Geometric M-theory backgrounds

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Motivation

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Aim: To classify/characterise supergravity backgrounds with classical geometric interpretation.

References for this talk

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- purely gravitational backgrounds of $D=11$ supergravity [hep-th/9904124]

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- parallelisable backgrounds in ten-dimensional string theory [hep-th/0305079, FO–Kawano–Yamaguchi hep-th/0308141]

Supergravity backgrounds

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- (M, g, S, Φ)

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$\exists$ parallel spinors does not imply Ricci-flatness

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Possible G follow from orbit decomposition of the spinor representation under $\mathrm{Spin}(1,n)$

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[Bryant math.DG/0004073]

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[Sorkin, Gross–Perry (1983); Han–Koh (1985)]

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d	$H \subset \mathrm{SO}(d)$	ν
10	$\mathrm{SU}(5)$	$\frac{1}{16}$
10	$\mathrm{SU}(2) \times \mathrm{SU}(3)$	$\frac{1}{8}$
8	$\mathrm{Spin}(7)$	$\frac{1}{16}$
8	$\mathrm{SU}(4)$	$\frac{1}{8}$
8	$\mathrm{Sp}(2)$	$\frac{3}{16}$

d	$H \subset \mathrm{SO}(d)$	ν
8	$\mathrm{Sp}(1) \times \mathrm{Sp}(1)$	$\frac{1}{4}$
7	G_2	$\frac{1}{8}$
6	$\mathrm{SU}(3)$	$\frac{1}{4}$
4	$\mathrm{SU}(2) \cong \mathrm{Sp}(1)$	$\frac{1}{2}$
0	$\{1\}$	1

[Wang (1989)]

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[Hull (1984)]

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$H \subset \mathrm{SO}(1,10)$	ν
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$\left(\mathrm{SU}(4) \ltimes \mathbb{R}^8\right) \times \mathbb{R}$	$\frac{1}{16}$
$\left(\mathrm{Sp}(2) \ltimes \mathbb{R}^8\right) \times \mathbb{R}$	$\frac{3}{32}$
$\left(\mathrm{Sp}(1) \ltimes \mathbb{R}^4\right) \times \left(\mathrm{Sp}(1) \ltimes \mathbb{R}^4\right) \times \mathbb{R}$	$\frac{1}{8}$
$\left(G_2 \ltimes \mathbb{R}^7\right) \times \mathbb{R}^2$	$\frac{1}{16}$
$\left(\mathrm{SU}(3) \ltimes \mathbb{R}^6\right) \times \mathbb{R}^3$	$\frac{1}{8}$
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[Leistner math.DG/0309274]

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- $\mathrm{AdS}_4 \times S^7$ and $\mathrm{AdS}_7 \times S^4$ in M-theory

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- $\mathrm{AdS}_5 \times S^5$ in IIB

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$$(M^4, g) \times (N^7, h) \qquad \text{and} \qquad F = f \, \mathrm{dvol}_g$$

Field equations:

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- (M, g) and (N, h) Einstein with scalar curvatures $\pm\frac{4}{3}f^2$ and $\mp\frac{7}{6}f^2$, respectively.

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with $\lambda = \pm\frac{f}{6}$ and $\pm\frac{f}{12}$, respectively.

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Task: To characterise the lorentzian and riemannian Einstein n-manifolds admitting geometric Killing spinors

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$\exists$ Killing spinors does not imply Einstein

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Killing spinors in (M, g) (with $\lambda = \pm\frac{1}{2}$)	$\leftrightarrow$	parallel spinors in the metric cone $(\bar{M}, \bar{g})$

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$$\bar{M} = M \times \mathbb{R}^+ \qquad \text{and} \qquad \bar{g} = dr^2 + r^2 g$$

[Bär (1993)]

If (M, g) is complete

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[Baum (2000), Baum–Leitner `math.DG/0305063`]

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For $d = 4, 5$ the most general such metric is known.
[hep-th/9904124]

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[Leitner math.DG/0302024]

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Others thus far lack a clear physical interpretation.

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This turns M into a Lie group with bi-invariant metric.

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[Cartan–Schouten (1926), Wolf (1970)]

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- compact simple Lie group with (a multiple of) the Killing form
- S^7 with the nearly parallel G_2 structure

[Cartan–Schouten (1926), Wolf (1970)]

Only the first two have $dH = 0$.

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[Cahen–Parker (1977)]

They are all Lie groups with bi-invariant metrics, whence $dH = 0$.

