# What is supersymmetry?

GAMES, 26 February 2016

# Space

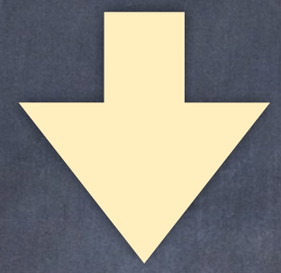
$(x, y, z)$ 3 dimensions

$$\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

euclidean distance

# ... and time

$t - t'$ time intervals



Dynamics!

# The universe according to Newton

events $E \longleftarrow A^3$

space of simultaneous events

$\downarrow$

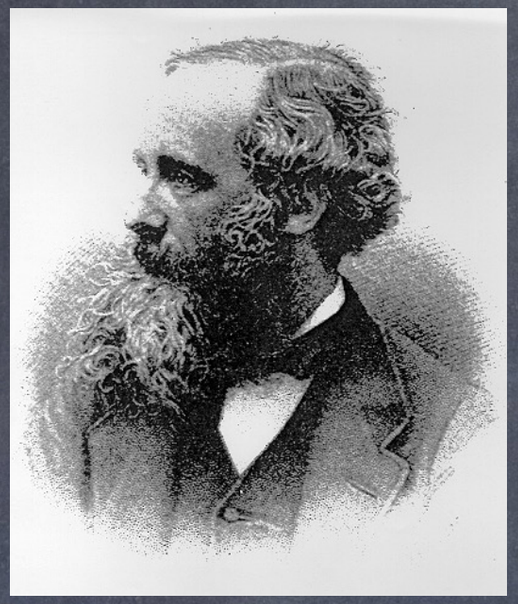
$A^1$

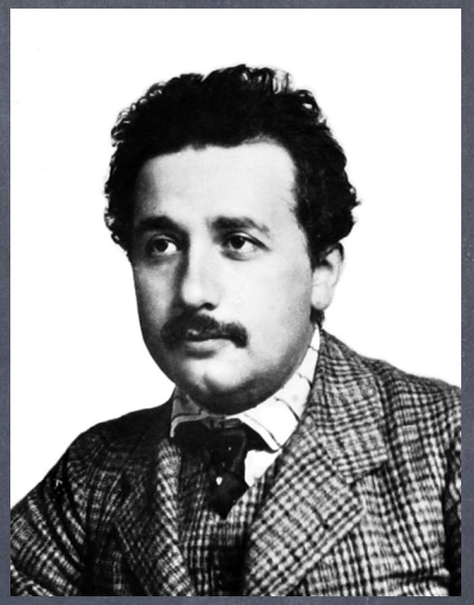
time

# Relativity

- invariant notions:
  - time difference between events
  - euclidean distance between simultaneous events
- automorphisms: galilean group:
  - translations in space and time
  - rotations in space
  - galilean boosts $(t, \boldsymbol{x}) \mapsto (t, \boldsymbol{x} + \boldsymbol{v}t)$

# Maxwell & Einstein

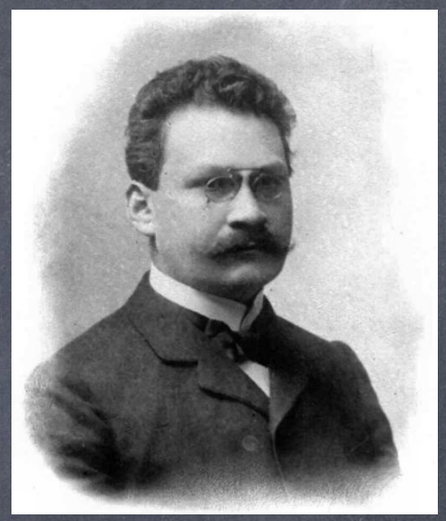




Newton's equations are galilean-invariant, but Maxwell's are <u>not</u>.

Einstein took Maxwell seriously and arrived at his <u>special theory of relativity</u>.

# Minkowski



"The views of space and time that I wish to lay before you have sprung from the soil of experimental physics and therein lies their strength."

"Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of both will retain an independent reality."

