GENERALIZATION OF EVERITT MANIFOLDS

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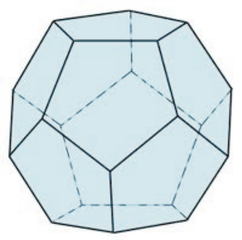
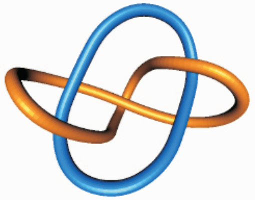
Spherical and hyperbolic dodecahedral spaces

The spherical ($\frac{2\pi}{3}$) and hyperbolic ($\frac{2\pi}{5}$) dodecahedra.



Seifert H., Weber C (1933) the 3 – manifolds (spherical and hyperbolic) by a pairwise identification of faces of the regular dodecahedra.

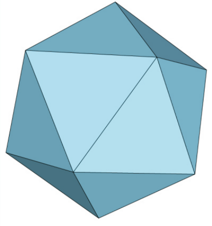
The Seifert–Weber dodecahedral hyperbolic manifold is the 5 – fold cyclic branched covering of the 3 – sphere branched over the Whitehead link



Best L.(1971), Lorimer P. (1992): Other manifolds arising from the $2\pi/5$–dodecahedron.

Hyperbolic icosahedral manifolds

Best L. (1971): Three hyperbolic manifolds, having the $\frac{2\pi}{3}$–icosahedron as the fundamental polyhedron.



Richardson J., Rubinstein J. (1982): the complete list of orientable 3–dimensional closed hyperbolic manifolds whose fundamental polyhedron is the $2\pi/5$–dodecahedron or the $2\pi/3$–icosahedron.

Hyperbolic icosahedral manifolds

Everitt B. (2004): the complete list of manifolds obtained from regular polyhedra.

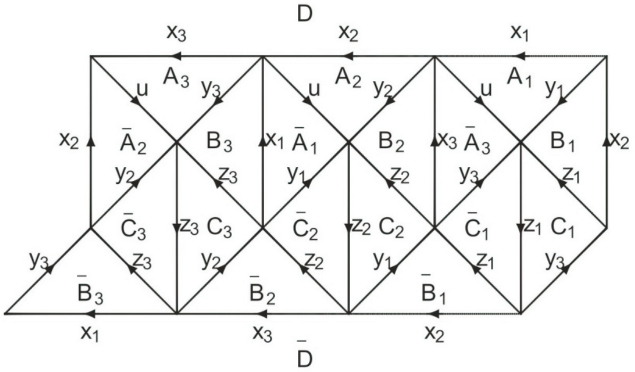
$M_{15},\ldots,M_{22}$ $-\frac{2\pi}{5}-$dodecahedron , $M_{23},\ldots,M_{28}$ $-$ $\frac{2\pi}{3}-$icosahedron.

M_{24} and M_{25} have symmetries of order 3.

Cavicchioli A, Spaggiari F, Telloni A (2009) the hyperbolic $3-$ manifolds, arising from an icosahedron with angle $2\pi/3$. M_{24} is the $3-$fold cyclic covering of the lens space $L(3,1)$ branched over some $2-$ component link.

The hyperbolic $2\pi/3$–icosahedron and the manifold $M_3(3,1)$

The simplicial complex P_3 has 20 faces, 30 edges and 12 vertices.

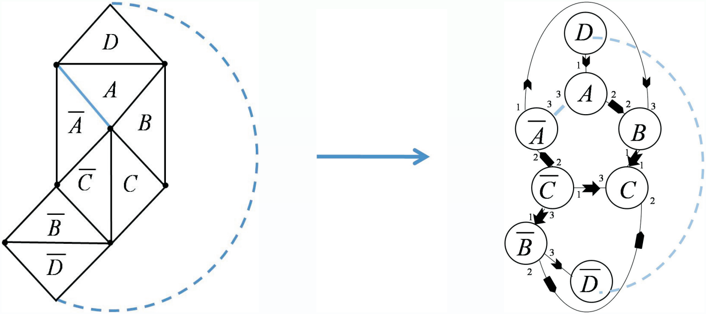


$$\mathbf{A_i} \to \mathbf{\bar{A}_i}, \quad \mathbf{B_i} \to \mathbf{\bar{B}_i}, \quad \mathbf{C_i} \to \mathbf{\bar{C}_i}, \quad \mathbf{D} \to \mathbf{\bar{D}}, \quad i = 1, 2, 3$$

