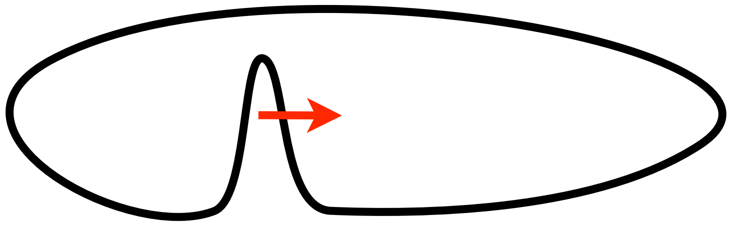


x



Algebraic Curves

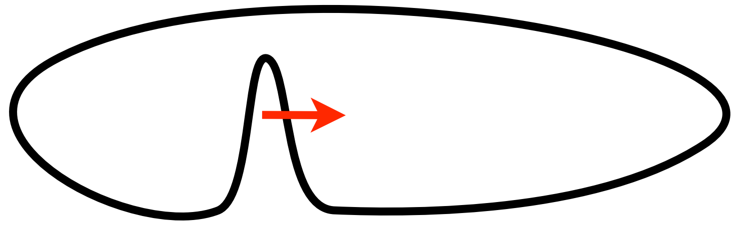
Classical motion



$$g(\sigma,\tau) \in PSU(2,2|4)$$

Algebraic Curves

Classical motion



$$g(\sigma,\tau) \in PSU(2,2|4)$$

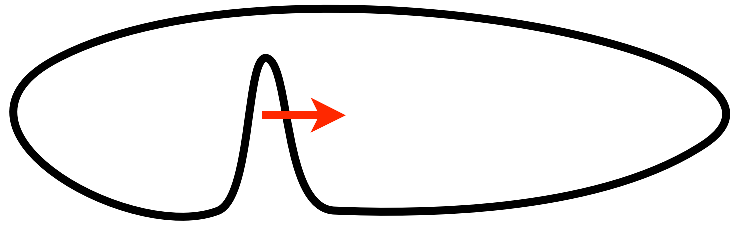
Flat Connection

[Bena, Polchinski, Roiban]

$$\exists\, A(g;x) \;\; \texttt{such that} \;\; dA - A\wedge A = 0$$

Algebraic Curves

Classical motion



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Flat Connection
[Bena, Polchinski, Roiban]

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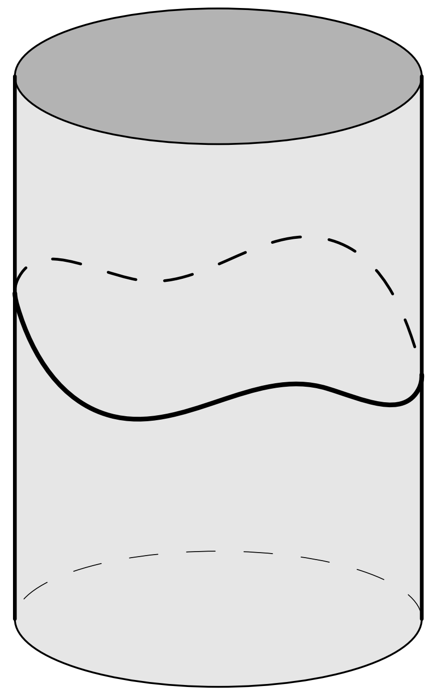
Monodromy and
quasi-momenta

[Kazakov, Marshakov, Minahan, Zarembo; Beisert, Kazakov, Sakai, Zarembo]



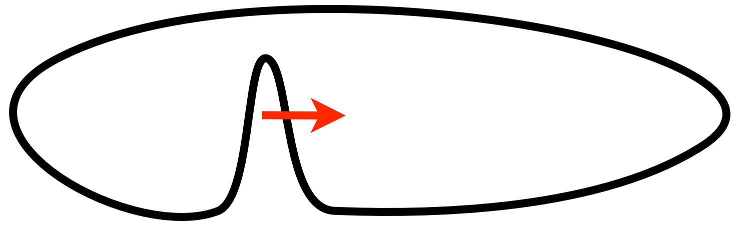
$$= \Omega(x) = P \exp \int A(g;x)$$

Eigenvalues $\{e^{ip_1}, \ldots, e^{ip_8}\}$ are conserved... for any x!!

Classical Strings ⟷ Algebraic Curves

Algebraic Curves

Classical motion



$g(\sigma, \tau) \in PSU(2,2|4)$

Flat Connec

[Bena, Polch

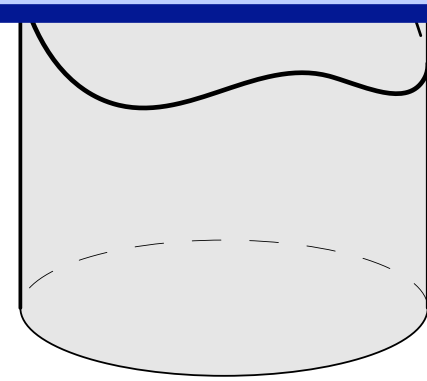
$A = 0$

More in couple of minutes in Benoit I

Monodrom
quasi-mon

$;x)$

[Kazakov, Marshakov, Minahan, Zarembo; Beisert, Kazakov, Sakai, Zarembo]



Eigenvalues $\{e^{ip_1}, \ldots, e^{ip_8}\}$ are conserved... for any x!!