# Spacetime

$A^4$ spacetime events

invariant notion: proper distance

$$(x-x')^2+(y-y')^2+(z-z')^2-(t-t')^2$$

modelled on a vector space $(V,\eta)$

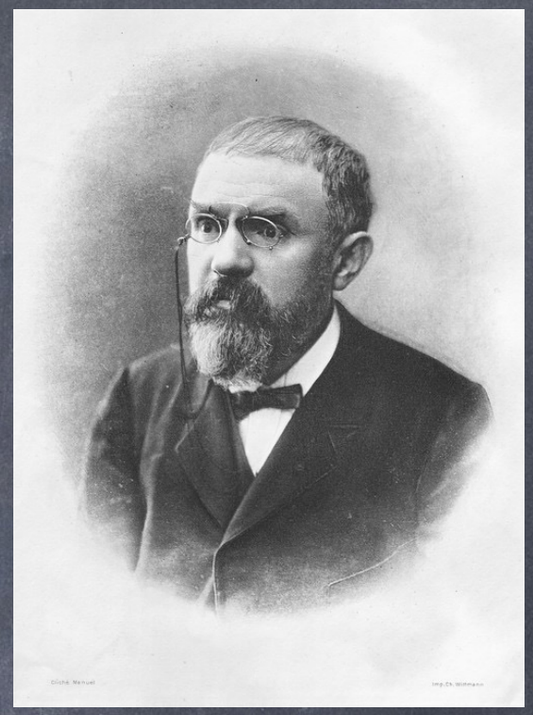
automorphisms: Poincaré group

# Poincaré algebra

$$\mathfrak{p} = \mathfrak{so}(V) \oplus V$$

translations

Lorentz transformations
(skewsymmetric endomorphisms)



$$[A, B] = AB - BA$$

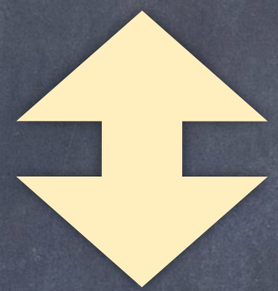
$$[A, v] = Av$$

$$A, B \in \mathfrak{so}(V)$$

$$v \in V$$

# Particles

Elementary particles

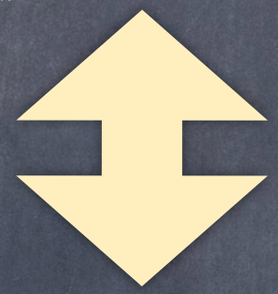


unitary irreps of $\mathfrak{p}$

(indexed by "mass" and "spin")

# Fields

unitary irreps of $\mathfrak{p}$



sections of homogeneous vector bundles on Minkowski spacetime, subject to field equations, such as...

$$\Box\varphi = -m^2\varphi \qquad \text{(Klein-Gordon)}$$

$$i\not{\partial}\psi = m\psi \qquad \text{(Dirac)}$$

# Supergroup?

- 1960s: is there a "supergroup" whose irreps contain Poincaré irreps of different masses and spins?

- 1967: No. (Coleman-Mandula)
- 1975: Yes, sort of. (Haag-Lopuszanski-Sohnius)

Requires redefining the very notion of symmetry!

# Lie superalgebras

$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ 2-graded vector space

$[-,-] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ (super skewsymmetric)

$$[X,Y] = -(-1)^{|X||Y|}[Y,X]$$

$\forall X \in \mathfrak{g}_0 \cup \mathfrak{g}_1 \quad [X,-]$ is a superderivation.

