

BMS Working seminar

Carrollian geometry: " $c \rightarrow 0$ " will hypersurfaces in Lorentzian mfd's

cf. Galilean geometry: " $c \rightarrow \infty$ " will reductions of Lorentzian mfd's



Examples

\mathbb{M}^{D+1} (D+1)-dim'l Minkowski spacetime (x^+, x^-, x^a) $[P_a, P_b] = \text{Sub H}$
 $x^+ = \text{const}$ Carrollian spacetime symmetric affine space of Carroll gr.

$2x^+x^- = \sum_a (x^a)^2$ Carrollian lightcone non-reductive homog space of $SO(1, D)$

$\mathbb{Q}_{\text{AdS}} \subset \mathbb{R}^{D,2}$ $-t_1^2 - t_2^2 + x_1^2 + \dots + x_D^2 = -R^2$
 $x_i = t_i$ (Carrollian AdS) symmetric homog space of \square

$\mathbb{Q}_{\text{AS}} \subset \mathbb{R}^{D+1,1}$ $-t_1^2 + x_1^2 + \dots + x_{D+1}^2 = R^2$
 $x_i = t_i$ (Carrollian dS) symmetric homog space of euclidean LA

$i: \mathcal{N} \hookrightarrow (M, g)$
↑ will hypers. ↑ Lorentzian $i^*g = h$ degenerate, co-rank 1
v nowhere-vanishing vector field $h(v, -) = 0$

Def. $\xi \in \mathfrak{X}(M)$ is **Carrollian Killing** if $L_\xi h = 0$ $[\xi, v] = 0$
 is **conformal Carrollian Killing** (at level $N \in \mathbb{N}$) if $L_\xi h = \lambda h$ $[\xi, v] = -\frac{\lambda}{N} v$ $\exists \lambda \in C^\infty(M)$.

cf. (M, g) Lorentzian $\xi \in \mathfrak{X}(M)$ is Killing if $L_\xi g = 0$
 is conformal Killing if $L_\xi g = \lambda g$ $\exists \lambda \in C^\infty(M)$

$\underline{\text{carr}}(M, h, v) = \{ \xi \in \mathfrak{X}(M) \mid \xi \text{ Carrollian Killing} \}$
 $\underline{\text{ccarr}}_N(M, h, v) = \{ \xi \in \mathfrak{X}(M) \mid \xi \text{ conf. carr. Killing at level } N \}$

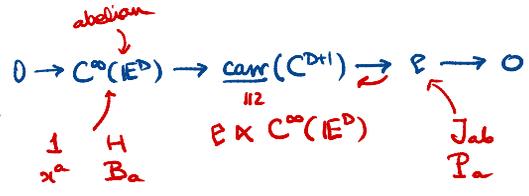
Both $\underline{\text{carr}}(M, h, v)$ & $\underline{\text{ccarr}}_N(M, h, v)$ are Lie subalgebras of $\mathfrak{X}(M)$.
 Unlike the Killing & Conf. Killing Lie algebras of (M, g) , they need not be finite-dimensional.

Examples

Carrollian Killing vector fields

$$C^{D+1} \quad \xi = T(x) \frac{\partial}{\partial t} + \zeta(x)$$

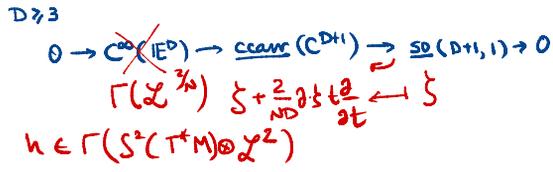
smooth fn on \mathbb{E}^D Killing vector of \mathbb{E}^D



Conformal carrollian vFs

$$C^{D+1} \quad \xi = \left(T(x) + \frac{2t}{ND} \frac{\partial \zeta^a}{\partial x^a} \right) \frac{\partial}{\partial t} + \zeta(x)$$

C^∞ fn on \mathbb{E}^D CKV on \mathbb{E}^D



- $D=2 \quad \text{CKV}(\mathbb{E}^2) \cong \mathcal{O}(C)$
- $D=1 \quad \text{CKV}(\mathbb{E}^1) \cong C^\infty(\mathbb{R})$

For the carrollian light cone $\mathcal{E}_+^{D+1} \subset M^{D+2}$, $\text{carr}(\mathcal{E}_+^{D+1}) \cong \mathfrak{so}(D+1, 1)$ at least for $D \geq 2$

but $\text{ccarr}(\mathcal{E}_+^{D+1}) \cong \Gamma(\mathcal{L}^{2/N}) \times \mathfrak{so}(D+1, 1)$

By contrast, the symmetries of the galilean structures are reminiscent of KM algebras.

$$0 \rightarrow C^\infty(\mathbb{R}_t, \mathbb{E}^D) \rightarrow \text{gal}^{D+1} \rightarrow \mathbb{R} \rightarrow 0$$

$H = \frac{\partial}{\partial t} \quad \leftarrow 1$

and

$$0 \rightarrow C^\infty(\mathbb{R}_t, \mathbb{E}^D) \rightarrow \text{cgal}^{D+1} \rightarrow C^\infty(\mathbb{R}_t) \rightarrow 0$$

The other \square -homogeneous spacetime: "AdS C^{D+1} " (h, v) coordinates (t, \vec{x})

"wronskian"

$$v = \frac{1}{\cosh(\tau)} \frac{\partial}{\partial t} \quad h = \underbrace{dx^2 + \sinh^2(\tau) g_{SO-1}}_{H^D}$$

Jac, P_a

$$\text{carr}(AdSC_{D+1}) \cong \mathfrak{so}(D, 1) \times C^\infty(H^D)$$

$1 \mapsto H$
 $x^a \mapsto B_a$

$$\text{ccarr}(AdSC_{D+1}) \cong \text{CKV}(H^D) \times \Gamma(\mathcal{L}^{2/N})$$

$\text{CKV}(H^{D \geq 3}) \cong \mathfrak{so}(D+1, 1)$ (conf. flatness)

$\text{CKV}(H^2) \cong \mathcal{O}(H)$

\hookrightarrow upper half \mathbb{C} -plane

$\text{CKV}(H^1) \cong C^\infty(\mathbb{R})$