Classical Strings ⟷ Algebraic Curves

Quantization

Quantization

=

Discretization!

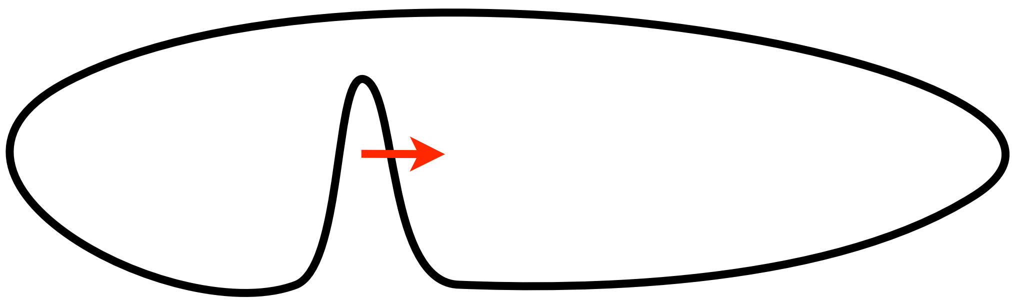
Cuts should be the condensation of the Bethe roots of some String Bethe Equations.

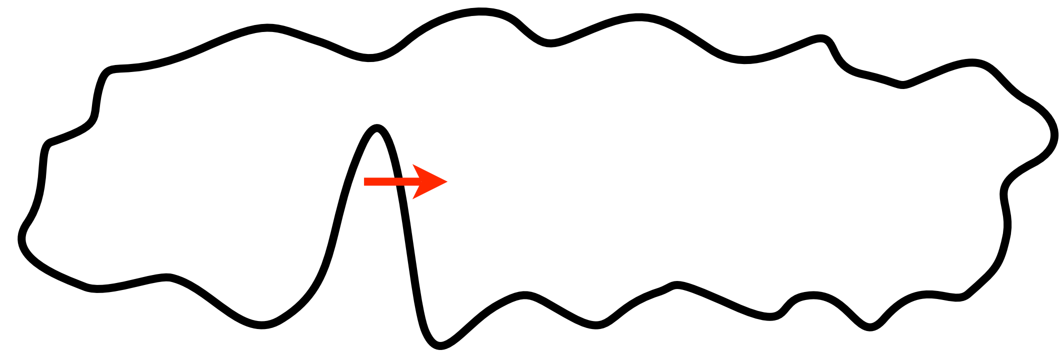
Quantization

After lunch in Pedro II

Cuts should be the condensation of the Bethe roots of some String Bethe Equations.