$$[X,[Y,Z]] = [[X,Y],Z] + (-1)^{|X||Y|}[Y,[X,Z]]$$

# Clifford algebra

$(V, \eta)$ lorentzian vector space

$C\ell(V)$ associative unital algebra

$$v^2 = -\eta(v,v)\mathbf{1}$$

$$C\ell(V) \cong \mathrm{End}(S) \qquad (S, \Omega)$$

$$\Omega(v \cdot s_1, s_2) = -\Omega(s_1, v \cdot s_2)$$

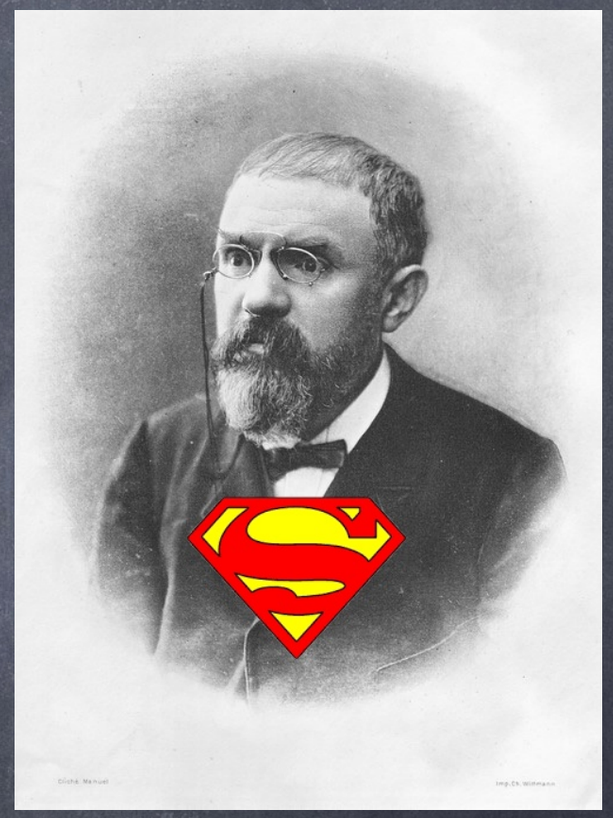
# Poincaré superalgebra

$$\mathfrak{s} = \underbrace{\mathfrak{so}(V) \oplus V}_{\mathfrak{s}_0} \oplus \underbrace{S}_{\mathfrak{s}_1}$$

$$\forall s \in S \quad [s, s] \in V$$

$$\eta([s, s], v) = \Omega(s, v \cdot s)$$

$[A, s]$ given by $\mathfrak{so}(V) \subset C\ell(V)$



# Superparticles?

- unitary irreps of Poincaré superalgebra decompose into unitary irreps of Poincaré algebra with the same mass but different spins, e.g.,
  - chiral multiplet: KG + Dirac (same mass)
  - vector multiplet: Maxwell + Weyl (massless)

# Superspace... the final frontier

- Poincaré algebra = infinitesimal automorphisms of Minkowski spacetime
- Poincaré superalgebra = infinitesimal automorphisms of Minkowski superspacetime

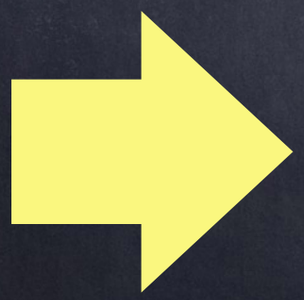
# Supermanifolds

- Dual to a manifold we have its algebra of functions: it is commutative
- The algebra of functions of a supermanifold is "mildly noncommutative": it has nilpotent elements

# Minkowski superspacetime

Algebra of functions

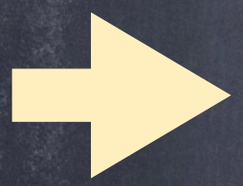
$$C^\infty(V) \otimes \Lambda^\bullet S$$



Supersymmetry adds "quantum" coordinates to the spacetime.

# Supersymmetric theories

- Supersymmetric field theories
  - Supersymmetric gauge theories
  - Supersymmetric sigma models:



characterisation of Kähler, Hodge, Calabi-Yau, hyperkähler, quaternionic Kähler,... manifolds

- Supergravity
- Superstrings, M-theory,...

# Supersymmetry in Mathematics

- Proof of Atiyah-Singer index theorem
- Proof of Morse inequalities
- Mirror symmetry
- Topological/cohomological field theories:
  - Donaldson-Witten
  - Gromov-Witten
  - Seiberg-Witten

>30 years of successes!

# SUSY2215

## SUSY: THE NEW HOPE

- QUANTUM MECHANICS AND QFT STILL HOLD
- THE ORBITAL COLLIDER STILL SEES NOTHING

**THREE CENTURIES OF TRIUMPH FOR SUSY AND STRINGS!**

**The seasonal trends**

**Extremely-weeny constrained SUSY**

**NSFWMSSM**

**FF3C10ACBA9-MSSM**

**MSSM retrograde**

**Anthropic landscaping and trimming it down**

**The problem of condensed matter: They still don't get it**

**Strings - The Perpetual Revolution**

**Number of free parameters: P or NP complete?**

***Invited seminar***

**How to ensure your model remains predictability-free**

***Forum***

**Is choice moral?**

**"Every time you choose a path of action, a multiverse is killed"**

***Special topic***

**If the universe is not supersymmetric is it necessarily existing?**

**The perpetual conference**

5 Jan - 5 Mar: Chamonix
15 Mar - 30 June: Hainan Island
1 July - 15 Sep: Wailea, Maui
15 Sep - 20 Nov: Jumeirah 1
21 Nov - 24 Dec: Hainan Island

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