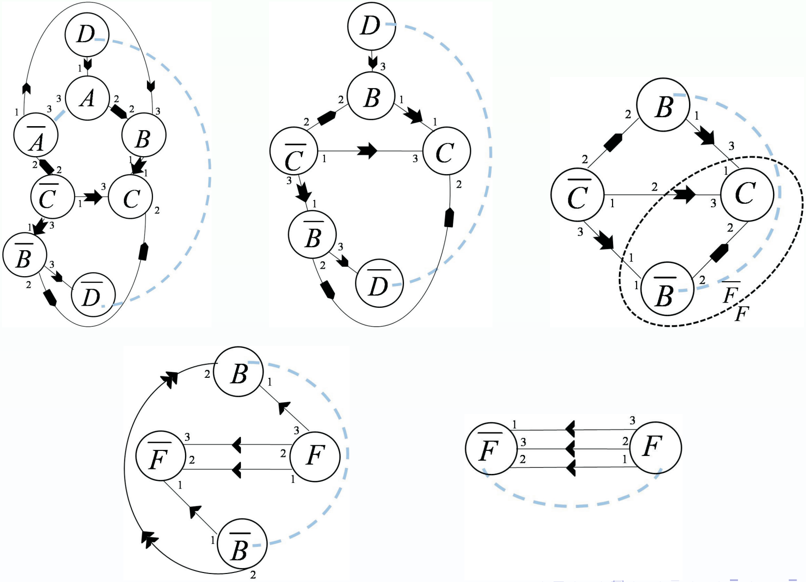
The manifold $M_3(3, 1)$

Theorem [Cavicchioli A., Spaggiari F., Telloni A.]

The manifold $M_3(3, 1)$ is an 3-fold cyclic covering of the lens space $L(3, 1)$ branched over a two-component link.

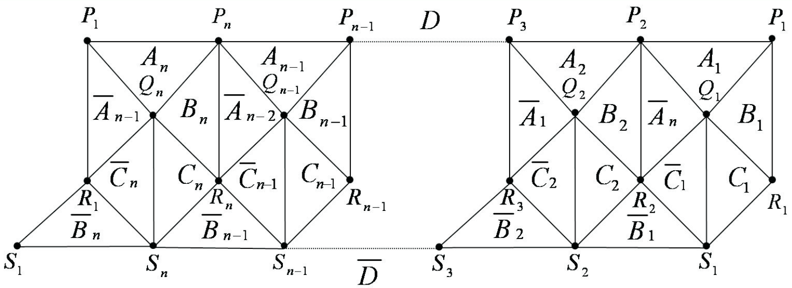


Proof [Kozlovskaya–Vesnin]



The family of manifolds $M_n(3,1)$

Consider the the simplicial complex P_n, $n \geqslant 3$. P_n has $6n+2$ faces, $10n$ edges, $4n$ vertices. In particular, P_3 is the icosahedron.



$\mathbf{A_i} \to \mathbf{\bar{A}_i}[P_iP_{i+1}Q_i \to R_{i+2}P_{i+2}Q_{i+1}]$, $\mathbf{B_i} \to \mathbf{\bar{B}_i}[R_iP_iQ_i \to S_iS_{i+1}R_{i+1}]$,

$\mathbf{C_i} \to \mathbf{\bar{C}_i}[Q_iR_iS_i \to S_iQ_iR_{i+1}]$, $\mathbf{D} \to \mathbf{\bar{D}}[P_1P_2\dots P_{n-1}P_n \to S_3S_4\dots S_1S_2]$

$i = 1, \dots, n$; subscripts mod n.

The family of manifolds $M_n(3,1)$

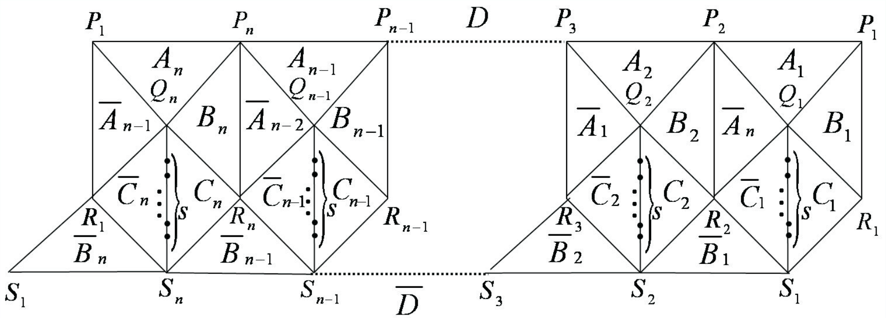
Theorem 1. For each $n \geqslant 1$ the quotient space $M_n(3,1) = \mathcal{P}_n/\varphi_n(3,1)$ is a manifold.