Semiclassics





$$\Omega_n^{ij}$$

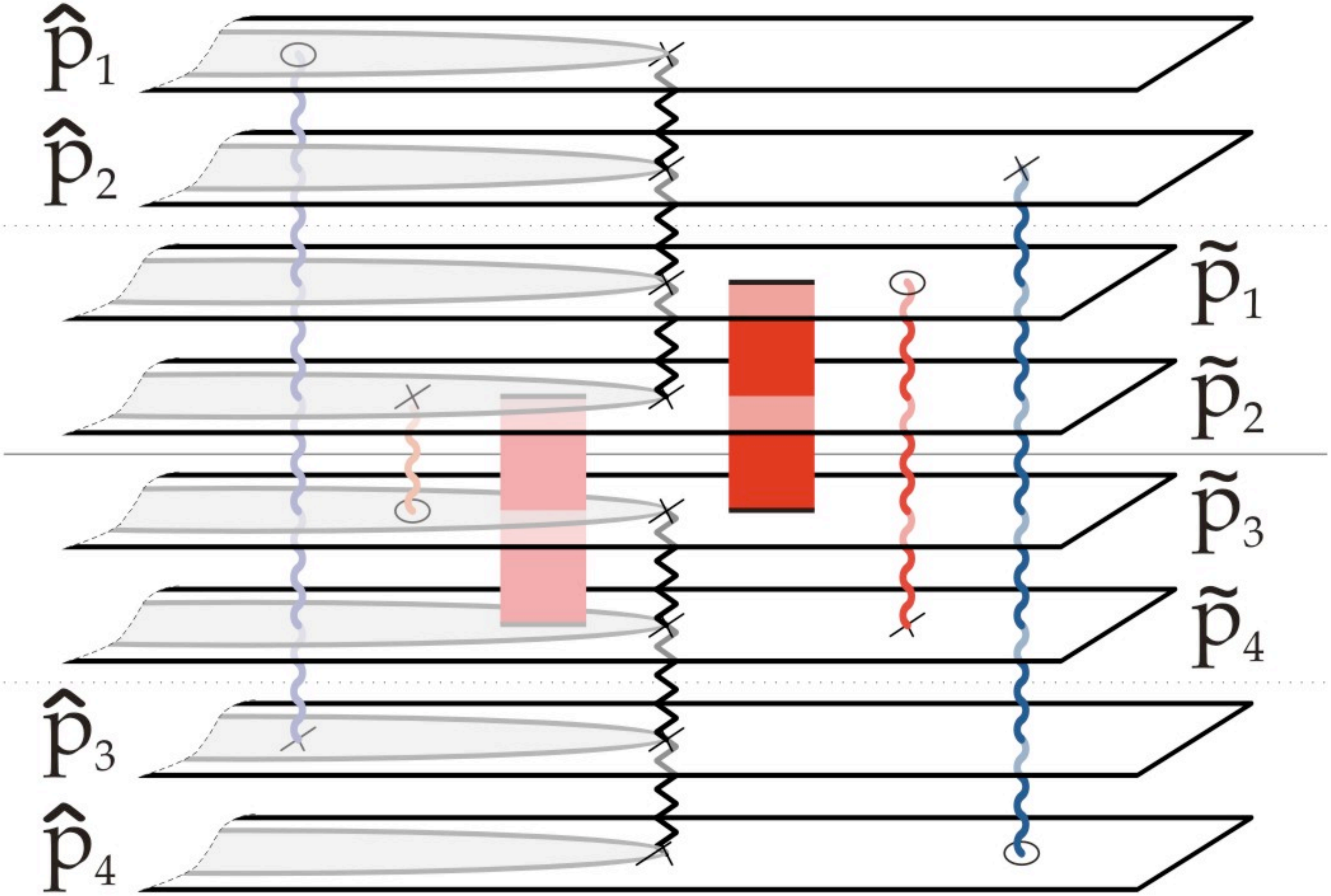
Polarization ij

Mode number n

$$\delta\epsilon_{1-loop} = \frac{1}{2}\sum_{n\in\mathbb{Z}}\sum_{(ij)}(-1)^{F_{ij}}\,\Omega_n^{ij}$$

[Gromov, PV; Vicedo, to appear]

Semiclassics



$$\tilde{p}_1^+ - \tilde{p}_3^- = 2\pi n$$

$$\tilde{p}_1(x_n^{\tilde{1}\tilde{4}}) - \tilde{p}_4(x_n^{\tilde{1}\tilde{4}}) = 2\pi n_{\tilde{1}\tilde{4}}$$
$$\tilde{p}_1(x_n^{\tilde{1}\hat{4}}) - \hat{p}_4(x_n^{\tilde{1}\hat{4}}) = 2\pi n_{\tilde{1}\hat{4}}$$

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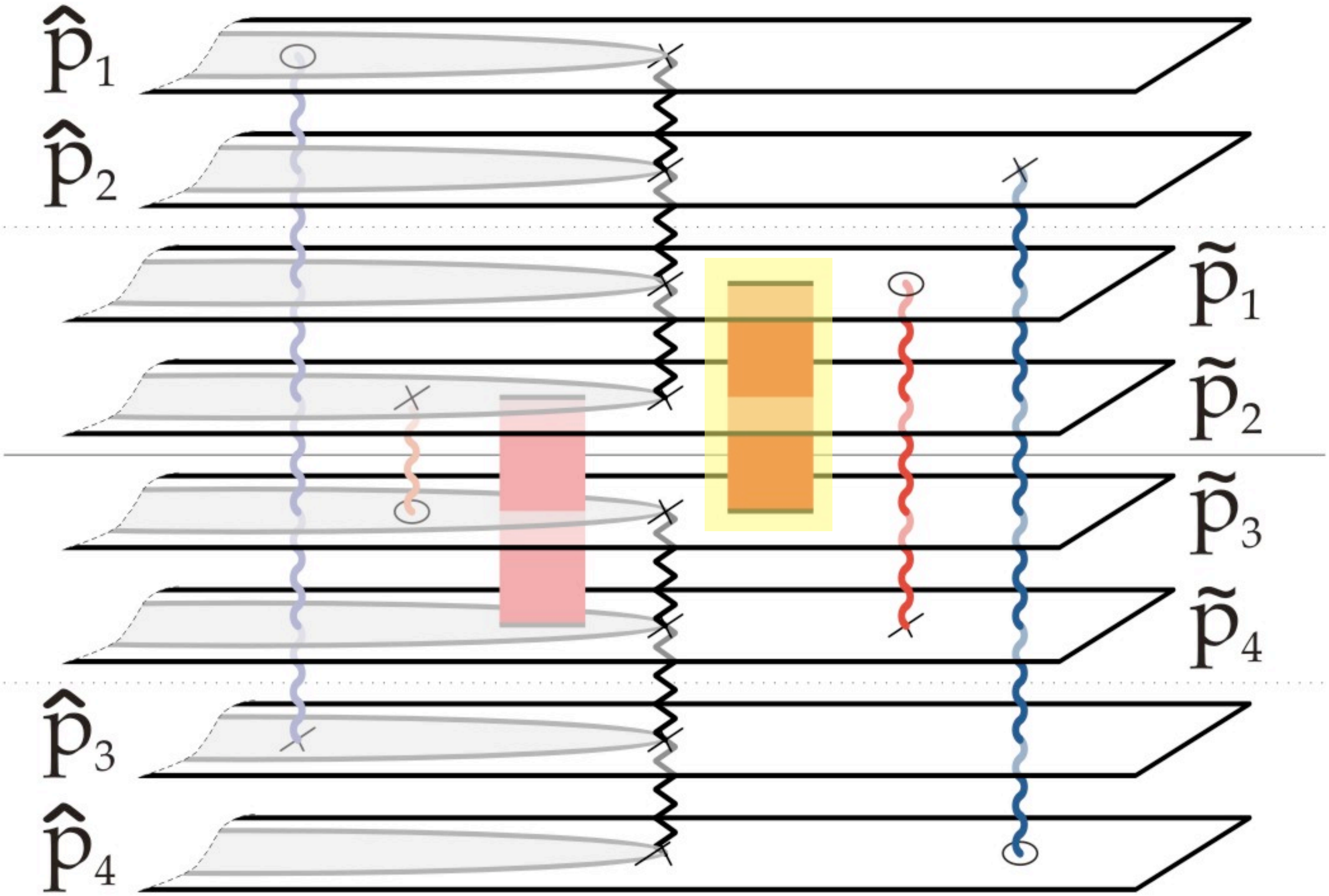
Semiclassics