Parallelisable building blocks

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All ten-dimensional lorentzian parallelisable spacetimes can be built out of:

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Space	Torsion
AdS_3	$dH = 0 \quad \lvert H\rvert^2 < 0$
$\mathbb{R}^{1,n},\ n \geq 0$	$H = 0$
$\mathbb{R}^n,\ n \geq 1$	$H = 0$
S^3	$dH = 0 \quad \lvert H\rvert^2 > 0$
S^7	$dH \neq 0 \quad \lvert H\rvert^2 > 0$
$\mathrm{SU}(3)$	$dH = 0 \quad \lvert H\rvert^2 > 0$
$\mathrm{CW}_{2n+2}(J)$	$dH = 0 \quad \lvert H\rvert^2 = 0$

Ten-dimensional parallelisable geometries

Ten-dimensional parallelisable geometries

$\mathrm{AdS}_3 \times S^7$

$\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4$

$\mathbb{R}^{1,0} \times S^3 \times S^3 \times S^3$

$\mathbb{R}^{1,2} \times S^7$

$\mathbb{R}^{1,6} \times S^3$

$\mathrm{CW}_{10}(J)$

$\mathrm{CW}_6(J) \times S^3 \times \mathbb{R}$

$\mathrm{CW}_4(J) \times S^3 \times S^3$

$\mathrm{CW}_4(J) \times \mathbb{R}^6$

$\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R}$

$\mathrm{AdS}_3 \times \mathbb{R}^7$

$\mathbb{R}^{1,1} \times \mathrm{SU}(3)$

$\mathbb{R}^{1,3} \times S^3 \times S^3$

$\mathbb{R}^{1,9}$

$\mathrm{CW}_8(J) \times \mathbb{R}^2$

$\mathrm{CW}_6(J) \times \mathbb{R}^4$

$\mathrm{CW}_4(J) \times S^3 \times \mathbb{R}^3$

Equations of motion

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The case of linear dilaton was analysed by Kawano and Yamaguchi.

Supersymmetry

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$$(d\phi + \frac{1}{2}H)\varepsilon = 0$$

For non-dilatonic backgrounds, this equation has solutions if and only if $|H|^2 = 0$; which restricts the possible geometries.

Spacetime	Supersymmetry
$\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R}$	16
$\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4$	16
$\mathrm{CW}_{10}(J)$	16,18(A),20,22(A),24(B),28(B)
$\mathrm{CW}_8(J) \times \mathbb{R}^2$	16,20
$\mathrm{CW}_6(J) \times \mathbb{R}^4$	16,24
$\mathrm{CW}_4(J) \times \mathbb{R}^6$	16
$\mathbb{R}^{1,9}$	32

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$$\begin{array}{ll}
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\mathbb{R}^{1,1} \times \mathrm{SU}(3) & \mathrm{CW}_6(J) \times \mathbb{R}^4 \\
\mathbb{R}^{1,3} \times S^3 \times S^3 & \mathrm{CW}_6(J) \times S^3 \times \mathbb{R} \\
\mathbb{R}^{1,6} \times S^3 & \mathrm{CW}_4(J) \times S^3 \times \mathbb{R}^3 \\
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[Kawano–Yamaguchi hep-th/0306038]

All these backgrounds are exact string backgrounds

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$\mathbb{R}^{1,1} \times \mathrm{SU}(3)$	$\mathrm{CW}_6(J) \times \mathbb{R}^4$
$\mathbb{R}^{1,3} \times S^3 \times S^3$	$\mathrm{CW}_6(J) \times S^3 \times \mathbb{R}$
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All these backgrounds are exact string backgrounds: coupling a WZW model for (M, g, H) to a Liouville field theory for ϕ.

Parallelisable heterotic backgrounds

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- action

$$\int_M e^{-2\phi} \left(R + 4|d\phi|^2 - \tfrac{1}{2}|H|^2 - \tfrac{N}{2}|F|^2 \right) \mathrm{dvol}_g$$

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- $\delta^{D,A}(e^{-2\phi} F) = 0$

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Parallelisable backgrounds with $F = 0$

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Linear dilaton: all backgrounds are $\frac{1}{2}$-BPS, whereas for constant dilaton one has:

Spacetime	Supersymmetry
$\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R}$	8
$\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4$	8
$\mathrm{CW}_{10}(J)$	8,10,12,14
$\mathrm{CW}_8(J) \times \mathbb{R}^2$	8,10
$\mathrm{CW}_6(J) \times \mathbb{R}^4$	8,12
$\mathrm{CW}_4(J) \times \mathbb{R}^6$	8
$\mathbb{R}^{1,9}$	16

Parallelisable backgrounds with $F \neq 0$

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$$
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$$

In all cases F is null: $F = du \wedge \theta$.

Some geometric limits

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- $\mathrm{CW}_{2n}(J) \rightsquigarrow \mathrm{CW}_{2n-2}(J') \times \mathbb{R}^2$

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- $\mathrm{CW}_{2n}(J) \rightsquigarrow \mathrm{CW}_{2n-2}(J') \times \mathbb{R}^2$ by allowing J to degenerate
- $\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4 \rightsquigarrow \mathrm{CW}_6(J) \times \mathbb{R}^4$

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- $\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4 \rightsquigarrow \mathrm{CW}_6(J) \times \mathbb{R}^4$, and $\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R} \rightsquigarrow \mathrm{CW}_8(J) \times \mathbb{R}^2$

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- $\mathrm{AdS}_3 \times S^3 \times \mathbb{R}^4 \rightsquigarrow \mathrm{CW}_6(J) \times \mathbb{R}^4$, and $\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R} \rightsquigarrow \mathrm{CW}_8(J) \times \mathbb{R}^2$ by taking a Penrose limit

Thank you.