Quotients of M-theory vacua

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- `hep-th/0402094`, to appear in *PRD*

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- we are interested in the orbit space M/Γ

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$\therefore$ can use orbifolds to study time-dependent phenomena in string theory; e.g., the nullbrane [Liu–Moore–Seiberg, hep-th/0206182]

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- $X, X' \in \mathfrak{g}$ give rise to equivalent quotients if and only if

$$\boxed{X' = \lambda g X g^{-1} \qquad g \in G \quad \lambda \in \mathbb{R}^\times}$$

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- equivariant under the isometry group G of (M, g)

[hep-th/9902066]

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- $(M^{1,n-1}, g)$ lorentzian $\implies$ $(\widehat{M}, \widehat{g})$ has signature $(2, n-1)$
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- the problem reduces to one of flat spaces!

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- we need to determine the elementary blocks

Elementary blocks for $\mathfrak{so}(2,p)$

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[Bañados–Henneaux–Teitelboim–Zanelli, gr-qc/9302012]

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- $(2, 2)$, self-dual boost $+$ antiself-dual rotation

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- $(2,2)$, self-dual boost + antiself-dual rotation

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$$B^{(2,3)}$$

- $(2,3)$, deformation of $B_{+}^{(2,2)}$ by a null rotation in a perpendicular direction

$$B^{(2,3)} = \begin{bmatrix} 0 & 1 & -1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- $(2, 4)$

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- and now we simply play LEGO !

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- Killing vectors on $\mathrm{AdS}_{1+p} \times S^q$ decompose

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whose norms add

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- it is convenient to distinguish Killing vectors according to norm

- everywhere non-negative norm

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  - ⋆ $\oplus_i B^{(0,2)}(\varphi_i)$

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  - ⋆ $B^{(2,2)}_{\pm} \oplus_i B^{(0,2)}(\varphi_i)$

- norm bounded below:
  - ⋆ $B^{(2,0)}(\varphi) \oplus_i B^{(0,2)}(\varphi_i)$, if p is even

- everywhere non-negative norm:
  - $\star$ $\oplus_i B^{(0,2)}(\varphi_i)$
  - $\star$ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, if $|\beta_1| = |\beta_2|$
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- arbitrarily negative norm

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- arbitrarily negative norm: the rest!

- ⋆ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, unless $|\beta_1| = |\beta_2| > 0$
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- ⋆ $B_{\pm}^{(2,2)}(\beta, \varphi) \oplus_i B^{(0,2)}(\varphi_i)$
- ⋆ $B^{(2,3)} \oplus_i B^{(0,2)}(\varphi_i)$
- ⋆ $B_{\pm}^{(2,4)}(\varphi) \oplus_i B^{(0,2)}(\varphi_i)$

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$$\gamma = \exp(LX)$$

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[FO–Leitner–Simón, to appear]

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- $\xi = \mp\boldsymbol{e}_{12} - \boldsymbol{e}_{13} \pm \boldsymbol{e}_{24} + \boldsymbol{e}_{34} + \frac{1}{2}(\theta_1 \pm \theta_2)(\boldsymbol{e}_{34} \mp \boldsymbol{e}_{12}) + \theta_1 R_{12} + \theta_2 R_{34}$

$\frac{1}{8}$-BPS quotients

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- all are amenable to conformal field theory!

Go forth and calculate!

Thank you.

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