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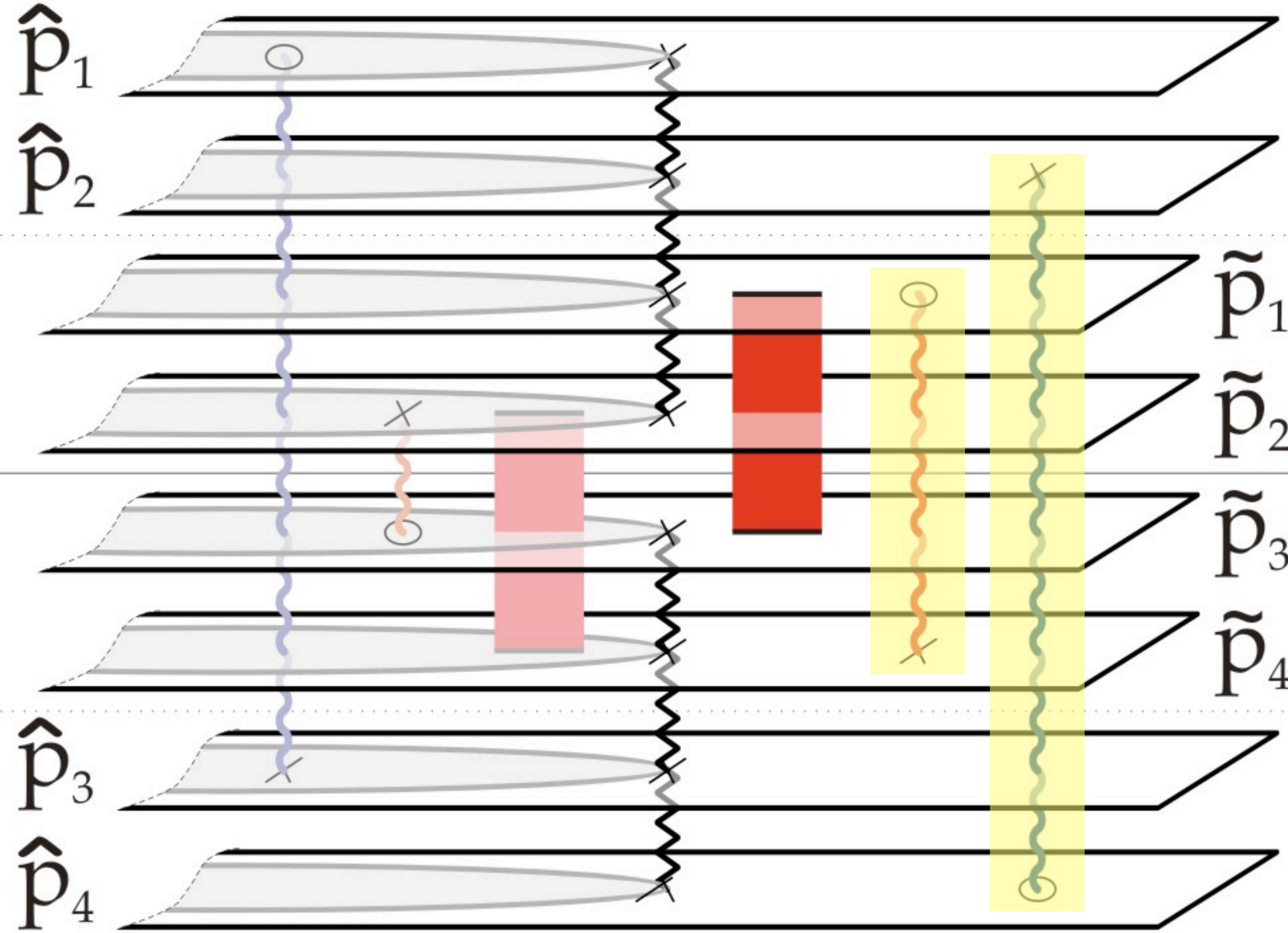