Theorem 2. The fundamental group of $M_n(3,1)$, $n \geqslant 1$, has the following geometric presentation:

$$\pi_1(M_n(3,1)) = \langle a_1, \ldots, a_n; b_1, \ldots, b_n; c_1, \ldots, c_n; d \quad | \quad a_1 a_2 \ldots a_n = 1, \\ a_i b_{i+2} d^{-1} = 1, \qquad a_i c_{i+1}^{-1} b_i^{-1} = 1, \qquad c_i^2 b_i^{-1} = 1, \qquad i = 1, \ldots, n \rangle.$$

Theorem 3. For each $n \geqslant 1$, the manifold $M_n(3,1)$ is an n-fold strongly-cyclic branched covering of the lens space $L_{3,1}$, branched over a 2-component link.

Consider the the simplicial complex $P_n(p)$, $n \geqslant 3$, $p = s + 3$. $P_n(p)$ has $6n + 2$ faces, $(10 + s)n$ edges, $(4 + s)n$ vertices.



We add vertices $T_i^1, T_i^2, \dots T_i^s$ on edges S_iQ_i, $i = 1, \dots, n$, where additional vertices are numerated in the direction from S_i to Q_i.

Define the pairwise identification $\varphi_n(p,q)$ of faces of $P_n(p)$ according to the following rule:

a_i: $\mathbf{A_i} \to \bar{\mathbf{A}}_\mathbf{i}$ $[P_i P_{i+1} Q_i \to R_{i+2} P_{i+2} Q_{i+1}]$,

b_i: $\mathbf{B_i} \to \bar{\mathbf{B}}_\mathbf{i}$ $[R_i P_i Q_i \to S_i S_{i+1} R_{i+1}]$,

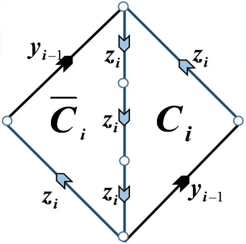
$$
c_i\colon \mathbf{C_i} \to \bar{\mathbf{C}}_\mathbf{i} \begin{array}{ll}
[Q_i R_i S_i T_i^1 \dots T_i^s \to T_i^s Q_i R_{i+1} S_i T_i^1 \dots T_i^{s-1}], & \text{if } q = 1; \\
[Q_i R_i S_i T_i^1 \dots T_i^s \to T_i^{s-1} T_i^s Q_i R_{i+1} S_i T_i^1 \dots T_i^{s-2}], & \text{if } q = 2;
\end{array}
$$

....

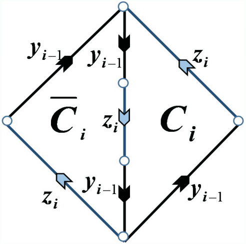
$$
\begin{array}{ll}
[Q_i R_i S_i T_i^1 \dots T_i^s \to T_i^1 \dots T_i^s Q_i R_{i+1} S_i], & \text{if } q = p-3; \\
[Q_i R_i S_i T_i^1 \dots T_i^s \to S_i T_i^1 \dots T_i^s Q_i R_{i+1}], & \text{if } q = p-2; \\
[Q_i R_i S_i T_i^1 \dots T_i^s \to R_{i+1} S_i T_i^1 \dots T_i^s Q_i], & \text{if } q = p-1;
\end{array}
$$

d : $\mathbf{D} \to \bar{\mathbf{D}}$ $[P_1 P_2 \dots P_{n-1} P_n \to S_3 S_4 \dots S_1 S_2]$.

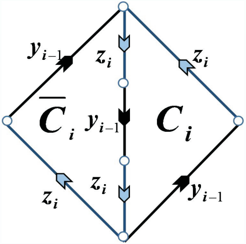
Manifolds $M_3(5, q)$, $q = 1, 2, 3, 4$



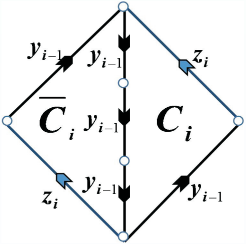
$M_3(5, 1)$



$M_3(5, 2)$



$M_3(5, 3)$

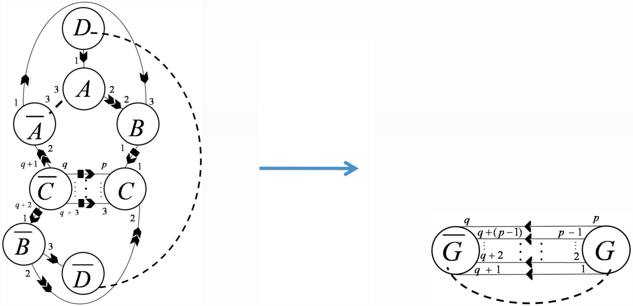


$M_3(5, 4)$

The family of manifolds $M_n(p, q)$

Теорема 5

The manifold $M_n(p, q)$ ($n \geqslant 1$ и $p \geqslant 3$, $0 < q < p$, $(p, q) = 1$), is an n–fold cyclic covering of the lens space $L(p, q)$ branched over a two-component link.