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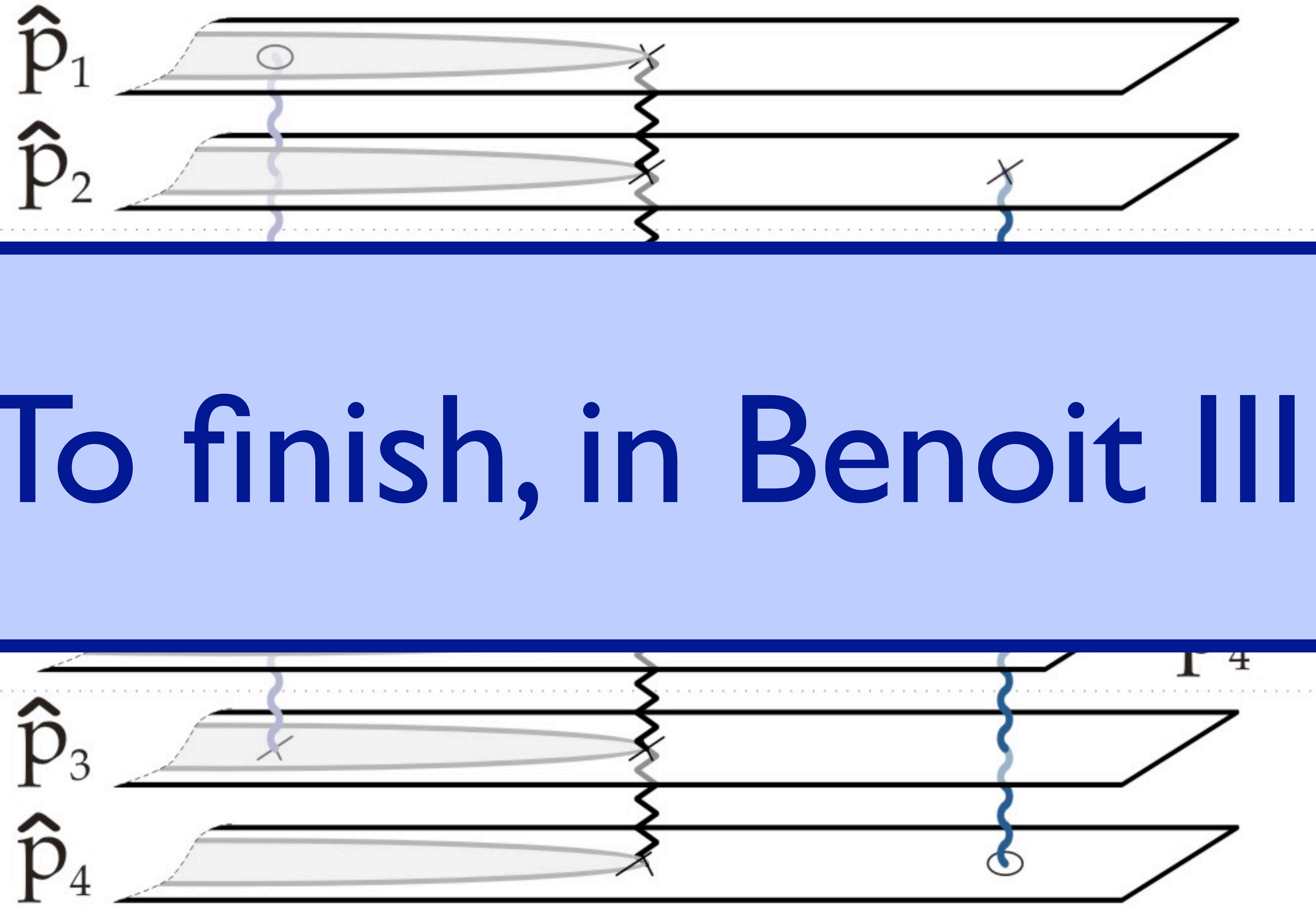
Semiclassics



To finish, in Benoit III

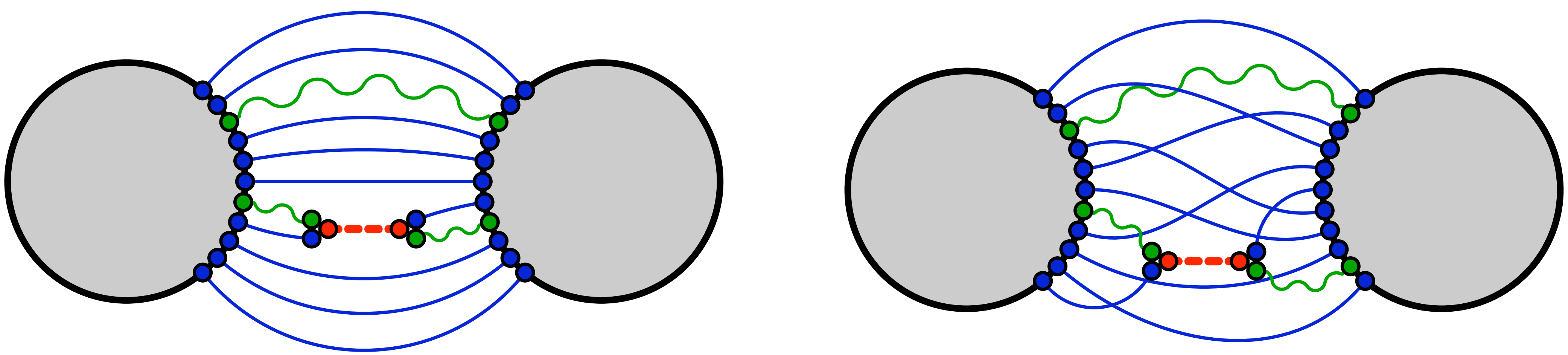
$$\tilde{p}_1^+ - \tilde{p}_3^- = 2\pi n$$

$$\tilde{p}_1(x_n^{\tilde{1}\tilde{4}}) - \tilde{p}_4(x_n^{\tilde{1}\tilde{4}}) = 2\pi n_{\tilde{1}\tilde{4}}$$
$$\tilde{p}_1(x_n^{\tilde{1}\hat{4}}) - \hat{p}_4(x_n^{\tilde{1}\hat{4}}) = 2\pi n_{\tilde{1}\hat{4}}$$

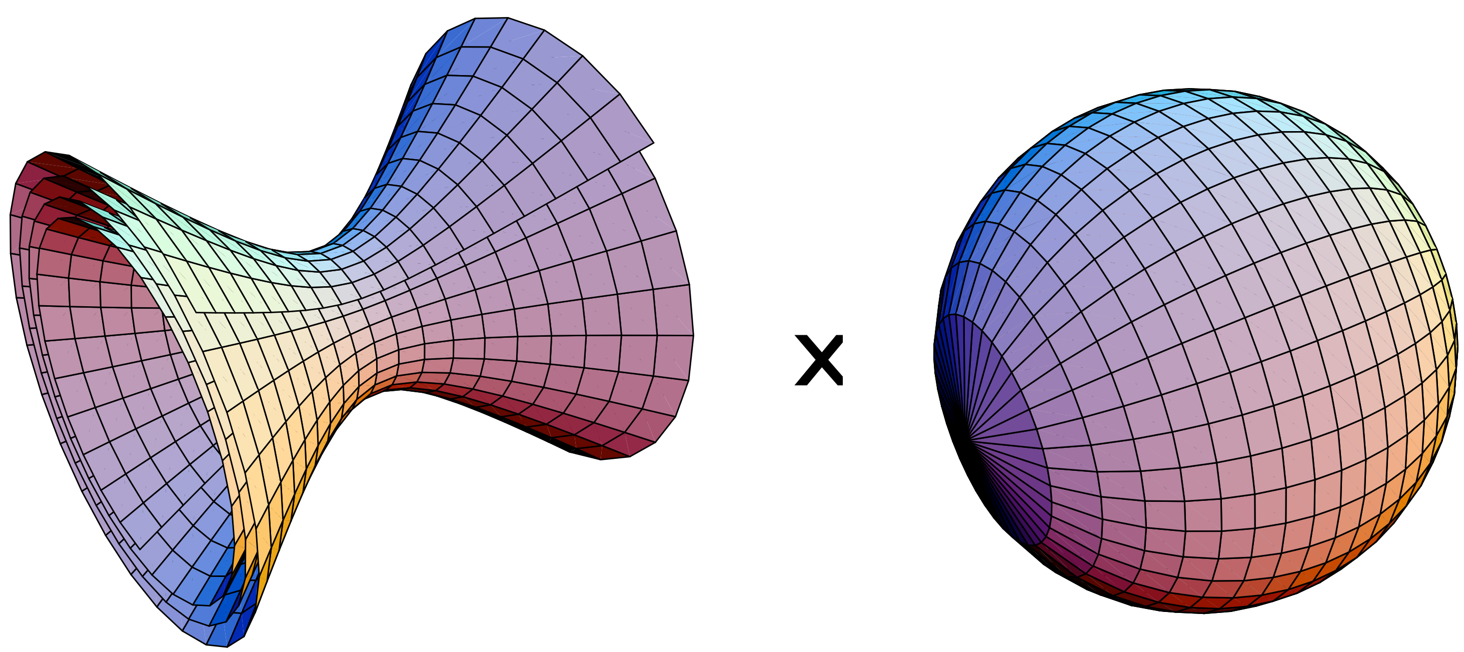
Integrability in AdS/CFT

Summary of the state of the art