Definitions:

Let p and q - be coprime integers ($p \geq 3$) and consider S^3 as the unit sphere in $\mathbb{C}^2$. Set group action $\mathbb{Z}/p\mathbb{Z}$:

$$\begin{cases} z \longrightarrow e^{\frac{2\pi i}{p}} z \\ \omega \longrightarrow e^{\frac{2\pi iq}{p}} \omega \end{cases}$$

The resulting quotient space of S^3 by group action is called the lens space $L(p, q)$

Definition:

A lens space $L(p,q)$ is the $3-$ manifold obtained by gluing the boundaries of two solid tori together such that the meridian of the first goes to a (p,q)-curve on the second, where a (p,q)-curve wraps around the longitude p times and around the meridian q times.
The lens space is that of a space resulting from gluing two solid tori together by a homeomorphism of their boundaries.

Definitions:

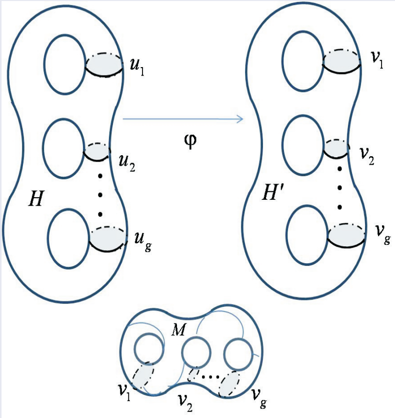
Any closed orientable 3 – manifold M can be presented as the union of two handlebodies with common boundary. Such a presentation is called the Heegaard splitting of M.
Assume that $M = H \cup H^{'}$ is a genus - g Heegaard splitting of a manifold M and $u = u_1, \ldots, u_g$ and $v = v_1, \ldots, v_g$ are meridian system for H, $H^{'}$, respectively. Then the triple (F, u, v) is called a Heegaard diagram of M.

Основные определения: диаграмма Хегора

Диаграмма Хегора

Любое замкнутое ориентируемое 3-мерное многообразие M можно представить в виде объединения двух полных кренделей с общим краем. Такое представление называется разбиением Хегора многообразия M.

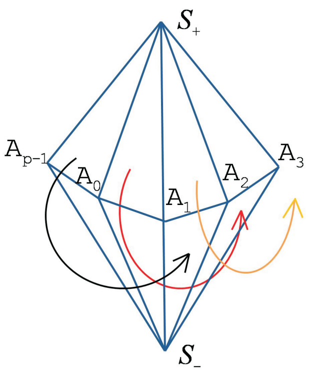
Пусть $M = H \cup H'$-разбиение Хегора рода g многообразия M и $u = u_1, \dots, u_g, v = v_1, \dots, v_g$ - системы меридианов H, H'. Тогда тройка (F, u, v) называется диаграммой Хегора многообразия M.



Основные определения: линзовое пространство

Линзовое пространство

Пусть $p \geqslant 3$, $0 < q < p$ и $(p, q) = 1$. Рассмотрим p – угольную бипирамиду. Обозначим через $A_0, A_1, \ldots, A_{p-1}$ вершины p – угольника, через S_+ и S_- - вершины конусов. Для каждого i склеим грань $A_i S_+ A_{i+1}$ с гранью $A_{i+q} S_- A_{i+q+1}$ (индексы берутся по модулю p). Получившееся пространство - линзовое пространство $L_{p,q}$.



manifold	volume	homology groups
$M_3(3,1)$	4.686034273803...	$\mathbb{Z}_9$
$M_4(3,1)$	9.702341514665...	$\mathbb{Z}_3 \oplus \mathbb{Z}_{12}$
$M_5(3,1)$	14.319926985892...	$\mathbb{Z}_5 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{15}$
$M_6(3,1)$	18.649157163789...	$\mathbb{Z}_3 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{18}$

manifold	volume	homology groups
$\overline{M}_3(3,1)$	4.686034273803...	$\mathbb{Z}_2 \oplus \mathbb{Z}_{18}$
$\overline{M}_4(3,1)$	3.970289623891...	$\mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_6$
$\overline{M}_5(3,1)$	14.319926985892...	$\mathbb{Z}_5 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_5$
$\overline{M}_6(3,1)$	14.004768920617...	$\mathbb{Z}_8 \oplus \mathbb{Z}_{72}$

program "Recognizer"