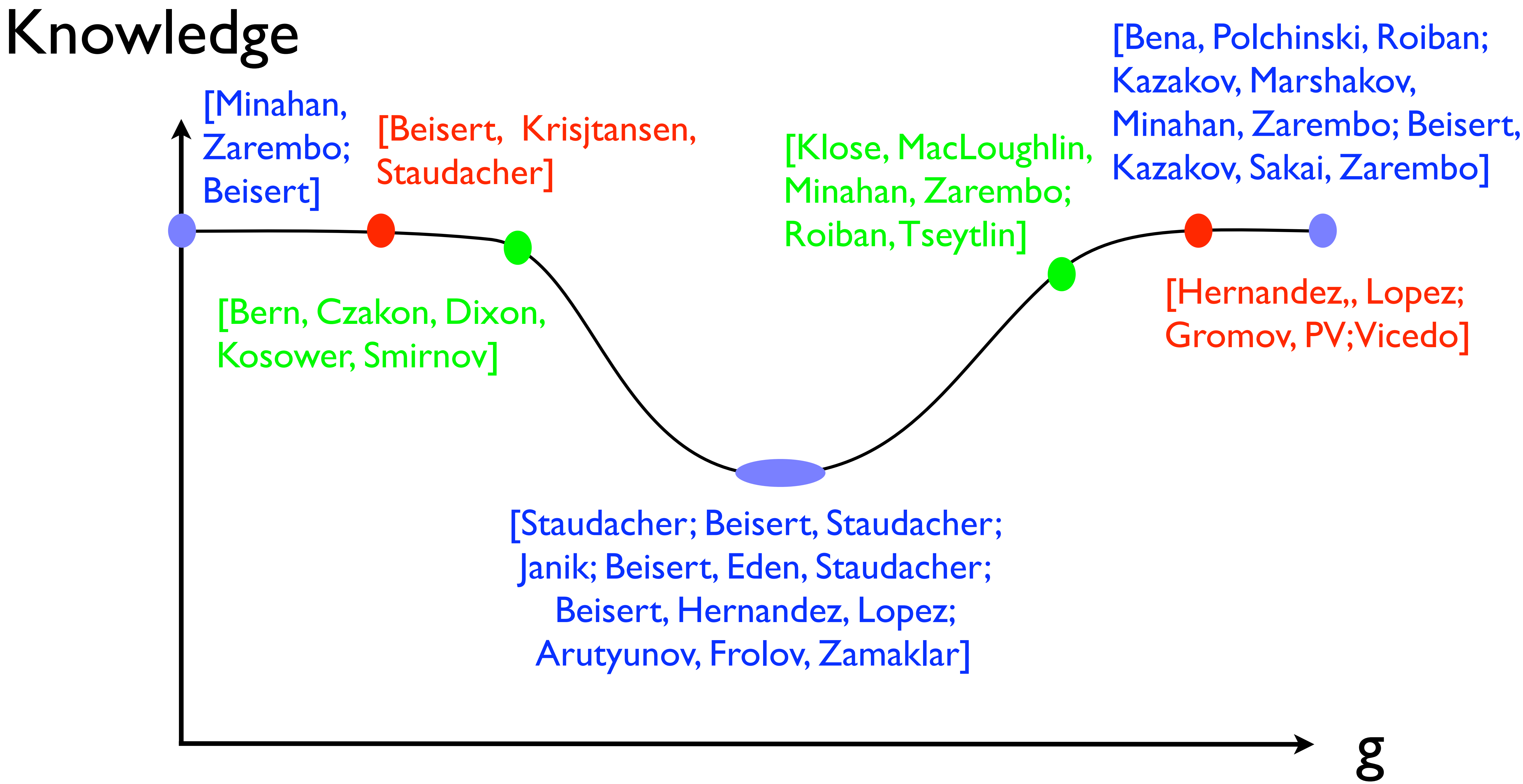
g



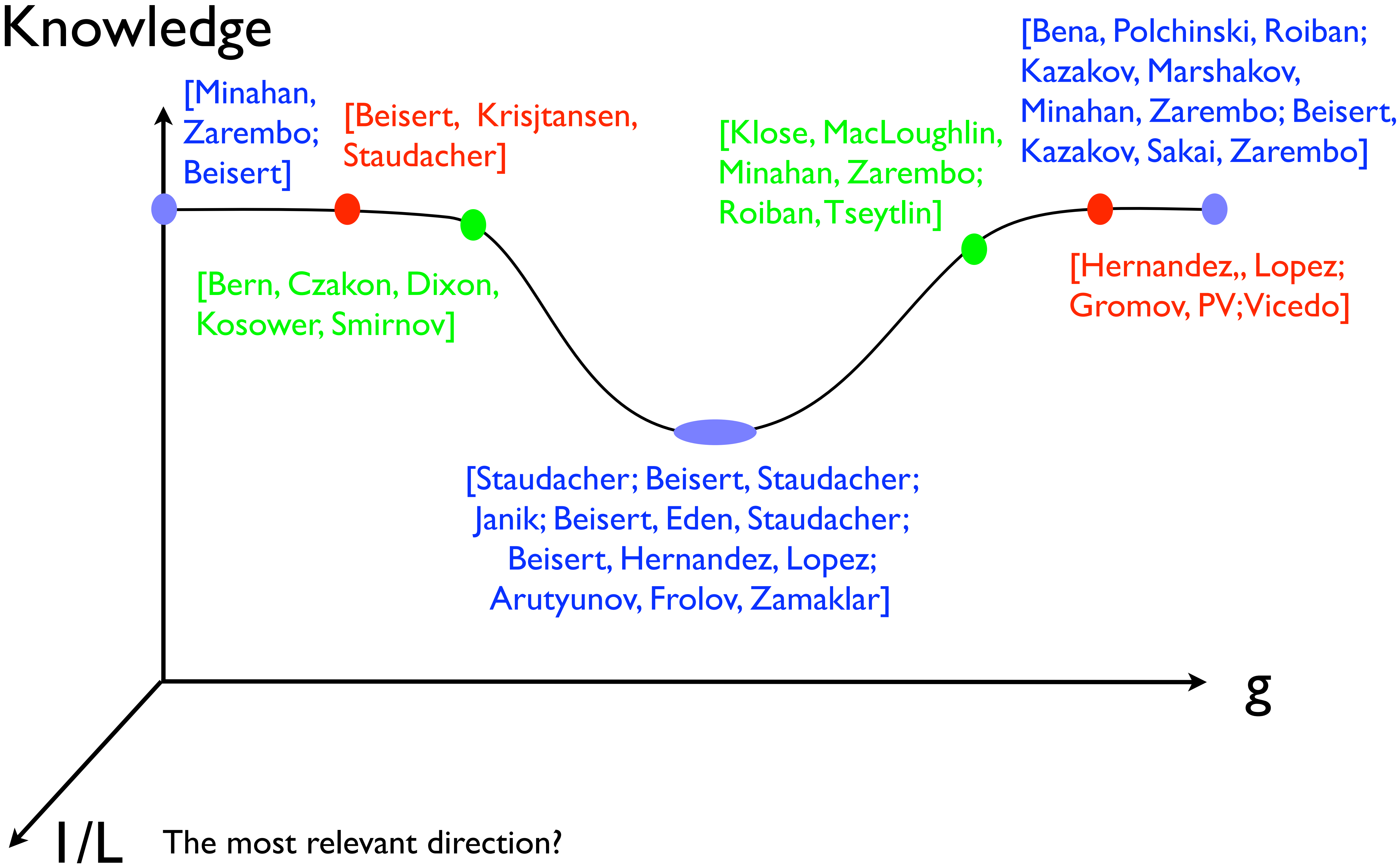
1/g



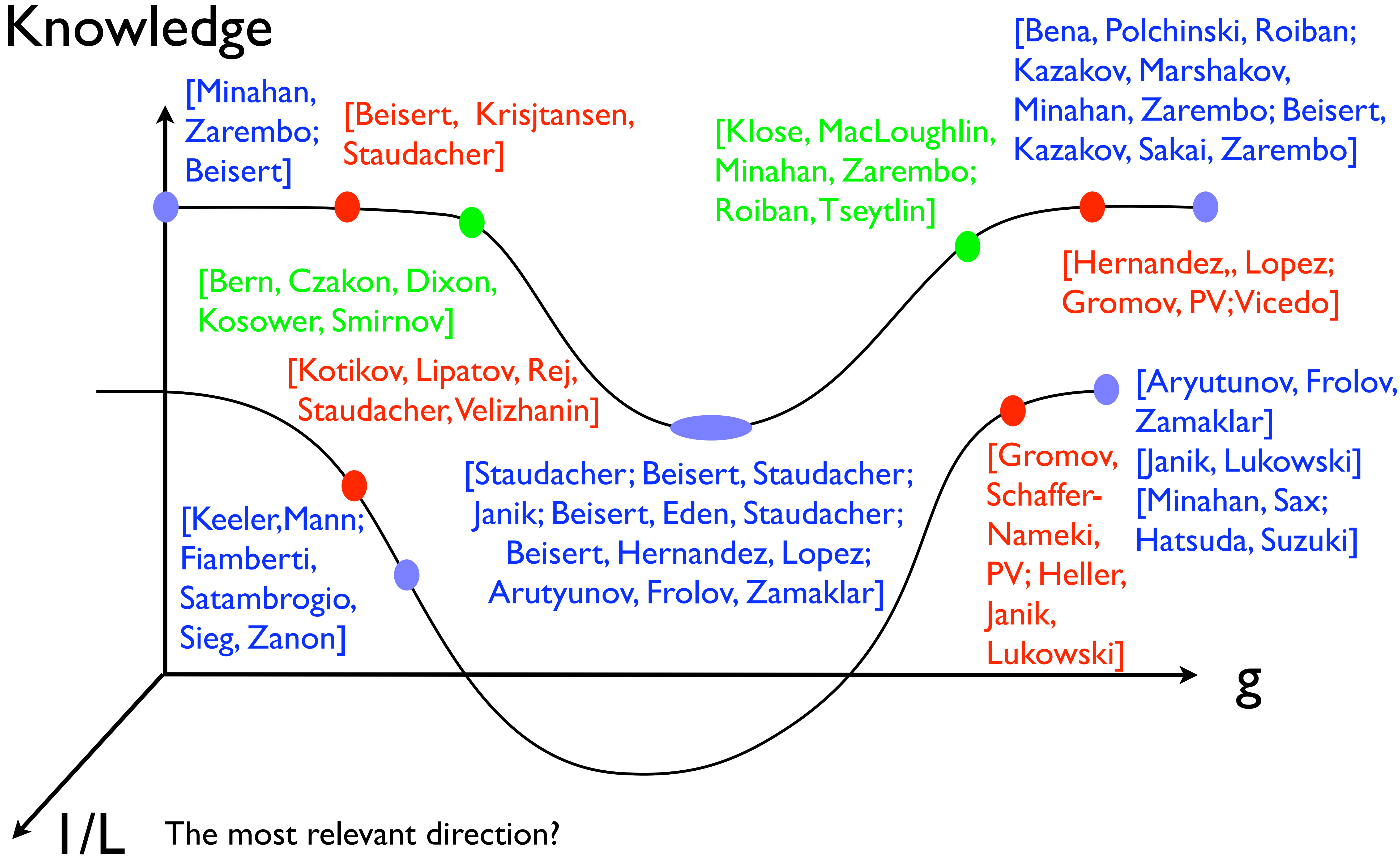
State of the art



State of the art



State of the art



State of the art